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# A five-dimensional model of varying fine structure constant

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**Abstract.** The cosmological variation of the fine structure constant $\alpha$ is explored from an effective theory, under the form of an improved version of the 5D Kaluza-Klein theory.

## 1. Introduction

Recent publications [1] report observations of distant quasars absorption lines, which may indicate a time variation of the fine structure constant $\alpha$. Since many theoretical arguments suggest that our present theories of physical interactions are not the ultimate ones, the possibility that new physics may be involved deserves serious attention. At the effective level, and in particular in astrophysical conditions, those can be manifest as a soft dependence of the 'constants' of the interactions with some parameters. For instance, the compactified 5D Kaluza–Klein (KK) theory predicts a variation of the effective $\alpha$ with respect to gravitational conditions, and thus with the cosmic time (seen as a parameter expressing the variation of the cosmic gravitational potential, through the Friedmann–Lemaître equations). More precisely, the effective fine structure constant is given by

$$\alpha_{\text{eff}} = \frac{\alpha}{\Phi^3}, \tag{1}$$

where $\Phi$ denotes the internal 5D KK scalar field. However, because of stability reasons [2], the effective action that we consider reads after dimensional reduction (in the Jordan–Fierz frame):

$$\begin{aligned} S = &- \int \sqrt{-g} \left[ \frac{c^4}{16\pi} \frac{\Phi}{G} R + \frac{1}{4} \varepsilon_0 \Phi^3 F_{\alpha\beta} F^{\alpha\beta} + \frac{c^4}{4\pi G} \frac{\partial_\alpha \Phi \partial^\alpha \Phi}{\Phi} \right] \mathrm{d}^4 x \\ &+ \frac{c^4}{4\pi G} \int \sqrt{-g} \Phi \left[ \frac{1}{2} \partial_\alpha \psi \partial^\alpha \psi - U - J\psi \right] \mathrm{d}^4 x, \end{aligned} \tag{2}$$

where $U = U(\psi)$ denotes the self-interaction potential of $\psi$ and $J$ its source term. The latter includes contributions from the matter and from $\Phi$, both proportional to the trace of their energy–momentum tensor, viz. $(8\pi G/3c^4) g(\psi, \Phi) T^\alpha_\alpha$. The necessity to recover the usual physics whenever the scalar fields are not excited requires $g(v, 1) = 0$, $U(v) = 0$ and $(\partial U/\partial\psi)(v) = 0$; $v$ defines the vacuum expectation value (VEV) of $\psi$.

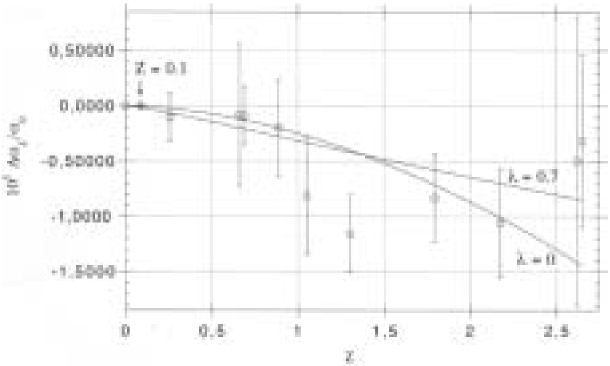


**Figure 1.** Observed data and predicted curve $\Delta\alpha_z/\alpha_0$ versus the redshift. The fits correspond respectively to $\chi^2 = 0.948$ ($\lambda = 0$) and $\chi^2 = 0.779$ ($\lambda = 0.7$) per dof, which seems to favor the late time $\lambda$ dominated cosmology. Including the Oklo bounds ($z = 0.1$) in the data set would yield respectively $\chi^2 = 0.882$ and $\chi^2 = 1.094$ per dof; $z = \frac{a(t_0)}{a(t_e)} - 1$, $\alpha_z = \alpha_{\rm eff}(t_e)$ and $\alpha_0 = \alpha_{\rm eff}(t_0)$. As one can see, the prediction (1) is found to be in good agreement with the observational data.

## 2. Cosmological solutions

Since we know that, for the effects examined here, the effective values are close to the usual ones, we will linearize the two scalar fields around their respective VEVs throughout. For cosmology, we assume spatially constant values of the fields and we follow their evolutions with respect to the cosmic time, $t$. Hence, $\psi = \psi(t)$ and $\Phi = \Phi(t)$. Thus, the cosmological equations reduce to the effective Friedmann equation (with a cosmological constant $\Lambda$)

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\,\rho - \frac{k\,c^2}{a^2} + \frac{\Lambda\,c^2}{3} + \frac{1}{3}\,\dot\psi^2 - \frac{1}{6}\,(\,\ddot\Phi\, + 6H\,\dot\Phi\,), \tag{3}$$

$$\frac{\ddot a}{a} = -\,\frac{4\pi G}{3}\left(\rho + 3\,\frac{P}{c^2}\right) + \frac{\Lambda\,c^2}{3} - \frac{1}{3}\,\dot\psi^2 - \frac{1}{6}\,(\,\ddot\Phi\, + 3H\,\dot\Phi\,), \tag{4}$$

and, for the scalar fields,

$$\ddot\psi + 3H\,\dot\psi = -\,\frac{8\pi G}{3}\,\beta_\psi\,v\left(\rho - 3\,\frac{P}{c^2}\right), \tag{5}$$

$$\ddot\Phi + 3H\,\dot\Phi = -\,\frac{1}{2}\,\dot\psi^2 + \frac{8\pi G}{3}\,\beta_\Phi\,v\left(\rho - 3\,\frac{P}{c^2}\right). \tag{6}$$

The dot denotes the derivative with respect to the cosmic time, $H = \dot a/a$ is the expansion rate (Hubble parameter), $a = a(t)$ is the scale factor, $k$ is the spatial curvature parameter and $P$ is the pressure. We have set $\beta_\psi = (\partial g/\partial\psi)(v, 1)$ and $\beta_\Phi = (\partial g/\partial\Phi)(v, 1)$. The smallness of the observed effects implies $|\beta_\Phi\,v| \ll 1$,

$|\beta_\psi v| \ll 1$, whereas the consistency of the model implies $|\dot{\psi}| \ll H$ and $|\dot{\Phi}| \ll H$ (all confirmed by the numerical calculations). As this is suggested by observations, we assume zero spatial curvature. It is found that both scalar fields remain equal to their respective VEV, during the radiation era and the cosmological constant era. As a consequence, $\alpha_{\text{eff}}$ identifies to the true fine structure constant during both eras.

## References

[1] J K Webb, *et al*, *Phys. Rev. Lett.* **87**, 091301 (2001)
[2] J-P Mbelek and M Lachièze-Rey, *Astron. Astrophys.* **397**, 803 (2003)