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Cancellation of global anomalies in spontaneously broken gauge theories

MARCO FABBRICHESI
INFN and Scuola Internazionale Superiore di Studi Avanzati, via Beirut 4, 34014 Trieste, Italy

Abstract. I discuss the generalization to global gauge anomalies of the familiar procedure for the cancellation of local gauge anomalies in effective theories of spontaneously broken symmetries

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1. Canceling the anomalies

In purely bosonic low-energy effective actions, anomalies are accounted for by the Wess–Zumino (WZ) term [1]. For a global symmetry group G spontaneously broken to an anomaly-free subgroup H, the WZ coupling is a non-invariant function of the corresponding Goldstone bosons. The same term can also be generalized to reproduce gauge anomalies when part or all of the group G is gauged [2]. From an effective field theory point of view, the WZ term can be seen as the remnant of massive fermions that have been integrated out in a microscopic theory [3].

The WZ term can also be viewed from a different perspective: an anomalous theory can be rendered gauge-invariant by coupling it to G-valued scalar fields and adding the WZ term to the action [4]. Thus, instead of *reproducing* the anomalies of an underlying theory, one can use the WZ term to *cancel* them. This construction is similar to the GS mechanism [5], which also requires the presence of additional degrees of freedom (but only works for reducible anomalies).

A variation of this anomaly-cancellation scheme occurs when the scalar fields are not introduced by hand but are already present in the theory. This can happen in the Higgs phase of a gauge theory [6]. The would-be Goldstone bosons—the angular components of the Higgs field—are the scalar fields that can be used to write the WZ term. This idea had already been put forward in [7] where the interplay between renormalizability and anomaly cancellation in spontaneously broken gauge theories was first discussed in an abelian Higgs model coupled to a chiral fermion in four dimensions.

The result of these analyses is that, as far as local gauge anomalies are concerned, effective theories, for which renormalizability is not a requirement, can be made anomaly free by the addition of an appropriate WZ term. No restriction on the fermion content of the theory is needed, provided that the first two conditions listed in the introduction are satisfied.

Even a theory free of perturbative anomalies can still be anomalous under gauge transformations that are not homotopic to the identity. This was discussed first for an $SU(2)$ theory in four dimensions in [8] and for other dimensions in [9] (see also [10]). These gauge anomalies are called global, or non-perturbative anomalies.

Extending the results of [4], it is always possible to cancel any global anomaly by coupling the theory to a G-valued scalar field and adding a suitable WZ term to the action [11]. We are thus led to ask: if the G gauge symmetry is spontaneously broken to H due to a Higgs mechanism, can one cancel the global anomalies of the low-energy effective theory by means of a WZ term written as a functional of the would-be Goldstone bosons? The answer is that this is possible [12] provided that:

1. the coset space G/H is reductive;
2. the fermion representations are free of *local* anomalies when restricted to the group H;
3. the fermion representations are free of *global* anomalies when restricted to the group H;
4. G can be embedded in a group K such that its homotopy group $\pi_d(K) = 0$ and the fermion representations can be extended to K without generating further anomalies of G.

Most of these conditions can be derived from the literature. In particular, conditions 1 and 2 have been derived for local gauge anomalies [13,14], while conditions 3 and 4 are closely related to the results of [2,15,9].

2. The standard model in six dimensions

In a recent paper [16], it was argued that, in the minimal non-supersymmetric version of the standard model in $d = 6$ dimensions, the requirement of canceling all gauge anomalies restricts the chiral fermion content of the theory. In particular, the cancellation of the $SU(2)$ global anomaly implies a restriction on the number N_g of matter families: $N_g = 0 \bmod 3$.

The field content of a single family is restricted by the requirement of the cancellation of all irreducible local anomalies—that is, by the requirement of having a vector-like theory of strong and gravitational interactions—while the $d = 6$ realization of the usual GS mechanism is invoked in order to eliminate reducible local anomalies.

A detailed inspection of the GS mechanism, performed in [17], showed that the addition of two GS 2-forms—supplemented by the use of the $U(1)_Y$ Goldstone boson in order to realize the generalized abelian GS of [18]—is sufficient to restore gauge invariance. After compactification of the two extra-dimensions, some pseudo-scalar fields, behaving like Peccei-Quinn axions, remain in the $d = 4$ effective field theory as remnants of the GS fields. Their presence solves the strong CP problem, but imposes a strong bound on the scale of the GS couplings and therefore a limit on the volume of the compact extra-dimensions, the fundamental scale of which turns out to be in the range of the usual GUT theories.

Is it possible to achieve the cancellation of the reducible anomalies with some mechanism which does not leave any axionic remnant in the low energy $d = 4$ effective theory?

The choice of fermion content guarantees that the model has no pure gravitational anomaly. The addition of a local Chern–Simons term shifts mixed gravitational anomalies into those of gauge. Fermions form a vector-like representation of the group $SU(3)_c$ with no contribution to the anomalies. We can thus identify the groups of the previous sections with those in [16]: $K = SU(4)_L \times U(1)_Y$, $G = SU(2)_L \times U(1)_Y$ and $H = U(1)_{\text{e.m.}}$. Since $\pi_6(G/H) = \mathbf{Z}_{12}$, we have enlarged G to K for which $\pi_6(K) = 0$.

Each $SU(2)$ doublet goes into the fundamental representation of $SU(4)$ (to be decomposed into a $SU(2)$ doublet plus singlets). In addition to the already-present singlets, more singlets are necessary in order to preserve the $U(1)_Y^4$ anomaly.

All the conditions stated in the previous section hold, and therefore the WZ term can be used to cancel all anomalies, as explained in the previous sections.

Thanks to this construction, no GS fields are needed; since there are no axions in the low energy $d = 4$ theory, their experimental bounds do not apply. However, all global anomalies are canceled as well, and therefore the interesting prediction on the number of families is lost.

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