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$N = 1$ super quantum chromodynamics and fractional branes

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Abstract. In this talk we show how to get the one-loop beta function and the chiral anomaly of $\mathcal{N} = 1$ super QCD from a stack of fractional N $D3$-branes localized inside the world-volume of $2M$ fractional $D7$-branes in the orbifold $C^3/(Z_2 \times Z_2)$. They are obtained by analyzing the classical supergravity background generated by such a brane configuration, in the spirit of the gauge/gravity correspondence.

Keywords. String; brane; duality.

1. Introduction

This talk is based on the results of the paper [1], that goes in the direction of achieving a deeper understanding of the so-called *gauge/gravity correspondence*, originating from the *complementarity* between two different but equivalent descriptions of the low-energy properties of D-branes. Indeed, on the one hand a D-brane can be described in terms of the gauge theory living on its world-volume; on the other hand it is a classical solution of the ten-dimensional supergravity (low-energy string effective theory). Hence one can exploit the classical geometrical properties of D-branes to get insight in the *dual* gauge theory and vice versa. This correspondence has led Maldacena to his famous conjecture [2], that provides an exact duality between a conformal and highly supersymmetric gauge theory and a (super)gravity theory. It is interesting to apply the gauge/gravity correspondence to less supersymmetric and non-conformal gauge theories using fractional branes [3] which live in an orbifold space, stuck at the orbifold fixed point. We consider a bound state of N coincident fractional $D3$-branes and $2M$ fractional $D7$-branes on the orbifold $R^{1,3} \times C^3/(Z_2 \times Z_2)$ yielding $\mathcal{N} = 1$ super QCD with M fundamental hypermultiplet [4]. The beta function and the chiral anomaly of this theory are reproduced by analyzing the asymptotic behaviour of the classical supergravity solution [5]. This is valid in the region far from the brane, where the ultraviolet properties of the gauge theory are reproduced.

2. Fractional Dp-branes on the orbifold $R^{1,3} \times C^3/(Z_2 \times Z_2)$

We consider fractional $D3$ and $D7$ branes on the orbifold $C^3/(Z_2 \times Z_2)$ in order to study the properties of $\mathcal{N} = 1$ supersymmetric gauge theories. The orbifold group acts on the directions $x^4, \dots, x^9$ transverse to the world-volume of the stack of the N $D3$-branes. The Z_2 group is characterized by two elements $\{1, h\}$, with $h^2 = 1$, hence the four elements of the tensor product $Z_2 \times Z_2$ are easily obtained. The non-trivial elements act on the complex vector $\vec{z} = (z^1 = x^4 + ix^5, z^2 = x^6 + ix^7, z^3 = x^8 + ix^9)$ of C^3 as: $h_1 = h \times 1 \Rightarrow z_1 \to z_1, z_2 \to -z_2, z_3 \to -z_3$; $h_2 = 1 \times h \Rightarrow z_1 \to -z_1, z_2 \to z_2, z_3 \to -z_3$; $h_3 = h \times h \Rightarrow z_1 \to -z_1, z_2 \to -z_2, z_3 \to z_3$. This orbifold is non-compact with the fixed point $z_1 = z_2 = z_3 = 0$, corresponding to the three C^2 points $z_1, z_2 = 0$, $z_1, z_3 = 0$ and $z_2, z_3 = 0$ each of them associated to a vanishing two-cycle $\mathcal{C}_2^{(i)}$ with $i = 1, 2, 3$. The low energy type IIB superstring theory spectrum consists of an *untwisted* and three *twisted* sectors generated by its p-form fields dimensionally reduced on the three vanishing two-cycles $\mathcal{C}_2^{(i)}$ $(i = 1, 2, 3)$. A fractional Dp-brane can be regarded as a $D(p+2)$-brane wrapped on the vanishing two-cycles and, being stuck at the orbifold fixed point, it couples to all twisted states. The Chan–Paton factor accompanying any state of the open string stretched between two fractional branes transforms according to an irreducible (one-dimensional) representation of the orbifold group. This means that we have four different kinds of fractional Dp-branes on this orbifold. We can get a pure $\mathcal{N} = 1$, $D = 4$ SYM with gauge group $SU(N)$ by means of a stack of N fractional $D3$-branes of the same kind. In order to get matter in the fundamental representation we consider a bound state of N fractional $D3$-branes and $2M$ fractional $D7$-branes of two different kinds, in equal number. Chiral matter is then provided by the open strings having one end attached to the $D3$-brane and the other one to the $D7$-brane. Physical states associated to these strings transform under the fundamental representation of the gauge group, while the number of $D7$-branes can be regarded as a flavor index. We have considered configurations in which the $D7$-branes extend in the directions $x^0, \dots, x^3$, $x^6, \dots, x^9$, i.e., partially along the orbifold, while the $D3$-branes are in the directions $x^0, \dots, x^3$, completely localized in the $D7$-brane world-volume. In this way we are able to study $\mathcal{N} = 1$ super QCD with M hypermultiplets. We have shown in [1] that choosing $2M$ $D7$-branes yields a consistent gauge theory free from gauge anomalies.

3. Supergravity analysis of the dual gauge theory

The two complementary ways of describing a Dp-brane can be related by studying its low-energy effective world-volume action in the supergravity background. If a fractional Dp-brane is thought as a $D(p+2)$-brane wrapped on the orbifold vanishing two-cycles, its world volume action reads:

$$S_{D(p)} = -\tau_{p+2} \int \mathrm{d}^{p+3}\xi \mathrm{e}^{\frac{p-1}{4}\phi} \sqrt{-\det \left[G_{\alpha\beta} + \mathrm{e}^{-\frac{\phi}{2}} \left(B_{\alpha\beta} + 2\pi\alpha' \mathcal{F}_{\alpha\beta} \right) \right]} + \tau_{p+2} \int \left(C \wedge \mathrm{e}^{B + 2\pi\alpha' \mathcal{F}} \right)_{p+3}, \tag{1}$$

where $\tau_p = \frac{(g_s\sqrt{\alpha'})^{-\frac{1}{2}}}{(2\pi\sqrt{\alpha'})^p}$. Specializing this action for $p = 3$ and expanding it up to quadratic terms in $\mathcal{F}$ one has

$$S_{D3} = -\frac{1}{g_{\rm YM}^2}\int \mathrm{d}^4x \frac{1}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} + \frac{\theta_{\rm YM}}{32\pi^2}\int \mathrm{d}^4x \mathcal{F}_{\alpha\beta}\tilde{\mathcal{F}}^{\alpha\beta},$$

where

$$\frac{1}{g_{\rm YM}^2} \equiv \frac{1}{16\pi^3 g_s\alpha'}\int_{\cup\mathcal{C}_2^{(i)}} \mathrm{e}^{-\phi}B_2; \quad \theta_{\rm YM} \equiv \tau_5 4\pi^4{\alpha'}^2\int_{\cup\mathcal{C}_2^{(i)}}(C_2 + C_0B_2). \tag{2}$$

These equations show the fundamental role played by the the twisted fields $B_2 = \sum_{i=1}^3 b^i\omega_2^{(i)}$ and $C_2 = \sum_{i=1}^3 c^i\omega_2^{(i)}$, being $\omega_2^{(i)}$ the volume form dual to the vanishing two-cycle $\mathcal{C}_2^{(i)}$. The next step is to plug into eq. (2) the supergravity background generated by the $D3/D7$ system. We have determined only the asymptotic behaviour for large distances of the classical solution [1] and this has been revealed to be sufficient for computing the gauge coupling constant and the θ angle of $\mathcal{N} = 1$ super QCD ($z_i = \rho_i\mathrm{e}^{i\theta_i}$):

$$\frac{1}{g_{\rm YM}^2} = \frac{1}{16\pi g_s} + \frac{1}{8\pi^2}\left(N\sum_{i=1}^3 \log\frac{\rho_i}{\epsilon} - M\log\frac{\rho_1}{\epsilon}\right),$$

$$\theta_{\rm YM} = -N\sum_{i=1}^3\theta_i + M\theta_1.$$

The coordinates $z_i = \rho_i\mathrm{e}^{i\theta_i}$ are holographically identified with the scalar components ϕ_i of the superfield Φ_i appearing in the cubic superpotential $W = \mathrm{Tr}\,(\Phi_1\,[\Phi_2,\Phi_3])$ [4]. The scale transformation $\phi_i \to \mu\phi_i$ and the $U(1)_R$ one $\phi_i \to \mathrm{e}^{i\frac{2}{3}\alpha}\phi_i$ induce on z_i the transformation $z_i \to \mu\mathrm{e}^{i2\alpha/3}z_i$, i.e. $\rho_i \to \mu\rho_i$ and $\theta_i \to \theta_i + 2/3\alpha$. The latter ones act on the gauge parameters as follows:

$$\frac{1}{g_{\rm YM}^2} \to \frac{1}{g_{\rm YM}^2} + \frac{3N - M}{8\pi^2}\log\mu, \quad \theta_{\rm YM} \to \theta_{\rm YM} - 2\alpha\left(N - \frac{M}{3}\right). \tag{3}$$

The first equation generates the one-loop β-function of $\mathcal{N} = 1$ super QCD with M hypermultiplets:

$$\beta(g_{\rm YM}) = -\frac{3N - M}{8\pi^2}g_{\rm YM}^3,$$

while the second one reproduces the chiral $U(1)$ anomaly.

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