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The gravity dual of the non-perturbative $N = 2$ supersymmetric Yang–Mills theory

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Abstract. The anomalous Ward identity is derived for $N = 2$ SUSY Yang–Mills theories, which is resulted out of wrapping of D_5 branes on supersymmetric two cycles. From the ward identity one obtains the Witten–Dijkgraaf–Verlinde–Verlinde equation and hence can solve for the pre-potential. This way one avoids the problem of enhancon which maligns the non-perturbative behaviour of the Yang–Mills theory resulted out of wrapped branes.

Keywords. Yang–Mills; supersymmetry; strings.

1. Introduction

Recently the gauge theory/gravity duality, which is commonly known as Ads/CFT duality is extended to non-conformal pure $\mathcal{N} = 1$ or $\mathcal{N} = 2$ supersymmetric Yang–Mills (SYM) theories [1]. So far the perturbative behaviour of the $\mathcal{N} = 2$ SYM is produced by this duality. The conventional folklore is that the instantons which are responsible for the non-perturbative part of the pre-potential are suppressed in the large N limit (since the gauge gravity duality is valid only in the large N limit). Also the non-perturbative strong coupling behaviour of SYM concerns the dilaton which is plagued with the singularity of 'enhancon' [2]. Here we establish the anomalous super conformal ward identity for $\mathcal{N} = 2$ SYM in the gravity dual picture. Further the super conformal Ward identity is written as Witten–Dijkgraaf–Verlinde–Verlinde (WDVV) [3] equation from which one can obtain the exact pre-potential.

2. The strategy

We start with type IIB little string theory e la' a collection of a large number of NS_5 brane in the vanishing string coupling limit which gives rise to $D = 6$ SYM [4]. Then we dimensionally reduce two of its spatial world volume in such a way that we retain $\mathcal{N} = 2$ SYM in the low energy limit. The NS_5 brane has $SO(4)$ R-symmetry as the normal bundle. When one identifies the $U(1)$ subgroup of the $SO(4)$ R-symmetry with the $U(1)$ spin connection of the two cycle which is compactified, one gets a covariant constant spinor and SUSY is retained, which is commonly known as twisting [1]. This is called wrapping of NS_5 brane on supersymmetric two cycles. If the compact space is a two-sphere, then there will be no extra hyper-multiplet and in the low energy limit i.e. in the scale much lower than the radius of the sphere we will get pure $\mathcal{N}=2$ SYM. Thus it amounts to consider a gauged $D=7$ supergravity solution and then lift it to get the solutions in ten dimensions. We use here the results of [5,6] classical solutions of $D=7$ gauged supergravity which is amenable to ten dimensional string theory.

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{\Phi}\Bigg[\mathrm{d}x_{1,3}^2 + \frac{z}{\lambda^2}\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right) + \frac{1}{\lambda^2}\,\mathrm{e}^{2x}\,\mathrm{d}z^2 + \frac{1}{\lambda^2}\left(\mathrm{d}\theta_1^2 + \frac{\mathrm{e}^{-x}}{f(x)}\cos^2\theta_1\,(\mathrm{d}\theta_2 + \cos\theta\,\mathrm{d}\varphi)^2 + \frac{\mathrm{e}^{x}}{f(x)}\sin^2\theta_1\,\mathrm{d}\theta_3^2\right)\Bigg], \tag{1}$$

where the dilaton is

$$\mathrm{e}^{2\Phi} = \mathrm{e}^{2z}\left[1 - \sin^2\theta_1\;\frac{1 + c\,\mathrm{e}^{-2z}}{2z}\right] \tag{2}$$

and

$$f(x) = \mathrm{e}^{x}\cos^2\theta_1 + \mathrm{e}^{-x}\sin^2\theta_1. \tag{3}$$

Also

$$\mathrm{e}^{-2x} = 1 - \frac{1 + c\,\mathrm{e}^{-2z}}{2z}, \tag{4}$$

where λ is the gauge coupling constant of the seven dimensional gauged supergravity and c is a parameter as the integration constant of the classical solution. For $c \geq -1$ the range of the radial variable is $z_0 \leq z \leq \infty$, where z_0 is the solution for $\mathrm{e}^{-2x(z_0)} = 0$. Here θ and φ are the angles of compact two-sphere with radius of compactification as $\frac{z}{\lambda^2}$ and θ_1, θ_2 and θ_3 are the angles of the transverse three-sphere. The conservation of the RR-charge on the transverse sphere S_3 fixes $\frac{1}{\lambda^2} = N\,g_s\,\alpha'$ for N number of D_5 branes with string coupling g_s. The D_5 brane action is given by

$$S = -\tau_5\int \mathrm{d}^6\xi\;\mathrm{e}^{-\Phi}\sqrt{-\det\left(G + 2\pi\alpha' F\right)} + \tau_5\int\left(\sum_n C^{(n)}\wedge\mathrm{e}^{2\pi\alpha' F}\right), \tag{5}$$

where F is the world volume gauge field and τ_5 is the brane tension. The BPS condition is fixed from the condition of the vanishing of the potential between two branes which gives θ_1 to be $\frac{\pi}{2}$. This condition makes the transverse boundary of the D brane to be a two dimensional space consisting of z and θ_3 which will eventually be the moduli space of $\mathcal{N}=2$ SYM.

We want to establish here the anomalous super conformal Ward identity. In the presence of gravity the trace anomaly

$$\langle\theta^\mu_\mu\rangle = \frac{1}{2}\frac{\beta(g)}{g^3}\left(F^a_{\mu\nu}\right)^2 + \frac{c(g^2)}{16\pi^2}\left(W_{\mu\nu\rho\sigma}\right)^2 - \frac{a(g^2)}{16\pi^2}\left(\tilde{R}_{\mu\nu\rho\sigma}\right)^2, \tag{6}$$

where $\beta(g)$ is the beta function of SYM, $a(g)$ and $c(g)$ are central functions near the criticality, $W_{\mu\nu\rho\sigma}$ is the Weyl tensor and $\tilde{R}_{\mu\nu\rho\sigma}$ is the dual of the curvature tensor. However this relation can be extracted from the two point functions of the energy momentum tensors [7]. In ref. [8], it is shown how to extract these functions from the absorption cross-section of soft dilatons or gravitons by the D branes. The probability of absorption is taken as the ratio of the flux near z_0 to the in-coming flux at very large z. This gives

$$\sigma = \frac{N^4}{128\pi^3}\left(z - z_0\right)^2\omega^3, \tag{7}$$

where ω is the frequency of the soft gluon. Here we see the presence of $(z - z_0)^2$ which if we write as in ref. [6] $\mathrm{e}^z = \rho$ we get a term $\left(\log\frac{\rho}{\rho_0}\right)^2$ in the cross-section signaling the asymptotic freedom or logarithmic coupling. Also, we see the presence of enhancon when z_0 is zero. This gives $\beta(g) = -\frac{N}{8\pi^2}g^3$. Similarly one can also calculate from $U(1)$ R-current the chiral anomaly. Combining trace anomaly, chiral anomaly and supertrace anomaly one can write the Ward identity as [9]

$$2\mathcal{F} - \mathcal{F}(\mathcal{A})'\mathcal{A} = \frac{N}{8\pi^2}\langle\mathrm{tr}\psi^2\rangle, \tag{8}$$

where $\mathcal{F}$ is the effective potential or the pre-potential, $\mathcal{A}$ is the chiral multiplet coupled to vector multiplet in the $\mathcal{N} = 2$ SYM and $\mathrm{tr}\,\psi^2$ is the anomaly multiplet for example $\mathrm{tr}\,F^2$ will correspond to θ^μ_μ. Here the bosonic component of $\mathcal{A}$ corresponds to $z - i\theta_3$ or in radial coordinate $v\mathrm{e}^{i\gamma}$. In the broken phase the branes will be distributed on a circle or $\mathcal{A}_i$ the eigenvalues of $\mathcal{A}$ which are $U(N)$ matrices will be distributed on a circle. The second part of eq. (8) will read as $\sum_i \frac{\partial\mathcal{F}}{\partial\mathcal{A}_i}\mathcal{A}_i$. One can, in principle, solve this equation in the large N limit and obtain the exact pre-potential in this limit [10].

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