PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. 62, No. 3 March 2004 pp. 711–715

Orientifolds of type IIA strings on Calabi-Yau manifolds

SURESH GOVINDARAJAN* and JAYDEEP MAJUMDER†
*Department of Physics, Indian Institute of Technology, Chennai 600 036, India
†Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855-0849, USA

Abstract. We identify type IIA orientifolds that are dual to M-theory compactifications on manifolds with G_2-holonomy. We then discuss the construction of cross-cap states in Gepner models.

1. Introduction

The advent of D-branes has led to a better understanding of dualities involving strong coupling limits. In particular, $\mathcal{N} = 1$ compactifications of the heterotic string (on Calabi–Yau manifolds) are no longer the only string theories of phenomenological interest. One such class is furnished by M-theory compactifications on seven-dimensional manifolds of G_2-holonomy which gives rise to four-dimensional theories with $\mathcal{N} = 1$ supersymmetry. When the G_2 manifolds have certain kinds of singularities, both non-abelian gauge groups as well as chiral fermions can appear.

Joyce has constructed manifolds of G_2 holonomy as $\mathbb{Z}_2$ orbifolds of a Calabi–Yau three-fold M: $X = (M \times S^1)/\sigma \cdot \mathcal{I}_1$, where σ is an anti-holomorphic involution of the CY^3 and $\mathcal{I}_1$ is the inversion of the S^1 [1]. One obtains a smooth manifold when the orbifold action has no fixed points. However, when there is a fixed point set Σ, one obtains a singular manifold [2,3]. The singularity can be smoothed out when $b_1(\Sigma) > 0$. The focus of this talk will be on the cases when there are fixed points.

Our working example of a Joyce manifold is the one obtained from the Fermat quintic given by the hypersurface $z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0$, in $\mathbb{CP}^4$(z_i are homogeneous coordinates of $\mathbb{CP}^4$). The anti-holomorphic involution σ is $z_i \to \bar{z}_i$ for $i = 1, \ldots, 5$. The fixed-point set Σ is an $\mathbb{RP}^3$, which is a special Lagrangian (sL) submanifold of the Fermat quintic [4]. Since $b_1(\mathbb{RP}^3) = 0$, the singularity of X, which is locally of the form $\Sigma \times \mathbb{C}^2/\mathbb{Z}_2$, *cannot* be resolved. Σ is actually one in a family of $5^4 = 625$ sL submanifolds of the Fermat quintic, all of whom are $\mathbb{RP}^3$'s. They are all fixed points of the anti-holomorphic involutions: $z_i \to \alpha^{n_i} \bar{z}_i$ with $\alpha^5 = 1$.

We will focus on obtaining the precise type IIA orientifold dual for M-theory compactification on this Joyce manifold. We will then proceed to study the orientifolding in the Gepner model corresponding to the Fermat quintic. This involves the construction of cross-cap states in Gepner models which we schematically discuss postponing details to a subsequent paper [5].

2. Obtaining the orientifold dual

M-theory compactified on $M \times S^1$ is dual to the type IIA compactification on M. Since the Joyce manifold X is an orbifold of $M \times S^1$, the type IIA dual can be obtained if we can identify the action of $\mathcal{I}_1$ on the type IIA side. But, $\mathcal{I}_1$ is not a symmetry of M-theory and thus cannot quite be identified with a symmetry on the type IIA side. However, the inversion of an even number of coordinates is a symmetry of M-theory. In the example of the quintic that we considered, Σ is the base of the SYZ T^3 fibration of the quintic and σ inverts the fibre [6]. Thus, $\sigma \cdot \mathcal{I}_1$ corresponds to the simultaneous inversion of four circles – three from the SYZ fibre and one from the S^1 fibre. This uniquely fixes the type IIA orientifold to be the second choice from the following two possibilities [7,8]: (Ω: worldsheet parity, F_L: space-time fermion number)

$$[\sigma \cdot \Omega] \qquad \text{or} \qquad [(-)^{F_L} \cdot \sigma \cdot \Omega].$$

It also turns out that only the second choice preserves $\mathcal{N} = 1$ supersymmetry [9,10]. This is easily understood by studying the action on the vertex operators involving the Ramond sector.

The spectrum of M-theory on $M \times S^1$ has $\mathcal{N} = 2$ supersymmetry in $d = 4$ and consists of: (a) the $\mathcal{N} = 2$ supergravity multiplet; (b) $h_{1,1}(M)$ abelian vector multiplets; and (c) $h_{2,1}(M) + 1$ hypermultiplets. The orbifolding breaks half the supersymmetry and the spectrum for a smooth Joyce manifold X (with Betti numbers b_3 and b_2) which consists of [11,12]: (a) the $\mathcal{N} = 1$ supergravity multiplet; (b) $b_2(X) = h^+_{1,1}(M)$ abelian vector multiplets; and (c) $b_3(X) = h_{2,1}(M) + h^-_{1,1}(M) + 1$ chiral multiplets, where $h^{\pm}_{1,1}(M)$ are the number of Kähler moduli that are even(odd) under σ. For the case when the orbifolding has fixed points, additional moduli appear corresponding to modes that smoothen the singularity.

For the Fermat quintic, $h_{2,1} = 101$; $h^+_{1,1} = 0$; $h^-_{1,1} = 1$ and the singularity cannot be resolved. The two fixed points are of the form $\Sigma \times \mathbb{R}^{3,1}$ and the singularity is locally like $\mathbb{R}^4/\mathbb{Z}_2$, i.e., it is an A_1 singularity – expect $U(1) \times U(1)$ enhanced gauge symmetry in M-theory. In the type IIA dual, we expect an $O6$-plane with the SO-projection. The RR-charge will be equal to the $\mathbb{R}^3/\mathbb{Z}_2$ orientifold plane in flat space. Based on this, we add four $D6$-branes wrapping $\Sigma \times \mathbb{R}^{3,1}$ implying a $SO(4)$ gauge symmetry. The rest of the talk will be towards checking if this geometric intuition can be realised in the orientifold of the Gepner model associated with the Fermat quintic.

3. Aspects of orientifolding

It is useful to understand how the M-theory spectrum on X must appear from the orientifold projection in the type IIA theory on M. Let $\widetilde{\Omega}$ denote the orientifolding

$\mathbb{Z}_2(=(-)^{F_L}\cdot\sigma\cdot\Omega$ in our example). Under its action, the states of the original type IIA theory fall into three representations (which we label by $\epsilon=0,\pm1$):

Real representations: $[\epsilon=+1]$. These have eigenvalue $+1$ and survive orientifold projection.
Pseudo-real representations: $[\epsilon=-1]$. These have eigenvalue -1 and are projected out.
Complex representations: $[\epsilon=0]$. Under the action of $\widetilde{\Omega}$, one state gets mapped to another. In such cases, one linear combination is projected out. In our example, it is easy to see that states that arise from the (c,c) and (a,a) rings (complex moduli of M) are in the complex representation while those that arise from the (a,c) and (c,a) rings (Kähler moduli of M) have $\epsilon=\pm1$.

The presence of orientifold planes leads to unoriented strings and hence unoriented surfaces. At 'one-loop', this adds a Klein bottle to the torus. The Klein bottle amplitude has two 'channels' related by the modular transformation: the *direct channel* $K(q)=\mathrm{Tr}(\widetilde{\Omega}\ q^{H_{cl}})=\sum_i\epsilon_i\chi_i(q)$ and the *transverse channel* $\tilde{K}(\tilde{q})=\langle C|\tilde{q}^{H_{cl}}|C\rangle=\sum_j\Gamma_i^2\chi_i(\tilde{q})$. We have assumed for simplicity that all states have a multiplicity of one. Thus, the direct channel amplitude encodes the orientifold projection.

In the CFT of unoriented strings, one first constructs a cross-cap state whose direct channel amplitude encodes the required projection. One general class of solutions has been provided by Pradisi–Sagnotti–Stanev [13]. The cross-cap state is

$$|C\rangle=\sum_i\Gamma_i\ |C:i\rangle\rangle=\sum_i\frac{P_{0i}}{\sqrt{S_{0i}}}\ |C:i\rangle\rangle,$$

where $|C:i\rangle\rangle$ are the Ishibashi basis for cross-cap states and $P\equiv\sqrt{T}ST^2S\sqrt{T}$. This plays the analogue of the S-matrix in Cardy's ansatz for the boundary states. The matrices $Y_{ij}^k\equiv\sum_m\frac{S_{mi}P_{mj}P_m^k}{S_{m0}}$ plays a role analogous to the fusion matrix for boundary states. They satisfy the fusion algebra: $Y_iY_j={N_{ij}}^kY_k$ with $Y_{00}^k=\epsilon_k$ determining the KB projection.

4. An application: $\mathcal{N}=2$ minimal models

The states in the minimal model of level k are labeled by (L,M,S) with $L=0,\ldots,k$, $M=0,\ldots,(2k{+}3)\bmod(2k{+}4)$, $S=0,1,2,3\bmod 4$ and $L{+}M{+}S=\mathit{even}$. There is an additional identification: $(L,M,S)\sim(k-L,M+k+2,S+2)$. Even S is the NS-sector and odd S is the R-sector. The S-matrix and P-matrix are schematically given by

$$\begin{aligned}
S_{LMS}^{\tilde{L}\tilde{M}\tilde{S}}&\propto\sin(L,\tilde{L})_k\ \mathrm{e}^{\frac{i\pi M\tilde{M}}{k+2}}\ \mathrm{e}^{\frac{-i\pi S\tilde{S}}{2}}\\
P_{LMS}^{\tilde{L}\tilde{M}\tilde{S}}&\propto\Bigg(\sin\frac{1}{2}(L,\tilde{L})_k\ \mathrm{e}^{\frac{i\pi M\tilde{M}}{(2k+4)}}\ \mathrm{e}^{\frac{-i\pi S\tilde{S}}{4}}\ \delta^{(2)}_{M+\tilde{M}+k}\ \delta^{(2)}_{S+\tilde{S}}\\
&\qquad+\mathrm{e}^{i\alpha_{LMS}}\ \sin\frac{1}{2}(k-L,\tilde{L})_k\ \mathrm{e}^{\frac{i\pi(M+k+2)\tilde{M}}{(2k+4)}}\ \mathrm{e}^{\frac{-i\pi(S+2)\tilde{S}}{4}}\ \delta^{(2)}_{M+\tilde{M}}\ \delta^{(2)}_{S+\tilde{S}}\Bigg),
\end{aligned}$$

where $(L, \tilde{L})_k = \pi(L+1)(\tilde{L}+1)/(k+2)$ and α_{LMS} is a phase that one needs to introduce to take care of the identification mentioned earlier [5,14,15]. The appearance of a Krönecker delta function in P-matrix implies that only $NSNS$ (or RR) states alone appear in the PSS cross-cap state.

5. Cross-cap states in the Gepner model

The Gepner model is obtained by tensoring copies of $\mathcal{N}=2$ minimal models (MM) such that total central charge is 9. For the quintic – tensor five copies of $k=3$ MM. Further, restrict to states that come from tensoring NS states with NS states and R with R from each minimal model and project onto states with total (including space-time sector) $U(1)$ charge an odd integer.

This suggests the following strategy for cross-cap states in the Gepner model: Take the tensor product of cross-cap states in the individual minimal model. Implement the Gepner projection on this cross-cap state. This is a natural guess for the cross-cap state in the Gepner model. But this cannot be the cross-cap that realises the type IIA orientifold! This is because PSS cross-cap state has contributions only from the $NSNS$ sector. This implies that its Ramond charge is zero. The direct channel KB amplitude is not supersymmetric.

Consider the two cross-cap states in a single MM,

$$|C:NSNS\rangle \equiv P_{000}^{LMS}\ |C:LMS\rangle\rangle$$

and

$$C:RR\rangle \equiv P_{011}^{LMS}\ |C:LMS\rangle\rangle$$

The first one is the PSS cross-cap state while the second one is the PSS cross-cap state associated with the simple current that is related to space-time supersymmetry. It contains only RR Ishibashi states. Then, we propose that the correct cross-cap state schematically takes the form

$$|C\rangle_{\text{Gepner}} = \mathcal{P}\left(\prod_{i=1}^{r}|C_i:NSNS\rangle + \prod_{i=1}^{r}|C_i:RR\rangle\right).$$

$\mathcal{P}$ imposes the $U(1)$ charge projection of Gepner. This is the cross-cap analogue of the Recknagel–Schomerus construction for boundary states in the Gepner model [16].

Now the cross-cap state clearly carries RR charge. It has all the terms to provide a supersymmetric KB amplitude. For the quintic, in fact, we find a full family of 625 distinct cross-cap states in agreement with the 625 anti-holomorphic involutions. More detailed checks such as the KB projection, tadpole cancellation etc. for specific examples will be discussed in the paper to appear soon [5]. A recent paper by Misra also discusses a type IIA orientifold of a Calabi–Yau threefold [17].

References

[1] D Joyce, *J. Diff. Geom.* **43**, 291, 329 (1996)
[2] J A Harvey and G W Moore, arXiv:hep-th/9907026

[3] H Partouche and B Pioline, *J. High Energy Phys.* **0103**, 005 (2001); arXiv:hep-th/0011130
[4] K Becker, M Becker and A Strominger, *Nucl. Phys.* **B456**, 130 (1995); arXiv:hep-th/9507158
[5] S Govindarajan and J Majumder (to appear)
[6] A Strominger, S T Yau and E Zaslow, *Nucl. Phys.* **B479**, 243 (1996); arXiv:hep-th/9606040
[7] A Sen, *J. High Energy Phys.* **9709**, 001 (1997); arXiv:hep-th/9707123
[8] J Majumder, *J. High Energy Phys.* **0201**, 048 (2002); arXiv:hep-th/0109076
[9] R Gopakumar and S Mukhi, *Nucl. Phys.* **B479**, 260 (1996); arXiv:hep-th/9607057
[10] S Kachru and J McGreevy, *J. High Energy Phys.* **0106**, 027 (2001); arXiv:hep-th/0103223
[11] G Papadopoulos and P K Townsend, *Phys. Lett.* **B357**, 300 (1995); arXiv:hep-th/9506150
[12] C Vafa and E Witten, *Nucl. Phys. Proc. Suppl.* **46**, 225 (1996); arXiv:hep-th/9507050
[13] G Pradisi, A Sagnotti and Ya S Stanev, *Phys. Lett.* **B354**, 279 (1995); arXiv:hep-th/9503207
G Pradisi, A Sagnotti and Ya S Stanev, *Phys. Lett.* **B356**, 230 (1995); arXiv:hep-th/9506014
[14] Y Hikida, *J. High Energy Phys.* **0211**, 035 (2002); arXiv:hep-th/0201175
[15] I Brunner and K Hori, arXiv:hep-th/0303135
[16] A Recknagel and V Schomerus, *Nucl. Phys.* **B531**, 185 (1998); arXiv:hep-th/9712186
[17] A Misra, arXiv:hep-th/0304209