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Black holes in brane worlds

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Abstract. A Kerr metric describing a rotating black hole is obtained on the three brane in a five-dimensional Randall–Sundrum brane world by considering a rotating five-dimensional black string in the bulk. We examine the causal structure of this space-time through the geodesic equations.

Keywords. Brane world; black holes.

The question of unification of the fundamental interactions in the context of string theory has provided a consistent framework for higher dimensional space-time. These extra spatial dimensions are assumed to be small and compactified at the Planck scale. However the hierarchy between the Planck and the electroweak scale has been an outstanding issue. Brane world models in which the gauge sector is restricted on a four dimensional hypersurface in a higher dimensional space-time offers a possible resolution.

The Randall–Sundrum (RS) [1] model resolves this by assuming a warped compactification where the hierarchy is driven by the warp factor and allows both small as well as large compact dimensions. This however requires another regulator brane which may be placed at infinity. The bulk space-time in these models are a slice of five-dimensional AdS space (for a single extra dimension S^1/Z_2).

For consistency it must be possible to describe usual four-dimensional GTR on the brane, in particular black hole and cosmological solutions. In this connection Chamblin *et al* [2] have shown that a four-dimensional Schwarzschild black hole on the brane with the usual singularity may be obtained from a bulk black string solution in a five-dimensional RS model. However the solution is also singular at the AdS horizon. Through the geodesic equations it may be seen that the AdS horizon singularity is reached only along bound geodesics and is a *p*–*p* curvature singularity. They argue that the solution is susceptible to the Gregory–Laflamme [3] instability near the horizon and pinches off earlier. Following this we have shown that a four-dimensional Kerr metric on the brane may be obtained from a five-dimensional bulk rotating black string solution [4].

The five-dimensional bulk rotating black string metric in the RS brane world is

$$\mathrm{d}s^2 = \frac{l^2}{(z_0+|w|)^2}\Big[-\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)\mathrm{d}t^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2$$

$$+ \Sigma \mathrm{d}\theta^2 + \frac{4aMr\sin^2\theta}{\Sigma}\mathrm{d}\phi\mathrm{d}t$$
$$+ \left[\frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma}\right]\sin^2\theta\mathrm{d}\phi^2 + \mathrm{d}w^2\Bigg], \quad (1)$$

where we have $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 + a^2 - 2Mr$. Here the location of the brane is at $z = z_0$ with the Z_2 reflection symmetry imposed there, $w = z - z_0$ is the AdS direction and l is the AdS length scale. The ADM mass and the angular momentum for the rotating black hole on the brane are then given as $M^* = Ml/z_0$ and $J^* = (l^2/z_0^2)Ma$ to an observer confined to the brane. The square of the curvature tensor is

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{1}{l^4}\left[40 + \frac{48M^2z^4}{\Sigma^6}(r^2 - a^2\cos^2\theta)(r^4 - 14a^2r^2\cos\theta + a^4\cos^4\theta)\right]. \quad (2)$$

This clearly shows the usual ring singularity of the Kerr metric at $r = 0$ and $\theta = pi/2$. However the solution is also singular at the AdS horizon $z = \infty$. It is necessary to obtain the geodesic equations to characterize the nature of this singularity.

Using the Killing isometries we derive the geodesic equations for t, ϕ, z in the equatorial plane $\theta = \pi/2$ as

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} = \frac{z^2}{l^2\Delta}\left[\left(r^2 + a^2 + \frac{2a^2M}{r}\right)E - \frac{2aM}{r}L\right], \quad (3)$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}\lambda} = \frac{z^2}{l^2\Delta}\left[\left(1 - \frac{2M}{r}\right)L + \frac{2aM}{r}E\right], \quad (4)$$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{1}{z^2}\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right) = -\frac{\sigma}{zl^2}. \quad (5)$$

Here $\sigma = 0$ for null and $\sigma = 1$ for the time-like geodesics. We choose the solution $z = -z_1\mathrm{cosec}(\lambda/l)$ for the z equation which reaches $z = \infty$, the AdS horizon. For this class the radial geodesic equation in the equatorial plane is

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2 + \frac{z^4}{l^4}\left[\left(\frac{L^2 - a^2E^2}{r^2} - \frac{2M}{r^3}(aE - L)^2 - E^2\right)\right] + \frac{l^2}{z_1^2}\frac{\Delta}{r^2} = 0. \quad (6)$$

It is now possible to introduce a new affine parameter ν for the null and time-like geodesics and rescale all the coordinates and constants in the radial equation by appropriate powers of z_1/l where z_1 is a constant. This removes the explicit z dependence and the equation reduces to that of a four-dimensional radial geodesic equation for a Kerr metric, with a mass $\tilde{M}$.

The behaviour of the geodesics near the AdS horizon may be now studied through the late time asymptotics of this equation. It is clearly seen that the curvature squared is singular only along the bound geodesics signaling a *p–p* curvature singularity. An explicit characterization of this singularity through the determination of curvature components in an orthonormal frame parallely propagated to the AdS horizon becomes computationally intractable in the case of the Kerr metric. Clearly the singularity at the AdS horizon is a pathology of this approach. However, the rotating black string metric in the five-dimensional bulk is expected to be subjected to the usual Gregory–Laflamme instability although an explicit calculation for axisymmetric cases is lacking. This would make the black string unstable near the AdS horizon but stable far away causing the horizon to pinch off and form a line of mini-black holes accumulating towards the AdS horizon [5]. It is shown in lower dimensions where exact AdS C-metrics are available that the solutions are well-behaved at the AdS horizon. So it is reasonable to conclude that the singularity at the AdS horizon is an artifact of the linearized approximation.

It must be understood that for consistency the RS model (or some variant) of it must be embedded in an appropriate string theory. Thus it requires a generalization of such models to higher dimensions. Generalization of the Randall–Sundrum construction to arbitrary $(N+1)$ dimensions with an appropriate co-dimension 1 brane is straightforward and may be easily extended to include the full non-linear equations for any Ricci flat metric.

In higher dimensions also GTR on an appropriate co-dimension brane needs to be reproduced for consistency, in particular higher dimensional black hole and cosmological solutions from a brane world perspective. The absence of exact C-metrics in dimensions $D \geq 3$ requires such studies to be based on the linearized approximation and to that end the CHR model is a reasonable approach despite the problem of the boundary singularity. In this context one of us (GS) have considered the description of higher dimensional black holes on a codimension 1 brane embedded in a $(N+1)$-dimensional RS brane world with a single AdS direction. In particular we consider the emergence of a generalization of the Kerr solution to N dimensions due to Myers and Perry [6] on the $(N-1)$-dimensional brane. The Myers–Perry solution shows interesting behaviour in the fact that the higher dimensions admit multiple rotations in the various coordinate planes. Furthermore, these solutions show a variety of horizon and singularity structures depending on the odd or even number of spatial dimensions. To this end we have considered a rotating black string in a $(N+1)$-dimensional brane world which intercepts the $(N-1)$ brane in a N-dimensional rotating black hole solution [7]. We have discussed various cases with multiple non-zero angular momentum parameters and even or odd number of spatial dimensions. We have also obtained the geodesic equations for the most general case and have discussed their consequences on the causal structure. It is observed that the pathological p–p curvature singularity at the AdS horizon persists. It is also conceivable that the solution is unstable at the AdS horizon due to the Gregory–Laflamme instability. Work is in progress to describe the most general four-dimensional Kerr–Newman metric for a rotating charged black hole in a RS brane world.

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