PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 62, No. 3
March 2004
pp. 703–706

Intersecting $p - p'$ branes in pp-wave background

ANINDYA BISWAS, ALOK KUMAR and KAMAL L PANIGRAHI
Institute of Physics, Bhubaneswar 751 005, India

Abstract. Several supergravity solutions corresponding to both Dp, as well as $Dp - Dp'$ systems, in NS-NS and R-R pp-wave background originating from $AdS_3 \times S^3 \times R^4$ are presented. The supersymmetry properties of these solutions are analysed along with a brief outline of the world sheet construction for the $p - p'$ branes.

Keywords. pp-Wave; D-branes.

pp-Wave space-times [1–3] yield exact classical background for string theory with wide applications to gauge theories. They are also known to be maximally supersymmetric solutions of supergravities in various dimensions [4], and are known to give rise to solvable string theories from world sheet point of view as well. The particular case of $AdS_5 \times S^5$ is of special interest, with applications to $N = 4$, $D = 4$ gauge theories in the limit of large conformal dimensions and R-charges [2]. As D-branes play an important role in understanding various dualities in string theory, it is desirable to study them in curved background in the presence of NS-NS and R-R fields. In AdS/CFT correspondence, D-branes correspond to, in the gauge theory side, defect on which a lower dimensinal conformal field theory lives. So it is often useful to have supergravity realization of D-branes in AdS space, in order to study the field theory dynamics in the presence of these objects. The CFT construction of D-branes in AdS space is rather difficult, but it is easier to treat this in pp-wave background. Explicit supergravity solution of D-branes, along with open string spectrum, has been studied in [5–10]. Dp-branes from world sheet point of view have already been obtained in NS-NS pp-wave background earlier [11]. D-branes intersecting at an angle in pp-wave background is also discussed [10,12]. Here we review the supergravity solutions of intersecting $p - p'$ branes along with its supersymmetry properties and open string spectrum in both NS-NS and R-R pp-wave background [10]. We would also like to point out that the branes presented here are instantonic branes, with the branes lying along both the light cone directions (x^+, x^-). For the classifications of branes in $AdS_5 \times S^5$ pp-wave background, see [9].

First we will present the supergravity solutions of Dp as well as $Dp - Dp'$ branes in pp-wave background originating from the Penrose limit of $AdS_3 \times S^3 \times R^4$ geometry with NS-NS and R-R flux. The method we will adopt here is as follows. We will start with an ansatz for the classical solutions of branes in the background that

Table 1. Branes and preserved supersymmetry.

Branes	World volume directions	Flux	SUSY
$(D1)$, $(D1-D5)$	$(+,-),(+,-,R^4)$	H_{+12}, H_{+34}	1/8
$(D1)$, $(D1-D5)$	$(+,-),(+,-,R^4)$	F_{+1256}, F_{+3456}	1/8

we have described earlier and then will solve for type-IIB field equations, Bianchi identities etc. The classical solutions of Dp ($p = 1, ..., 5$) branes are discussed in the original paper [10]. For completeness however, we will present the supergravity solutions of $D1-D5$ system, as an example of $p-p'$ branes. The metric, dilation and the field strengths of such a system is given by

$$\begin{aligned}
\mathrm{d}s^2 &= (f_1 f_5)^{-1/2}\left(2\mathrm{d}x^+\mathrm{d}x^- - \mu^2\sum_{i=1}^{4} x_i^2(\mathrm{d}x^+)^2\right) \\
&\quad + \left(\frac{f_1}{f_5}\right)^{1/2}\sum_{a=5}^{8}(\mathrm{d}x^a)^2 + (f_1 f_5)^{1/2}\sum_{i=1}^{4}(\mathrm{d}x_i)^2, \\
\mathrm{e}^{2\phi} &= \frac{f_1}{f_5}, \quad H_{+12} = H_{+34} = 2\mu, \quad F_{+-i} = \partial_i {f_1}^{-1}, \quad F_{mnp} = \epsilon_{mnpl}\partial_l f_5, \quad (1)
\end{aligned}$$

with f_1 and f_5 satisfying the Green function equation for $D1$ and $D5$ branes in the transverse space. One can check that the above ansatz solve the type IIB-field equations. By applying T-duality along R^4: $(x^5, ..., x^8)$ directions, one can obtain other D-brane bound state solutions such as $D2-D4$, $D3-D3'$ etc. The supergravity solutions for the branes in RR background can be obtained by applying S-duality and T-duality symmetries of string theory, with the constant three form field strength being replaced by the constant five form field strength. For example, the solutions in RR *pp*-wave background is obtained by applying S-duality followed by two T-dualities along the x^5 and x^6 directions of the brane solutions in NS-NS *pp*-wave background, which is summarized in table 1. For details of the brane construction in R-R *pp*-wave background see [10].

The supersymmetry properties of the branes constructed in this paper, can be analysed by solving the type-IIB killing spinor equations explicitly [13]. Without going into the calculational details, we summarize below the allowed brane configurations in the *pp*-wave background that are considered here and the supersymmetry preserved by them.

So the branes and their bound states constructed in this article preserve 1/8 supersymmetry. It is interesting to note that the brane solutions constructed in [6,8], where the brane was lying along $AdS_3 \times S^3$, preserve 3/8 supersymmetry. For detailed calculations and implications, see [8,10].

Now we discuss the $(D1-D5)$-brane system, constructed earlier, from the point of view of first quantized string theory in Green–Schwarz formalism, in light-cone gauge. The equations of motion and the boundary conditions, that we will be following are as follows:

Bosons

$$\partial_+\partial_- x_{i_1} + m^2 x_{i_1} - m\epsilon^{i_1 j_1}(\partial_- x^{j_1} - \partial_+ x^{j_1}) = 0, \quad \partial_+\partial_- x_\alpha = 0, \tag{2}$$

$$\partial_\sigma x^\alpha \Big|_{\sigma=0} = \partial_\tau x^\alpha \Big|_{\sigma=\pi} = 0, \quad x^i \Big|_{\sigma=0,\pi} = \text{constant}. \tag{3}$$

Fermions

To consider the equations of motion for fermions we would like to note that the matrix $M = -\frac{1}{2}(\gamma^{12} + \gamma^{34})$ [3], splits the fermions in the following manner: $S_{\rm L} \to (\tilde{S}_{\rm L}, \hat{S}_{\rm L}), S_{\rm R} \to (\tilde{S}_{\rm R}, \hat{S}_{\rm R})$. The equations of motion and boundary conditions for the fermions read as follows:

$$\partial_+({\rm e}^{2m\tau}\tilde{S}_{\rm R}) = 0, \quad \partial_-({\rm e}^{-2m\tau}\tilde{S}_{\rm L}) = 0, \quad \partial_+\hat{S}_{\rm R} = 0, \quad \partial_-\hat{S}_{\rm L} = 0, \tag{4}$$

$$\tilde{S}_{\rm L}\big|_{\sigma=0} = -\tilde{S}_{\rm R}\big|_{\sigma=0}, \quad \tilde{S}_{\rm L}\big|_{\sigma=\pi} = \tilde{S}_{\rm R}\big|_{\sigma=\pi}, \quad \hat{S}_{\rm L}\big|_{\sigma=0,\pi} = \hat{S}_{\rm R}\big|_{\sigma=0,\pi}. \tag{5}$$

Referring to the above equations of motion and boundary conditions, it is straightforward to write down the mode expansion and therefore the canonical commutation relation and hence the Hamiltonian of the system. For calculational details, see [10,11]. The open string construction in *R-R pp*-wave background can be followed by realizing that this background is T-dual to the NS-NS *pp*-wave background, that is presented earlier. We however would like to point out that as the branes described here preserve less than 1/2 supersymmetry, some of the restrictions imposed, using zero mode consideration [5] do not apply directly here.

We conclude by pointing out that it will be interesting to study the gauge theory duals of the branes presented in this paper by using operators such as 'defects'. It will also be interesting to examine these branes and their supersymmetry from the world volume analysis. One could possibly look at the black hole physics using the $(D1 - D5)$ system presented here in an attempt to understand their properties.

References

[1] R Penrose, Any space-time has a plane wave as a limit, in: *Differential geometry and relativity* (Reidel, Dordrecht, 1976) pp. 271–275
G T Horowitz and A A Tseytlin, *Phys. Rev.* **D51**, 2896 (1995); hep-th/9409021
R Gueven, *Phys. Lett.* **B482**, 255 (2000); hep-th/0005061
R R Metsaev, *Phys. Lett.* **B468**, 65 (1999); hep-th/9908114; *Nucl. Phys.* **B625**, 70 (2002); hep-th/0112044
R R Metsaev and A A Tseytlin, *Phys. Rev.* **D65**, 126004 (2002); hep-th/0202109
M Blau, J Figuero-O'Farrill, C Hull and G Papadopoulos, *J. High Energy Phys.* **0201**, 047 (2000); hep-th/0110242
M Blau, J Figuero-O'Farrill and G Papadopoulos, *Class. Quantum Gravit.* **19**, L87 (2000); hep-th/0201081; *Class. Quantum Gravit.* **19**, 4753 (2002); hep-th/0202111

[2] D Berenstein, J Maldacena and H Nastase, *J. High Energy Phys.* **0204**, 013 (2002); hep-th/0202021

[3] J G Russo and A A Tseytlin, *J. High Energy Phys.* **0204**, 021 (2002); hep-th/0202179

[4] P Meessen, *Phys. Rev.* **D65**, 087501 (2002); hep-th/0111031
M Cvetic, H Lu and C N Pope, hep-th/0203082
J P Gauntlett and C M Hull, *J. High Energy Phys.* **0206**, 013 (2002); hep-th/0203255
[5] A Dabholkar and S Parvizi, *Nucl. Phys.* **B641**, 223 (2002); hep-th/0203231
[6] A Kumar, R R Nayak and Sanjay, *Phys. Lett.* **B541**, 183 (2002); hep-th/0204025
[7] K Skenderis and M Taylor, *J. High Energy Phys.* **0206**, 025 (2002); hep-th/0204054; hep-th/0211011, 0212184
[8] M Alishahiha and A Kumar, *Phys. Lett.* **B542**, 130 (2002); hep-th/0205134
[9] P Bain, P Meessen and M Zamaklar, *Class. Quantum Gravit.* **20**, 913 (2003); hep-th/0205106
[10] A Biswas, A Kumar and K L Panigrahi, *Phys. Rev.* **D66**, 126002 (2002); hep-th/0208042
[11] Y Michishita, *J. High Energy Phys.* **0210**, 048 (2002); hep-th/0206131
[12] R R Nayak, hep-th/0210230
[13] J H Schwarz, *Nucl. Phys.* **B226**, 269 (1983)
S F Hassan, *Nucl. Phys.* **B568**, 145 (2000); hep-th/9907152