PRAMANA — journal of physics © Indian Academy of Sciences Vol. 62, No. 3 March 2004 pp. 695–698

D-branes in *pp*-wave background

ALOK KUMAR, RASHMI R NAYAK and SANJAY
Institute of Physics, Bhubaneswar 751 005, India

Abstract. We show the existence of classical solutions of D-branes as well as a system of $D3$-branes oriented at an arbitrary angle with respect to each other, in a six-dimensional *pp*-wave background obtained from $AdS_3 \times S^3 \times R^4$, with $R - R$ and $NS - NS$ 3-from flux. The world volume coordinate of $D5$-brane lies along the six-dimensional *pp*-wave directions, whereas the *pp*-wave direction is transverse to the system of $D3$-branes. We also present more D-brane bound state solutions by applying T-duality symmetries. The system of $D3$-branes oriented at an arbitrary angle is shown to preserve 1/16 supersymmetries. Finally a brief discussion of the open string construction is presented for both the cases.

Keywords. *pp*-wave; D-branes.

Study of string theory in *pp*-wave background has been a subject of wide interest. 'Penrose' limit plays an important role in obtaining such solutions, as any Einstein gravity admits a plane wave geometry in these limits [1]. String theory in this background is easy to handle due to the presence of natural light cone gauge and can be exactly solvable in Green–Schwarz formalism [2]. The exact solvability of string theory in these backgrounds has been used to establish the duality between string and gauge theories [3]. D-branes, known as non-perturbative and extended objects, also survive in this limit. Explicit supergravity solutions of these objects and their open string spectrum, have been discussed at length in [4–14]. These objects also play an important role in understanding the duality between string and gauge theories. For example in the existing AdS/CFT duality, if one adds D-brane in the bulk its dual correspond to a defect conformal field theory.

In view of the importance of BPS branes (preserving certain amount of supersymmetries) in understanding the non-perturbative dynamics of string theories and their dualities, in this article, we present classical solutions of various brane configurations in the *pp*-wave background originating from $AdS_3 \times S^3$ type of geometry. First, we will represent the $D5$-brane solution obtained from a known intersecting brane solution by applying Penrose limit as given in [6] and their open string construction in Green–Schwarz formalism. Next, we will show the existence and stability of a system of $D3$-branes oriented at certain angle (α), by writing down an ansatz and solving the corresponding type-IIB field equations in *pp*-wave background. This brane configuration is shown to preserve 1/16 supersymmetries.

We start by writing down the supergravity solution of $D5$-brane in *pp*-wave background. To obtain the curved $D5$-brane solution with the world volume as

$AdS_3 \times S^3$, we take the known supergravity solution of system of $1_{NS} + 5_{NS} + 5'_{NS}$, given in [15] and then by taking the Penrose limit as described in [6], the metric, dilaton and the antisymmetric tensor fields of $D5$-brane in *pp*-wave background read as follows:

$$\mathrm{d}S^2 = H_5'^{-1/2}[\mathrm{d}u\mathrm{d}v - \mu^2 \sum_{i=1}^{4} {z^i}^2 \mathrm{d}u^2 + \sum_{i=1}^{4} \mathrm{d}z^i \mathrm{d}z^i] + H_5'^{1/2}(\mathrm{d}y^2 + y^2 \mathrm{d}{\Omega_3'}^2),$$
$$\mathrm{e}^{2\phi} = H_5'(y)^{-1}, \quad F_{+12} = F_{+34} = 2\mu, \quad F_{mnp} = \epsilon_{mnpl}\partial_l H_5'(y). \tag{1}$$

By applying S-duality symmetry in the above $D5$-brane solution, one can get the $NS5$-brane solution in *pp*-wave background. More D-brane bound states can be obtained by applying T-duality symmetry. For example, T-duality along one of the transverse direction of $D5$-brane solution results in $D6$-brane solution in *pp*-wave background and more than one T-duality will generate higher brane solutions. Applying a mixing between one longitudinal and one transverse direction of $D6$-brane as described in [16], followed by a T-duality, one can generate the $D5-D7$ bound state as well. For details, see [8].

We now proceed to construct the $D5$-brane, from the point of view of a first quantized string theory in Green–Schwarz formalism in light-cone gauge. The relevant boundary conditions are:

$$\begin{aligned} \text{Bosons: Neumann } & \partial_\sigma X^i|_{\sigma=0,\pi} = 0, \quad \text{where } i = 1, ..., 4, \\ \text{Dirichlet } & \partial_\tau X^\alpha|_{\sigma=0,\pi} = 0, \quad \text{where } \alpha = 5, ..., 8. \end{aligned} \tag{2}$$

$$\text{Fermions: } S_{\mathrm{L}}|_{\sigma=0,\pi} = \Omega S_{\mathrm{R}}|_{\sigma=0,\pi}, \quad [\Omega, \gamma] = 0, \quad \Omega M \Omega = M. \tag{3}$$

The explicit form of Ω in our case is given by $\Omega = \gamma^{5678}$ and leads to the following boundary conditions on $(\tilde{S}_{\mathrm{L}}, \hat{S}_{\mathrm{L}})$ and $(\tilde{S}_{\mathrm{R}}, \hat{S}_{\mathrm{R}})$:

$$\tilde{S}_{\mathrm{L}}|_{\sigma=0,\pi} = -\tilde{S}_{\mathrm{R}}|_{\sigma=0,\pi}, \quad \hat{S}_{\mathrm{L}}|_{\sigma=0,\pi} = \hat{S}_{\mathrm{R}}|_{\sigma=0,\pi}. \tag{4}$$

It is now straightforward to write down the mode expansions, the canonical commutation relations and the corresponding Hamiltonian of the system. For details, see [8].

Now we present the classical solution of a system of two $D3$ branes oriented at certain $SU(2)$ angle (α), in $NS-NS$ plane wave background. The classical solution of such a system is given by

$$\begin{aligned} \mathrm{d}s^2 = \sqrt{1+X}\Bigg\{ \frac{1}{1+X}\Bigg(& 2\mathrm{d}x^+\mathrm{d}x^- - \mu^2 \sum_{i=1}^{4} x_i^2 \, (\mathrm{d}x^+)^2 \\ & + (1+X_2)\big[(\mathrm{d}x^5)^2 + (\mathrm{d}x^7)^2\big] + (\mathrm{d}x^6)^2 + (\mathrm{d}x^8)^2 \\ & + X_1 \left[(\cos\alpha \mathrm{d}x^5 - \sin\alpha \mathrm{d}x^6)^2 + (\cos\alpha \mathrm{d}x^7 + \sin\alpha \mathrm{d}x^8)^2\right] \Bigg) + \sum_{i=1}^{4} (\mathrm{d}x^i)^2 \Bigg\}, \end{aligned}$$

$$H_{+12} = H_{+34} = 2\mu, \quad F^{(5)}_{+-68i} = \partial_i \Big\{ \frac{X_2 + X_1 \cos^2\alpha + X_1 X_2 \sin^2\alpha}{(1+X)} \Big\}, \quad \mathrm{e}^{2\phi_b} = 1,$$

$$F^{(5)}_{+-58i} = -F^{(5)}_{+-67i} = \partial_i\left\{\frac{X_1\cos\alpha\sin\alpha}{(1+X)}\right\},\ F^{(5)}_{+-57i} = -\partial_i\left\{\frac{(X_1+X_1X_2)\sin^2\alpha}{(1+X)}\right\}, \tag{5}$$

and X is given by

$$X = X_1 + X_2 + X_1X_2\sin^2\alpha, \tag{6}$$

where X_1 and X_2 are the harmonic functions in the common transverse space. To start with, two $D3$-branes are parallel to each other and lying along x^+, x^-, x^6, x^8. By applying an $SU(2)$ rotation, the second brane will be rotated by a certain angle α and will lie in the x^+, x^-, x^5, x^7. At the same time these branes are delocalized along $x^5 - x^6$ and $x^6 - x^8$ planes respectively. This solution solves the type-IIB field equations. In the limit $\mu = 0$, the above solution represent the flat space case [17,18]. For $X_2 = 0$, the above solution reduces to a single $D3$-brane oriented at an angle in *pp*-wave background. At $\alpha = \pi/2$, the above-described solution corresponds to a system of two $D3$-branes intersecting orthogonally along a string. By applying various S-duality and T-duality transformations one can generate more bound states of D-branes in $NS-NS$ and $R-R$ *pp*-wave backgrounds. For details, see [14]. Now we briefly outline the open string construction of the $D3$-brane system, presented from the supergravity point of view, following [17]. Boundary conditions are:

$$D3_1: \quad \partial_\sigma x^{+,-,6,8} = 0, \quad \partial_\tau x^{i,5,7} = 0, \quad i = 1,...,4. \tag{7}$$

$$\begin{aligned} D3_2: \quad & \partial_\sigma(\cos\alpha\ x^{6,8} \mp \sin\alpha\ x^{5,7}) = 0, \\ & \partial_\tau(\pm\sin\alpha\ x^{6,8} + \cos\alpha\ x^{5,7}) = 0, \end{aligned} \tag{8}$$

with x^+, x^- and x^i satisfying the usual Neumann and Dirichlet boundary conditions respectively. In eqs (7) and (8), $D3_1$ and $D3_2$ denote the unrotated and the rotated branes. Referring to the equation of motions given in [11], with the above boundary conditions, it is straightforward to write down the mode of expansion and hence the canonical commutation relation. We, however skip the details here.

The supersymmetry of the above-mentioned branes are obtained by solving the type-IIB killing spinor equations explicitly [20,10,14]. Skipping all the calculational details, we summarize in table 1 the allowed brane configurations and amount of supersymmetries preserved by them in *pp*-wave background.

In this article, we present the classical solution and briefly the open string construction of $D5$-branes in *pp*-wave background. They preserve 3/8 supersymmetries [10]. We also show the existence and stability of a system of $D3$-branes oriented by

Table 1. Branes and preserved supersymmetry.

Branes	World volume directions	Flux	SUSY
$(D5)$	$(+, -, 1, 2, 3, 4)$	F_{+12}, F_{+34}	3/8
$(D3_1 - D3_2)$	$(+, -, x^5, x^6, x^7, x^8)$	H_{+12}, H_{+34}	1/16

$SU(2)$ rotations in $NS - NS$ and $R - R$ *pp*-wave background. They preserve 1/16 of the supersymmetries. These configurations are also studied as the probe branes in the given background. They are shown to preserve 1/4 supersymmetry [21,14] of the world volume. For details of calculations, see [14]. It will be interesting to generalize these to arbitrary $SU(N)$ rotations. As there is an appropriate representation of world volume gauge fields by means of the angle between the rotated branes, it may be interesting to study them further in *pp*-wave background.

References

[1] R Penrose, Any space-time has a plane wave as a limit, in: *Differential geometry and relativity* (Reidel, Dordrecht, 1976) pp. 271–275

[2] R R Metsaev, *Nucl. Phys.* **B625**, 70 (2002), hep-th/0112044
R R Metsaev and A A Tseytlin, *Phys. Rev.* **D65**, 126004 (2002), hep-th/0202109
J G Russo and A A Tseytlin, *J. High Energy Phys.* **0204**, (2002)

[3] D Berenstein, J Maldacena and H Nastase, hep-th/0202021
N Itzhaki, I R Klebanov and S Mukhi, *J. High Energy Phys.* **0203**, 048 (2002), hep-th/0202153
L A Pando Zayas and J Sonnenschein, hep-th/0202186

[4] G T Horowitz and A A Tseytlin, *Phys. Rev.* **D51**, 2896 (1995), hep-th/9409021

[5] R Gueven, *Phys. Lett.* **B482**, 255 (2000), hep-th/0005061

[6] M Blau, J Figuero-O'Farrill and G Papadopoulos, hep-th/0202111

[7] A Dabholkar and S Parvizi, *Nucl. Phys.* **B641**, 223 (2002), hep-th/0203231

[8] A Kumar, R R Nayak and Sanjay, *Phys. Lett.* **B541**, 183 (2002), hep-th/0204025

[9] K Skenderis and M Taylor, *J. High Energy Phys.* **0206**, 025 (2002), hep-th/0204054

[10] M Alishahiha and A Kumar, *Phys. Lett.* **B542**, 130 (2002), hep-th/0205134

[11] Y Michishita, *J. High Energy Phys.* **0210**, 048 (2002), hep-th/0206131

[12] A Biswas, A Kumar and K L Panigrahi, *Phys. Rev.* **D66**, 126002 (2002), hep-th/0208042

[13] M Cvetic, H Lu, C N Pope and K S Stelle, hep-th/0209193

[14] Rashmi R Nayak, hep-th/0210230 (to appear in *Phys. Rev. D*)

[15] G Papadopoulos, J Russo and A A Tseytlin, *Class. Quantum Gravit.* **17**, 1713 (2000), hep-th/9911253

[16] J C Breckenridge, G Michaud and R C Myers, *Phys. Rev.* **D55**, 6438 (1997), hep-th/9611174

[17] M Berkooz, M R Douglas and R G Leigh, *Nucl. Phys.* **B480**, 265 (1996), hep-th/9606139

[18] J C Breckenridge, G Michaud and R C Myers, *Phys. Rev.* **D56**, 5172 (1997), hep-th/9703041

[19] N Hambli, *Phys. Rev.* **D56**, 2369 (1997), hep-th9703179

[20] J H Schwarz, *Nucl. Phys.* **B226**, 269 (1983)
S F Hassan, *Nucl. Phys.* **B568**, 145 (2000), hep-th/9907152

[21] E Bergshoeff, R Kallosh, T Ortin and G Papadopoulos, *Nucl. Phys.* **B502**, 149 (1997), hep-th/9705040