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Possibility of extracting the weak phase γ from $\Lambda_b \to \Lambda D^0$ decays

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Abstract. We explore the possibility of extracting the weak phase γ from pure tree decays $\Lambda_b \to \Lambda(D^0, \overline{D^0}, D^0_{\rm CP})$ in a model independent way. We find the CP violating phase γ can be determined cleanly without any hadronic uncertainties. Furthermore, these decays are free from the penguin pollution and neither tagging nor time dependent studies are required for the extraction.

Keywords. CP violation; non-leptonic baryon decays.

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The standard model (SM) provides a simple description of the phenomenon of CP violation through the complex CKM matrix [1]. The major goal of B-factories is to test Kobayashi–Maskawa mechanism of CP violation within the SM and to detect possible new sources of CP violation beyond it. In the SM, the phenomenon of CP violation can be established if we can measure accurately the three angles (α, β and γ) of the CKM unitarity triangle, which add up to 180°. Among these three angles, the most difficult to measure is the angle γ. To this end, various interesting proposals [2] have been made with a view to obtain γ with lesser or no hadronic uncertainties. The aim is to check all possible decay modes and try to determine the angle γ as cleanly as possible. In this note we would like to show that the angle γ can also be extracted from the pure tree decays of the Λ_b baryon i.e, $\Lambda_b \to \Lambda\{D^0, \overline{D^0}, D^0_{\rm CP}\}$ [3]. The advantage of these decay modes is that these are free from penguin pollution and the amplitudes are of similar sizes. Furthermore, neither tagging nor time dependent studies are required for these decay modes, so γ can be extracted cleanly without hadronic uncertainties.

Let us now write the decay amplitudes for the processes $\Lambda_b \to \Lambda\{D^0, \overline{D^0}, D^0_{\rm CP}\}$. Both these amplitudes proceed via the colour suppressed tree diagrams only and thus are free from penguin pollutions. The amplitude for $\Lambda_b \to \Lambda D^0$ arises from the quark level transition $b \to c\bar{u}s$ and hence has no weak phase in the Wolfenstein parametrization, while the amplitude $\Lambda_b \to \Lambda\overline{D^0}$ arises from $b \to u\bar{c}s$ transition and carries the weak phase ${\rm e}^{-i\gamma}$. The amplitudes also have the strong phases ${\rm e}^{i\delta_1^i}$ and ${\rm e}^{i\delta_2^i}$, where $i = S$ or P. Thus we can write the decay amplitude for $\Lambda_b \to \Lambda D^0$

process as

$$A_1 = \mathrm{Amp}(\Lambda_b \to \Lambda D^0) = S_1 \mathrm{e}^{i\delta_1^S} + P_1 \mathrm{e}^{i\delta_1^P} \boldsymbol{\sigma}\cdot\hat{\mathbf{q}}$$
$$= \mathrm{e}^{i\delta_1^S}\left(S_1 + P_1 \mathrm{e}^{i\Delta_1}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\right) , \tag{1}$$

where S_1 and P_1 are magnitudes of the S and P wave amplitudes and $\Delta_1 = \delta_1^P - \delta_1^S$, is the relative strong phase between them. From the decay mode $\Lambda_b \to \Lambda D^0$, one can extract the three observables S_1, P_1 and Δ. Similarly, the amplitude for $\Lambda_b \to \Lambda \overline{D^0}$ as

$$A_2 = \mathrm{Amp}(\Lambda_b \to \Lambda \overline{D^0}) = \mathrm{e}^{-i\gamma}(S_2 \mathrm{e}^{i\delta_2^S} + P_2 \mathrm{e}^{i\delta_2^P} \boldsymbol{\sigma}\cdot\hat{\mathbf{q}})$$
$$= \mathrm{e}^{-i\gamma}\mathrm{e}^{i\delta_2^S}\left(S_2 + P_2 \mathrm{e}^{i\Delta_2}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\right) . \tag{2}$$

Thus from this decay mode we can extract another set of three observables S_2, P_2 and Δ_2. Now, the amplitudes for the corresponding CP conjugate processes are given as

$$\bar{A}_1 = \mathrm{Amp}(\bar{\Lambda}_b \to \bar{\Lambda}\overline{D^0}) = \bar{S}_1 \mathrm{e}^{i\delta_1^S} + \bar{P}_1 \mathrm{e}^{i\delta_1^P} \boldsymbol{\sigma}\cdot\hat{\mathbf{q}}$$
$$= \mathrm{e}^{i\delta_1^S}\left(\bar{S}_1 + \bar{P}_1 \mathrm{e}^{i\Delta_1}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\right) ,$$
$$\bar{A}_2 = \mathrm{Amp}(\bar{\Lambda}_b \to \bar{\Lambda} D^0) = \mathrm{e}^{i\gamma}(\bar{S}_2 \mathrm{e}^{i\delta_2^S} + \bar{P}_2 \mathrm{e}^{i\delta_2^P} \boldsymbol{\sigma}\cdot\hat{\mathbf{q}})$$
$$= \mathrm{e}^{i\gamma}\mathrm{e}^{i\delta_2^S}(\bar{S}_2 + \bar{P}_2 \mathrm{e}^{i\Delta_2}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}). \tag{3}$$

from which, the observables $\bar{S}_{1,2}$, $\bar{P}_{1,2}$ and $\Delta_{1,2}$ can be determined.

We now consider the decay modes $\Lambda_b \to \Lambda D^0_\pm$, where $D^0_\pm$ denote the neutral D meson even/odd CP states, defined as $D^0_\pm = (D^0 \pm \overline{D^0})/\sqrt{2}$. The CP even state D^0_+ can be identified by the CP even decay products such as $\pi^+\pi^-$ and K^+K^-, whereas the CP odd state D^0_- can be identified by the CP odd products such as $K_S\pi^0$, $K_S\rho^0$, $K_S\omega$ and $K_S\phi$. One can use either of these two CP eigenstates for the extraction of γ. Here we are considering the CP even eigenstate (D^0_+), however, the same argument will also hold for the CP odd state (D^0_-). The amplitude for $\Lambda_b \to \Lambda D^0_+$ is thus given as

$$A_+ = \mathrm{Amp}(\Lambda_b \to \Lambda D^0_+) = S_+ \mathrm{e}^{i\delta_+^S} + P_+ \mathrm{e}^{i\delta_+^P} \boldsymbol{\sigma}\cdot\hat{\mathbf{q}}$$
$$= \mathrm{e}^{i\delta_+^S}\left(S_+ + P_+ \mathrm{e}^{i\Delta_+}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\right) , \tag{4}$$

where S_+, P_+ are the magnitudes of the S and P wave amplitudes with phases $\mathrm{e}^{i\delta_+^S}$ and $\mathrm{e}^{i\delta_+^P}$. It should be noted that these phases contain both strong and weak components. Thus from this decay mode we can extract the observables S_+, P_+ and $\Delta_+ = \delta_+^P - \delta_+^S$. One can also write the amplitude A_+ as

$$A_+ = \frac{1}{\sqrt{2}}[A_1 + A_2] = \frac{1}{\sqrt{2}}[\mathrm{e}^{i\delta_1^S}(S_1 + S_2 \mathrm{e}^{i(\sigma_+^S - \gamma)})$$
$$+\mathrm{e}^{i\delta_1^P}(P_1 + P_2 \mathrm{e}^{i(\sigma_+^P - \gamma)})\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}], \tag{5}$$

where $\sigma_+^{S,P} = \delta_2^{S,P} - \delta_1^{S,P}$. Thus comparing eqs (4) and (5) we obtain

$$S_+ e^{i\delta_+^S} = \frac{1}{\sqrt{2}} e^{i\delta_1^S} (S_1 + S_2 e^{i(\sigma_+^S - \gamma)}),$$
$$P_+ e^{i\delta_+^P} = \frac{1}{\sqrt{2}} e^{i\delta_1^P} (P_1 + P_2 e^{i(\sigma_+^P - \gamma)}). \tag{6}$$

Now the amplitude for the corresponding CP conjugate process, i.e., $\bar{\Lambda}_b \to \bar{\Lambda} D_+^0$ is given as

$$\bar{A}_+ = \mathrm{Amp}(\bar{\Lambda}_b \to \bar{\Lambda} D_+^0) = \bar{S}_+ e^{i\delta_+^{\bar{S}}} + \bar{P}_+ e^{i\delta_+^{\bar{P}}} \boldsymbol{\sigma} \cdot \hat{\mathbf{q}}$$
$$= e^{i\delta_+^{\bar{S}}} (\bar{S}_+ + \bar{P}_+ e^{i\bar{\Delta}_+} \boldsymbol{\sigma} \cdot \hat{\mathbf{q}}) \ . \tag{7}$$

The observables obtained from this decay mode are $\bar{S}_+$, $\bar{P}_+$ and $\bar{\Delta}_+ = \bar{\delta}_+^P - \bar{\delta}_+^S$. Writing the amplitude in terms of $\bar{S}$ and $\bar{P}$ observables, we obtain the relations similar to eq. (6) as

$$\bar{S}_+ e^{i\delta_+^{\bar{S}}} = \frac{1}{\sqrt{2}} e^{i\delta_1^S} (\bar{S}_1 + \bar{S}_2 e^{i(\sigma_+^S + \gamma)}),$$
$$\bar{P}_+ e^{i\delta_+^{\bar{P}}} = \frac{1}{\sqrt{2}} e^{i\delta_1^P} (\bar{P}_1 + \bar{P}_2 e^{i(\sigma_+^P + \gamma)}). \tag{8}$$

We now use eqs (6) and (8) to obtain the weak phase γ. To derive the analytic expression for γ, we define the combinations of observables

$$X = \frac{2S_+^2 - S_1^2 - S_2^2}{2S_1 S_2}, \quad \bar{X} = \frac{2\bar{S}_+^2 - \bar{S}_1^2 - \bar{S}_2^2}{2\bar{S}_1 \bar{S}_2}. \tag{9}$$

Here we have considered only the S wave components, but similar combinations can also be derived from the P wave observables and one can use either set, for the extraction of γ. Thus, in this method we need to know only the magnitudes of S and P waves but not the phase difference between them. It is now very simple to see that one can obtain an expression for γ from eq. (9) as $2\gamma = \arccos \bar{X} - \arccos X$, with some discrete ambiguities. One can also obtain the value of $\sin^2\gamma$ from eq. (9) via the relation

$$\sin^2\gamma = \frac{1}{2}\left[1 - X\bar{X} \pm \sqrt{(1-X^2)(1-\bar{X}^2)}\right] \ . \tag{10}$$

One of the sign in eq. (10) will give the correct value of $\sin^2\gamma$, while the other will give the value of the strong phase $\sin^2\sigma_+^S$. Using the non-relativistic quark model, the branching ratio for the processes $\Lambda_b \to \Lambda\{D^0, \bar{D}^0\}$ are found to be

$$\mathrm{BR}(\Lambda_b \to \Lambda D^0) = 4.56 \times 10^{-6} \ , \quad \text{and} \quad \mathrm{BR}(\Lambda_b \to \Lambda \overline{D^0}) = 8.29 \times 10^{-7}. \tag{11}$$

Although the branching ratios for these decay modes are quite small, they could be observed in the future B experiments. Now let us study the experimental feasibility of such decay modes. The B TeV experiment, with luminosity $2 \times 10^{32} \mathrm{cm}^{-2}\mathrm{s}^{-1}$, will produce 2×10^{11} $b\bar{b}$ hadrons per 10^7 sec of running. If we assume the production fraction as $\overline{B_d} : B^- : \overline{B_s} : \Lambda_b = 0.375 : 0.375 : 0.15 : 10$ we expect around 2×10^{10} numbers of Λ_b baryon per year of running at BTeV. If we take the branching ratios as: $\mathrm{BR}(\Lambda_b \to \Lambda D^0) \sim 4.5 \times 10^{-6}$, $\mathrm{BR}(D^0 \to K^-\pi^+$ and $K^-\pi^+\pi^-\pi^+) = 0.12$, the reconstruction efficiency as 0.05 and the trigger efficiency level as 0.9, we expect to get 486 ΛD^0 events per year.

To summarize, here we have shown that the decay modes $\Lambda_b \to \Lambda\{D^0, \overline{D^0}, D^0_{\mathrm{CP}}\}$ appear to be ideally suited for the clean determination of the angle γ. We have considered only the SM contributions to the decay processes. So if the extracted value of γ differs from its value constrained by the SM, then this would be an indication of the existence of new physics.

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