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Radiative stability of neutrino-mass textures

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Abstract. Neutrino-mass textures proposed at high scales are known to be unstable against radiative corrections especially for nearly degenerate mass eigenvalues. We find a mechanism in a class of gauge models including 2HDM where the RG constraints can be evaded. Consequently, a high-scale texture can match the low-energy data or be reproduced at low energies.

Keywords. Bi-maximal texture; renormalisation group.

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A major challenge to particle physics at present is the theoretical understanding of experimentally observed neutrino anomalies. Whereas the observed mixing between quarks are small, experimental data favor two large mixings in the neutrino sector. A possible origin of two large neutrino mixings for $\nu_e - \nu_\mu$ and $\nu_\mu - \nu_\tau$ but small mixing for $\nu_e - \nu_\tau$ could be unification of mixings of quarks and neutrinos at the GUT see-saw scale and radiative magnification to bi-large mixings at the weak scale [1]. An outstanding problem with bi-maximal neutrino-mass textures with degenerate eigenvalues is the instability of the masses and mixing angles due to radiative corrections [2] which spoil their prospects for the neutrinoless double beta decay and the observed neutrino anomalies [3].

In this paper we show that there is a class of gauge models such as the 2HDM where the RG constraint can be evaded. Such models have two different matrices contributing to the physically relevant neutrino mass matrix in the flavor basis. As a result of evasion of the RG-constraints, a high-scale texture can be reproduced or made to match the experimental data neutrino at low energies. We illustrate this choosing 2HDM and bi-maximal mass texture [4] as example.

In a class of two Higgs doublet models (2HDM) there are two doublets, Φ_u and Φ_d with VEVs $v_u/\sqrt{2} = v\sin\beta/\sqrt{2}$ and $v_d/\sqrt{2} = v\cos\beta/\sqrt{2}$. Whereas Φ_u couples to up-quarks and Φ_d couples to down-quarks and charged leptons, unlike SM or MSSM, there are two neutrino-mass operators, K^{I} and K^{II}. The resulting two matrices, m^{I} and m^{II} add up to generate the physically relevant Majorana-neutrino-mass matrix, $m = -(1/4)\left(K^{\mathrm{I}}v_u^2 + K^{\mathrm{II}}v_d^2\right) \equiv m^{\mathrm{I}} + m^{\mathrm{II}}$. The relevant RGEs and their one-loop solutions for $\mu < M_N$ ($t = \ln\mu < t_0$) are

$$
\begin{aligned}
16\pi^2\frac{\mathrm{d}m^{\mathrm{I}}}{\mathrm{d}t} &= \left\{-3g_2^2+2\lambda_2+2\,\mathrm{Tr}\left(3Y_U^\dagger Y_U\right)\right\}m^{\mathrm{I}} \\
&\quad +\frac{1}{2}\left[\left(Y_E^\dagger Y_E\right)m^{\mathrm{I}}+m^{\mathrm{I}}\left(Y_E^\dagger Y_E\right)^T\right]+2\lambda_3 m^{\mathrm{II}}, \\
16\pi^2\frac{\mathrm{d}m^{\mathrm{II}}}{\mathrm{d}t} &= \left\{-3g_2^2+2\lambda_1+2\,\mathrm{Tr}\left(3Y_D^\dagger Y_D+Y_E^\dagger Y_E\right)\right\}m^{\mathrm{II}} \\
&\quad -\frac{3}{2}\left[\left(Y_E^\dagger Y_E\right)m^{\mathrm{II}}+m^{\mathrm{II}}\left(Y_E^\dagger Y_E\right)^T\right]+2\lambda_3^* m^{\mathrm{I}}, \\
m_{ij}^{\mathrm{I}}(t) &= a_{ij}^{\mathrm{I}}(t)m_{ij}^{\mathrm{I}}(0), \quad a_{ij}^{\mathrm{I}}(t)=I_{g_2}^{-3/2}I_{\lambda_2}I_{\mathrm{top}}^3\left(I_iI_j\right)^{1/4}R_{ij}, \qquad (1) \\
m_{ij}^{\mathrm{II}}(t) &= a_{ij}^{\mathrm{II}}(t)m_{ij}^{\mathrm{II}}(0), \quad a_{ij}^{\mathrm{II}}(t)=I_{g_2}^{-3/2}I_{\lambda_1}I_b^3 I_\tau\left(I_iI_j\right)^{-3/4}\tilde{R}_{ij}. \qquad (2)
\end{aligned}
$$

Here $m_{ij}^{\mathrm{I,II}}(0)=m_{ij}^{\mathrm{I,II}}(t_0)$,

$$
\begin{aligned}
I_l &= \exp\left(\frac{1}{8\pi^2}\int_{t_0}^{t}h_l^2\mathrm{d}t\right), \quad (l=e,\mu,\tau,\mathrm{top},b) \\
I_{g_k} &= \exp\left(\frac{1}{8\pi^2}\int_{t_0}^{t}g_k^2\mathrm{d}t\right), \quad (k=1,2) \\
I_{\lambda_k} &= \exp\left(\frac{1}{8\pi^2}\int_{t_0}^{t}\lambda_k\mathrm{d}t\right), \quad (k=1,2) \\
R_{ij} &= \exp\left[\frac{1}{8\pi^2}\int_{t_0}^{t}\left(m_{ij}^{\mathrm{II}}/m_{ij}^{\mathrm{I}}\right)\lambda_3\mathrm{d}t\right], \\
\tilde{R}_{ij} &= \exp\left[\frac{1}{8\pi^2}\int_{t_0}^{t}\left(m_{ij}^{\mathrm{I}}/m_{ij}^{\mathrm{II}}\right)\lambda_3^*\mathrm{d}t\right]. \qquad (3)
\end{aligned}
$$

Any texture at the highest scale for the physically relevant Majorana-neutrino-mass matrix

$$m(0)=m^{\mathrm{I}}(0)+m^{\mathrm{II}}(0), \qquad (4)$$

never determines both the matrices $m^{\mathrm{I}}(0)$ and $m^{\mathrm{II}}(0)$. We further impose the criterion that the texture matches the experimental data $m_{ij}^{(e)}(t_Z)$ at the lowest scale by demanding that

$$m_{ij}^{(e)}(t_Z)=a^{\mathrm{I}}(t_Z)m_{ij}^{\mathrm{I}}(0)+a^{\mathrm{II}}(t_Z)m_{ij}^{\mathrm{II}}(0). \qquad (5)$$

Solutions of (4) and (5) now determine both $m^{\mathrm{I}}(0)$ and $m^{\mathrm{II}}(0)$ in terms of the high-scale neutrino-mass texture, $m(0)$,

$$
\begin{aligned}
m_{ij}^{\mathrm{I}}(0) &= (a_{ij}^{\mathrm{II}}(t_Z)m_{ij}(0)-m_{ij}^{(e)}(t_Z))/d_{ij}, \\
m_{ij}^{\mathrm{II}}(0) &= (m_{ij}^{(e)}(t_Z)-a_{ij}^{\mathrm{I}}(t_Z)m_{ij}(0))/d_{ij} \\
d_{ij} &= a_{ij}^{\mathrm{II}}(t_Z)-a_{ij}^{\mathrm{I}}(t_Z). \qquad (6)
\end{aligned}
$$

As an example we study RG evolution of the bi-maximal texture with triply degenerate masses at the see-saw scale $M_N \simeq 10^{13}$ GeV [3].

$$m(0) = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix} m_0. \tag{7}$$

Using the mass eigenvalues $m_1 = -0.2$ eV, $m_2 = 0.20045$ eV, $m_3 = 0.2075$ eV, $s_{12} = 0.6946$, $s_{13} = 0.0$, and $s_{23} = 0.6950$, at M_Z we construct the experimental mass matrix

$$m^{(e)}(t_Z) = \begin{bmatrix} -0.044716 & 0.722025 & -0.690426 \\ 0.722025 & 0.501718 & 0.477908 \\ -0.690426 & 0.477908 & 0.544506 \end{bmatrix} m_0, \tag{8}$$

where $m_0 \simeq 0.2$ eV. We note that 7 matches with 8 provided the parameters $m^{\mathrm{I}}(0)$ and $m^{\mathrm{II}}(0)$ are as given below:

$$m^{\mathrm{I}}(0) = \begin{bmatrix} 0.112140 & -0.257854 & 3.041252 \\ -0.257854 & -0.160003 & -2.126483 \\ 3.041252 & -2.126483 & -0.151038 \end{bmatrix} m_0, \tag{9}$$

$$m^{\mathrm{II}}(0) = \begin{bmatrix} -0.112140 & 0.964961 & -2.334145 \\ 0.964961 & 0.660003 & 1.626483 \\ -2.334145 & 1.626483 & 0.651038 \end{bmatrix} m_0. \tag{10}$$

Similarly for any other texture there are corresponding values for $m^{\mathrm{I}}(0)$ and $m^{\mathrm{II}}(0)$ that match the experimental data. It is to be noted that similar procedure can be adopted to reproduce a high-scale texture at the weak scale. For this purpose $m_{ij}^{(e)}(t_Z)$ in (5), (6) is to be replaced by $m_{ij}(0)$ and the corresponding values of $m^{\mathrm{I}}(0)$ and $m^{\mathrm{II}}(0)$ are numerically different from those given in (9) and (10).

Using type-II see-saw mechanism but with left-handed Higgs triplet masses lower than the see-saw scale it might be possible to implement this mechanism in the SM or MSSM.

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