PRAMANA
— journal of
physics

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Vol. 62, No. 3
March 2004
pp. 639–642

T invariance of Higgs interactions in the standard model

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Abstract. In the standard model, the Cabibbo–Kobayashi–Maskawa matrix, which incorporates the time-reversal violation shown by the charged current weak interactions, originates from the Higgs–quark interactions. The Yukawa interactions of quarks with the physical Higgs particle can contain further complex phase factors, but nevertheless conserve T, as shown by constructing the fermion T transformation and the invariant Euclidean fermion measure.

PACS Nos 11.30.Er; 14.80.Bn

The quark mass terms arising from Yukawa interaction with Higgs fields as a result of a Higgs vacuum expectation value are complex and of the form $\bar{q}_{\rm L} M q_{\rm R} + \bar{\tilde{q}}_{\rm L} \tilde{M} \tilde{q}_{\rm R} + {\rm hc}$, where $q, \tilde{q}$ stand for quarks with charge $+2/3$ and $-1/3$ respectively. On diagonalization of $M, \tilde{M}$, by $SU(3) \times SU(3)$ matrices (without any $U(1)$ factors to avoid QCD anomalies),

$$\begin{aligned} q_{\rm L} &\to A_{\rm L}^{-1} q_{\rm L}, \quad q_{\rm R} \to A_{\rm R}^{-1} q_{\rm R}, \\ \tilde{q}_{\rm L} &\to \tilde{A}_{\rm L}^{-1} \tilde{q}_{\rm L}, \quad \tilde{q}_{\rm R} \to \tilde{A}_{\rm R}^{-1} \tilde{q}_{\rm R}, \end{aligned} \tag{1}$$

the W-interactions pick up a matrix $A_{\rm L} \tilde{A}_{\rm L}^{-1} \equiv C$, the Cabibbo–Kobayashi–Maskawa matrix, which may be complex and T-violating. The diagonalized mass terms may continue to be complex [1] and of the form $\bar{\psi} m \exp(i\theta' \gamma_5)\psi$. The physical Higgs–quark interaction terms are proportional to these mass terms and thus of the form $\bar{\psi}\phi \exp(i\theta'\gamma_5)\psi$. The γ_5 phase factors are usually believed to violate both P and T.

It is not widely known that these mass (and interaction) terms possess parity invariance [2] with a modified quark transformation. Here we shall demonstrate time-reversal invariance. A Euclidean space-time version of this invariance also exists. This is necessary in setting up the Euclidean functional integral. A measure for quark functional integration can be constructed to be invariant under this Euclidean transformation in addition to strong and electromagnetic gauge transformations. This means that the time-reversal invariance has no QCD or even QED anomaly. Any breaking of the invariance must be from other sources.

The requirement of time-reversal invariance of a fermionic action amounts to demanding that

$$\mathcal{T}\mathcal{L}(t)\mathcal{T}^{-1} = \mathcal{L}(-t). \tag{2}$$

Under the antilinear transformation $\mathcal{T}$, the gauge fields $A_0(t) \to A_0(-t)$, $A_i(t) \to -A_i(-t)$, while the fermion fields are taken to transform as

$$\mathcal{T}\psi(t)\mathcal{T}^{-1} = T\psi(-t) \tag{3}$$

with suitable matrices T. If there exists a T preserving (2), one has time-reversal invariance, otherwise it is broken.

For real quark mass, we consider $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$, where we keep in D only vector gauge interactions with photons and gluons. The weak interactions are understood to be treated separately in a perturbative manner. One finds that the matrices T have to obey

$$T^\dagger T = 1, \quad -T^\dagger\gamma^{0*}\gamma^{i*}T = \gamma^0\gamma^i, \quad T^\dagger\gamma^{0*}T = \gamma^0. \tag{4}$$

In the standard representation of gamma matrices, γ^2 is purely imaginary, while the rest are real. Then one finds $T = i\gamma^1\gamma^3$, the standard T-transformation for fermions.

For complex mass terms, the situation changes: the Lagrangian $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m\mathrm{e}^{i\theta'\gamma_5})\psi$ leads to the requirement

$$T^\dagger T = 1, \quad -T^\dagger\gamma^{0*}\gamma^{i*}T = \gamma^0\gamma^i, \quad T^\dagger\gamma^{0*}\mathrm{e}^{-i\theta'\gamma_5^*}T = \gamma^0\mathrm{e}^{i\theta'\gamma_5}. \tag{5}$$

These are satisfied by

$$T = i\mathrm{e}^{i\theta'\gamma_5}\gamma^1\gamma^3.$$

To investigate the measure, one goes to the Euclidean metric. Then one has a *linear* time-inversion instead of the antilinear time-reversal; $A_0(t) \to -A_0(-t)$, $A_i(t) \to A_i(-t)$. Further, $\psi, \bar{\psi}$ are independent fields, so that there are two transformation matrices $T, \bar{T}$:

$$\psi(t) \to T\psi(-t), \quad \bar{\psi}(t) \to \bar{\psi}(-t)\bar{T}. \tag{6}$$

γ_0 is also altered: all gamma matrices become antihermitian and can be made hermitian by absorbing i. In this situation, the conditions for invariance are

$$-\bar{T}\gamma^0 T = \gamma^0, \quad \bar{T}\gamma^i T = \gamma^i, \quad \bar{T}T = 1, \tag{7}$$

yielding $\bar{T} = T = i\gamma^1\gamma^2\gamma^3$ when the mass term is real, and

$$-\bar{T}\gamma^0 T = \gamma^0, \quad \bar{T}\gamma^i T = \gamma^i, \quad \bar{T}\mathrm{e}^{i\theta'\gamma_5}T = \mathrm{e}^{i\theta'\gamma_5}, \tag{8}$$

leading to

$$T = i\mathrm{e}^{-i\theta'\gamma_5}\gamma^1\gamma^2\gamma^3, \quad \bar{T} = i\mathrm{e}^{i\theta'\gamma_5}\gamma^1\gamma^2\gamma^3, \tag{9}$$

when the mass term is complex.

We come now to the construction of a measure for fermion fields in Euclidean space-time and define the functional integral by

$$Z = \int \mathcal{D}A \prod_n \int \mathrm{d}a_n \prod_n \int \mathrm{d}\bar{a}_n \mathrm{e}^{-S}. \tag{10}$$

For a real quark mass term, $a_n, \bar{a}_n$ are defined [1] by

$$\psi = \sum_n a_n \phi_n, \quad \bar{\psi} = \sum_n \bar{a}_n \phi_n^\dagger, \tag{11}$$

with ϕ_n being eigenfunctions of $i\gamma^\mu D_\mu$.

For a complex mass term, it turns out to be necessary [3] to expand the quark fields as

$$\psi = \mathrm{e}^{-(i\beta\gamma^5/2)} \sum_n a_n \phi_n, \quad \bar{\psi} = \sum_n \bar{a}_n \phi_n^\dagger \mathrm{e}^{-(i\beta\gamma^5/2)}, \tag{12}$$

with the phase β to be determined. This is a β-dependent family of measures. Note that θ', β can be removed from the functional integral by transforming $a, \bar{a}$ at the expense of changes in the $\theta F\tilde{F}$ term. The effective parity and T violation parameter turns out to be

$$\bar{\theta} = \theta - \theta' + \beta. \tag{13}$$

β is fixed by noting that under the time-inversion operation for gauge fields,

$$\phi_n(x_0, \vec{x}) \to i\gamma^1\gamma^2\gamma^3 \phi_n(-x_0, \vec{x}), \tag{14}$$

$$\phi_n^\dagger(x_0, \vec{x}) \to \phi_n^\dagger(-x_0, \vec{x}) i\gamma^1\gamma^2\gamma^3, \tag{15}$$

so that

$$\begin{aligned} \left(\mathrm{e}^{-(i\beta\gamma^5/2)}\phi_n(x_0, \vec{x})\right) &\to i\mathrm{e}^{-i\beta\gamma^5}\gamma^1\gamma^2\gamma^3 \left(\mathrm{e}^{-(i\beta\gamma^5/2)}\phi_n(-x_0, \vec{x})\right), \\ \left(\phi_n^\dagger(x_0, \vec{x})\mathrm{e}^{-(i\beta\gamma^5/2)}\right) &\to \left(\phi_n^\dagger(-x_0, \vec{x})\mathrm{e}^{-(i\beta\gamma^5/2)}\right) i\mathrm{e}^{i\beta\gamma^5}\gamma^1\gamma^2\gamma^3. \end{aligned} \tag{16}$$

The T-invariance of the measure or $a_n, \bar{a}_n$ requires the consistency of the transformation matrices in (16) with (9). Like parity invariance [3], this is achieved only with

$$\beta = \theta', \tag{17}$$

for which we have the interesting consequence [3] that

$$\bar{\theta} = \theta - \theta' + \beta = \theta, \tag{18}$$

providing a resolution of the strong T or strong CP problem [2].

We have shown that even when the mass term is complex, it is possible to define a time-reversal transformation which keeps it (as well as the kinetic terms and vector gauge interactions) invariant. This time-reversal is not anomalous because it (or its euclidean version) can be preserved by an appropriate quark measure. As regards the interaction of the quark with the physical Higgs particle, the phase in this Yukawa term is the same as the phase in the mass term which is generated from the vacuum expectation value of the Higgs field. Consequently, the time-reversal transformation defined in this paper also keeps the quark–Higgs interaction term invariant. Time-reversal is of course expected to be broken elsewhere: in the charged current weak interactions through the CKM matrix and in the gluon sector through the vacuum angle θ if it is non-zero. But the phase θ' in the quark mass term left after diagonalization by $SU(3) \times SU(3)$ matrices does not break T.

References

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