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Supersymmetry breaking with extra dimensions

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Abstract. This talk reviews some aspects of supersymmetry breaking in the presence of extra dimensions. The first part is a general introduction, recalling the motivations for supersymmetry and extra dimensions, as well as some unsolved problems of four-dimensional models of supersymmetry breaking. The central part is a more focused introduction to a mechanism for (super)symmetry breaking, proposed first by Scherk and Schwarz, where extra dimensions play a crucial role. The last part is devoted to the description of some recent results and of some open problems.

Keywords. Supersymmetry; supergravity; extra dimensions.

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1. General introduction

As a solid starting point for any speculation, we can take the standard model (SM) of strong and electroweak interactions coupled to Einstein gravity, an effective theory which accounts for all the observed interactions at the presently available energies. On the one hand, such theoretical framework is unsatisfactory: besides the large number of parameters, there is no explanation for the origin and the stability of two different mass scales that are many orders of magnitude smaller than the Planck scale of gravitational interactions, $M_{\mathrm{P}} \equiv (8\pi G_{\mathrm{N}})^{-1/2} \sim 2\times10^{18}$ GeV. The hierarchy problem refers to the Fermi scale of weak interactions, $M_{\mathrm{weak}} \equiv G_{\mathrm{F}}^{-1/2} \sim 250$ GeV. The cosmological constant problem refers to the scale of the vacuum energy density, $\Lambda_{\mathrm{cosm}} \sim 10^{-4}\,$eV. Why are the two ratios $M_{\mathrm{weak}}/M_{\mathrm{P}}$ and $\Lambda_{\mathrm{cosm}}/M_{\mathrm{P}}$ so incredibly small? On the other hand, even after many impressive recent experimental results, especially for neutrinos and heavy quarks, the SM provides a more than adequate description of flavour physics (after the obvious modifications to its minimal version needed to account for neutrino masses and oscillations): the only price to pay is the large number of parameters fitted to experiment.

At present, the most popular extension of the SM is the minimal supersymmetric standard model (MSSM), which incorporates explicitly but softly broken supersymmetry (SUSY). For some standard reviews of SUSY and of the MSSM, with lists of original references, see e.g. refs [1] and [2], respectively. The main virtue of the

MSSM is to ameliorate the hierarchy problem, linking the Fermi scale to the scale of the SUSY-breaking mass splittings, $M_{\rm weak} \sim \Delta m_{\rm SUSY}$. However, there is still no dynamical explanation of the large hierarchy between $\Delta m_{\rm SUSY}$ and $M_{\rm P}$, which is introduced 'by hand' via the soft SUSY-breaking terms. As for the cosmological constant problem, the improvement is irrelevant: the natural value of the cosmological constant scale in the MSSM is $\Lambda_{\rm cosm} \sim \sqrt{\Delta m_{\rm SUSY} \cdot M_{\rm P}}$, a big improvement over the SM expectation $\Lambda_{\rm cosm} \sim M_{\rm P}$, but still very far from the experimental value. Moreover, the MSSM brings in a new, severe flavour problem, associated with the very large number of parameters of its soft SUSY-breaking sector. While the MSSM is useful for parametrizing the exciting possibility of SUSY particles at the Fermi scale, it is clear that to make theoretical progress we must move to models where SUSY breaking is spontaneous rather than explicit.

1.1 *Four-dimensional models with spontaneously broken supersymmetry*

Any realistic model of the fundamental interactions cannot ignore gravity. The only consistent way of coupling SUSY to gravity is to promote the former to a local symmetry, i.e., to consider supergravity. The minimal conceivable framework is then a four-dimensional (4D) model with simple ($N = 1$) local supersymmetry. Such a model must obviously include the MSSM supermultiplets: matter chiral multiplets, with spin-1/2 quarks and leptons plus their complex spin-0 superpartners, the squarks and the sleptons; two Higgs chiral multiplets, with two scalar Higgs doublets of opposite hypercharge plus their spin-1/2 superpartners, the higgsinos; the vector multiplets of $SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}$, containing the SM gauge bosons plus their spin-1/2 superpartners, the gauginos. In addition, at least two other supermultiplets must be present: one is the gravitational multiplet, describing the massless graviton with helicities $h = \pm 2$ and the massless gravitino (the gauge fermion of local supersymmetry) with $h = \pm 3/2$; the other is the multiplet whose spin-1/2 component is the goldstino $\widetilde{G}$ (the Goldstone fermion of spontaneously broken SUSY, which provides the $h = \pm 1/2$ degrees of freedom to the massive gravitino), accompanied by its real spin-1 or complex spin-0 bosonic superpartners.

The general, 'kinematical' aspects of spontaneous SUSY breaking are well-understood, both in the global and in the local case: in a $N = 1$, 4D theory with chiral and vector supermultiplets, the order parameters controlling SUSY breaking are the VEVs of their F and D auxiliary fields, which give a positive semi-definite contribution to the scalar potential. For SUSY breaking to be compatible with a flat space-time background, the inclusion of gravity is essential, since in supergravity the scalar potential reads

$$V = ||F||^2 + ||D||^2 - ||H||^2. \tag{1}$$

The three terms $||F||^2$, $||D||^2$ and $||H||^2$ are positive-semidefinite, and controlled by the auxiliary fields of the chiral, vector and gravitational supermultiplets, respectively. The first two terms have different expressions but identical roles in local and global supersymmetry; the third one, peculiar to supergravity, has the universal property that $\langle ||H||^2 \rangle = 3\, m_{3/2}^2 M_{\rm P}^2$, where $m_{3/2}$ is the gravitino mass.

As will be clear in a moment, to generate phenomenologically acceptable masses for the SUSY partners of ordinary particles, a realistic model must have

$$\Lambda_{\rm SUSY} \equiv \langle ||F||^2 + ||D||^2 \rangle^{1/4} \gtrsim M_{\rm weak}. \tag{2}$$

On the other hand, to satisfy the bounds on the cosmological constant, a realistic model must also have

$$\Lambda_{\rm cosm} \equiv \langle V \rangle^{1/4} \lesssim 10^{-4}\ {\rm eV} \sim M_{\rm weak}^2 / M_{\rm P}. \tag{3}$$

It is then obvious that, when discussing the vacuum energy, the gravitational contribution to the scalar potential must be essentially identical to the non-gravitational one. Therefore, gravitational effects are likely to be crucial for the vacuum selection (given the vacuum, however, there are situations in which gravity can be neglected to discuss the spectrum and the interactions relevant for accelerator experiments).

On a SUSY-breaking vacuum, the goldstino is a linear combination of the spin-1/2 fermions in the chiral and vector multiplets, weighted by the VEVs of the corresponding F and D auxiliary fields. The mass splittings in the different sectors are schematically given by

$$\Delta m_{\rm SUSY}^2 \sim \lambda\, \Lambda_{\rm SUSY}^2, \tag{4}$$

where λ is the effective coupling of the goldstino multiplet to the sector under consideration. For SUSY to address the hierarchy problem, it is customary to require that the mass splittings among the MSSM states be $\Delta m_{\rm SUSY} \sim M_{\rm weak}$. However, this is not sufficient to fix $\Lambda_{\rm SUSY}$ or, equivalently, $m_{3/2}$ (to an excellent approximation, $\Lambda_{\rm SUSY} = \sqrt{3\, m_{3/2} M_{\rm P}}$): according to the numerical values of the effective couplings λ, different possibilities arise. If the goldstino couplings are of gravitational strength, $\lambda \sim M_{\rm weak}/M_{\rm P}$, then we must have a high scale of SUSY breaking, $\Lambda_{\rm SUSY} \sim \sqrt{M_{\rm weak} M_{\rm P}}$, and $m_{3/2}^2 \sim M_{\rm weak}$; if instead the goldstino couplings are of order one, $\lambda \sim 1$, then we must have a low scale of SUSY breaking, $\Lambda_{\rm SUSY} \sim M_{\rm weak}$, and $m_{3/2}^2 \sim M_{\rm weak}^2/M_{\rm P}$; intermediate cases are of course possible and discussed in the literature.

The generic structure of $N = 1$, 4D supergravity leaves a number of unsolved problems. The most important one is the vacuum energy, both at the classical and at the quantum level: its generic size is $\mathcal{O}(m_{3/2}^2 M_{\rm P}^2)$, better than in non-SUSY quantum theories of gravity, but still much worse than what is required by experiment. Also, we still lack a dynamical explanation of the hierarchy $m_{3/2}/M_{\rm P} \lesssim 10^{-15}$: if the only explicit mass scale in the theory is $M_{\rm P}$, without fine-tuning or some special properties we would expect $m_{3/2} \sim M_{\rm P}$, not $m_{3/2} \lesssim M_{\rm weak}$. In addition, the universality of the soft mass terms for squarks and sleptons (or an equivalent condition for solving the SUSY flavour problem) is not guaranteed in the presence of the most general couplings of supergravity to matter. In summary, a generic $N = 1$, 4D supergravity theory is unsatisfactory: since we are dealing with an effective non-renormalizable theory, with intrinsic limitations in its predictive power, it is difficult to imagine that a compelling solution to these problems can be found without some input from a more fundamental theory.

An appealing class of supergravity models, which suggest a possible solution to the general problems of $N = 1$, 4D supergravity, are the so-called no-scale models [3]. Incidentally, they were inspired from the very beginning by the properties of extended $N > 1$ supergravities, which are in turn related, by dimensional reduction, to higher-dimensional ($D > 4$) supergravities.

Before illustrating the main features of no-scale models, we need to recall some useful formulae of $N = 1$, 4D supergravity. Leaving aside, for simplicity, the discussion of gauge superfields and their interactions, the theory is defined by a real function of the chiral superfields,

$$G(\varphi, \overline{\varphi}) = K(\varphi, \overline{\varphi}) + \log |w(\varphi)|^2, \tag{5}$$

conventionally decomposed in a Kähler potential K and a superpotential w. In the standard supergravity notation, $M_{\rm P} = 1$ and $w_m \equiv \partial w / \partial \varphi^m$, $K_{\overline{m}n} \equiv \partial^2 K / \partial \overline{\varphi}^{\overline{m}} \partial \varphi^n$, ... In particular, the Kähler metric $K_{\overline{m}n}$ controls the generalized kinetic terms for the fermions and the scalars in the chiral supermultiplets. The classical scalar potential is then given by

$$V = {\rm e}^G \left(G_m G^{m\overline{n}} G_{\overline{n}} - 3 \right), \tag{6}$$

and 4D Minkowski vacua correspond to $\langle V \rangle = 0$. As for SUSY breaking, the auxiliary field F_m is proportional to G_m, which in turn is proportional to $w_m + K_m w$. The field-dependent gravitino mass $m_{3/2}$ is

$$m_{3/2}^2 = {\rm e}^G = |w|^2 {\rm e}^K. \tag{7}$$

The simplest form of no-scale model contains a single chiral superfield T, whose Kähler potential $K = -3 \log(T + \overline{T})$ parametrizes a $SU(1,1)/U(1)$ manifold. Obviously, the allowed field configurations are such that $T + \overline{T} > 0$. If the superpotential is T-independent, $w = k \neq 0$, then the classical potential V is flat, $V \equiv 0$, for any allowed value of T. Moreover, any such configuration corresponds to spontaneously broken SUSY, $F_T \neq 0$. The gravitino mass, $m_{3/2}^2 = |k|^2/(T + \overline{T})^3$, which controls the SUSY-breaking scale, slides along such a flat direction.

The most interesting feature of the no-scale structure is that it guarantees a classically vanishing cosmological constant, but it is not the only one. When coupled to MSSM chiral multiplets, denoted here generically with C, no-scale models do allow for universal SUSY-breaking masses: it is sufficient to have a Kähler potential with a scaling law of the form $\Delta K = (T + \overline{T})^{-n} |C|^2$, then universal scalar masses of the form $\widetilde{m}_C^2 = (n-1) m_{3/2}^2$ are generated [4]. If quantum corrections to the effective potential can be shown to be $\mathcal{O}(m_{3/2}^4)$, not $\mathcal{O}(m_{3/2}^2 M_{\rm P}^2)$ as expected on general grounds, then the $m_{3/2}/M_{\rm P}$ hierarchy can be generated via dimensional transmutation [5], as in the Coleman–Weinberg mechanism. The crucial ingredients are the quantum corrections due to the light fields: the interplay between gauge and Yukawa couplings may generate an effective infrared fixed point for the quantum effective potential, $V_{\rm eff}[m_{3/2}(T), H]$, where H is the generic MSSM Higgs fields (for a more complete discussion, see [6]). At this 4D level, however, we cannot hope to understand the origin of the required special properties of K and w, and we have no control over ultraviolet quantum corrections. We shall see that further progress can occur with the help of extra dimensions.

1.2 *Extra dimensions*

Theories formulated in more than four space-time dimensions have been discussed for several decades, starting from the historical papers by Kaluza and Klein on a possible unified theory of gravity and electromagnetism, and continuing with the attempts at finding realistic compactifications of eleven-dimensional supergravity (for reviews and references on these traditional aspects of extra dimensions, with and without supersymmetry, see [7]). Nowadays, the possible existence of extra dimensions is strongly motivated by superstring theories (for reviews and references, see [8]), our present best candidates for a consistent quantum theory unifying gravity with all other interactions. Moreover, developments in field theory and superstring theories have opened up many unexpected phenomenological possibilities (for reviews on these recent developments, see [9]). For example, the standard way of hiding the extra dimensions and making them phenomenologically viable was compactification; we have recently learned that this mechanism can be combined with other mechanisms that lead to the localization of some of the fields on lower-dimensional subspaces (branes) immersed in the full higher-dimensional space (bulk). In some cases, the new developments have prompted very intriguing reformulations (not yet solutions) of the hierarchy problem.

To get a feeling of how extra dimensions may be used to generate hierarchical ratios of scales, we can consider the simple case of one, flat extra dimension, and start with pure 5D Einstein gravity, with coordinates $x^M \equiv (x^m, x^5 \equiv y)$. We can then compactify such a theory on the circle S^1, identifying the points of coordinates y and $y + 2\pi\kappa$ (κ is an arbitrary constant with the dimension of a length, it is not restrictive to set it equal to the 5D Planck length), and decomposing the 5D metric G_{MN} according to the 4D Lorentz group as follows:

$$G_{MN} = \begin{pmatrix} \varphi^{-1} g_{mn} - \varphi^2 A_m A_n & \varphi^2 A_m \\ \varphi^2 A_n & \varphi^2 \end{pmatrix}. \tag{8}$$

It can be easily checked that the constant background defined by

$$\langle g_{mn} \rangle = \eta_{mn}, \quad \langle A_m \rangle = 0, \quad \langle \varphi \rangle = r, \tag{9}$$

solves the 5D vacuum Einstein's equations for any value of the physical compactification radius $R = \kappa r$. Moreover, as will be discussed in more detail below, the resulting 4D spectrum is characterized by the compactification scale, associated with the inverse radius R^{-1}. To discuss a possible hierarchy of scales, then, we must discuss the additional dynamics, not contained in the classical Einstein's equations, that may fix the value of R while preserving 4D Lorentz invariance, in the full 5D theory including gravity. This is a highly non-trivial problem, which keeps busy a significant fraction of the theoretical community: several interesting ideas have been proposed recently, but no completely satisfactory model exists (for a complementary point of view, more details and references to recent work, see [10]).

2. Symmetry breaking by coordinate-dependent compactifications

We are now ready for a more focused introduction to the special (super)symmetry breaking mechanisms that can occur in field (and string) theories with extra spatial dimensions. These are the so-called coordinate-dependent compactifications, first proposed by Scherk and Schwarz [11], which provide an elegant and efficient mechanism for mass generation and (super)symmetry breaking. The basic idea is very simple: if we twist the periodicity conditions in the compact extra dimensions by a symmetry of the action (or, more generally, of the equations of motion), then, from a 4D point of view, this twist induces mass terms that break the symmetries with which it does not commute.

The simplest illustrative example is the 5D theory of a free, massless complex scalar, described by the Lagrangian density

$$\mathcal{L} = \left(\partial^M \phi\right)^\dagger \left(\partial_M \phi\right). \tag{10}$$

Such theory has the global $U(1)$ symmetry:

$$\phi' = \mathrm{e}^{-i\beta}\, \phi\,, \qquad (\beta \in \mathbf{R}). \tag{11}$$

We choose now to compactify the theory on a circle of radius R, identifying the points with coordinates y and $y + 2\pi R$, for any value of the y coordinate. The standard Kaluza–Klein compactification requires strict periodicity conditions on the fields:

$$\phi(x, y + 2\pi R) = \phi(x, y)\,. \tag{12}$$

The field ϕ can then be expanded in Fourier series as

$$\phi(x,y) = \sum_{n \in Z} \varphi_n(x) \mathrm{e}^{iny/R}. \tag{13}$$

Substituting the above expansion in the action, acting with the y-derivatives and finally integrating over y, we find 4D masses of the form:

$$m_n^2 = \frac{n^2}{R^2}\,, \qquad (n \in Z)\,, \tag{14}$$

which give the standard Kaluza–Klein spectrum. However, the global $U(1)$ symmetry of the action allows for a more general choice of periodicity conditions, which may involve a twist by a phase β:

$$\phi(x, y + 2\pi R) = \mathrm{e}^{-i\beta}\phi(x,y). \tag{15}$$

To respect this twisted periodicity, the Fourier expansion of ϕ must be suitably modified:

$$\phi(x,y) = \mathrm{e}^{-i\beta y/2\pi R} \sum_{n \in Z} \varphi_n(x) \mathrm{e}^{iny/R}\,. \tag{16}$$

We are then led to a shifted Kaluza–Klein spectrum for the different 4D modes:

$$m_n^2 = \left(\frac{n}{R} - \frac{\beta}{2\pi R}\right)^2, \qquad (n \in Z)\,. \tag{17}$$

Early applications [12] of the mechanism focused on extended supergravity theories, for example those obtained from 11D supergravity in the limit of generalized dimensional reduction, i.e., neglecting all heavy Kaluza–Klein modes. The first phenomenological studies [13] had to face the problem of generating a chiral 4D spectrum, as in the SM or in the MSSM. Nowadays, we know how to solve such a problem: the easiest way out is to mix coordinate-dependent compactifications with orbifolds [14], where space-time points connected by a (non-freely acting) discrete symmetry of the action are identified.

The full higher-dimensional features of coordinate-dependent compactifications were only taken into account in the first string models with spontaneous SUSY breaking [15–17]. Chiral heterotic string orbifold models were formulated, with many desirable features [17]: gauge and gravity fields were in the bulk, matter fields were localized at the orbifold fixed points; the classical SUSY-breaking spectrum had vanishing masses for the matter scalars and universal contributions, equal to the gravitino mass, to the bulk gaugino masses and to the equivalent of the MSSM μ-term; the effective 4D supergravity theories were found to be no-scale models, with classically vanishing vacuum energy and the compactification radius R as a classical flat direction. SUSY breaking was recognized to be spontaneous, and the goldstinos identified with the internal components of the 5D gravitinos.

Coming back to generic features of SUSY breaking by coordinate-dependent compactifications, it was very important to realize that the non-locality of the order parameter (as in the case of gauge-symmetry breaking via Wilson loops [18]) improves the ultraviolet behavior of symmetry-breaking quantities [15,19]. Such behavior would be out of theoretical control if we were to consider the reduced 4D effective theory for the light modes only, instead of the compactified higher-dimensional one, with its full tower of Kaluza–Klein excitations.

After many years, coordinate-dependent compactifications found several different superstring realizations [20], and there is by now a vast literature on related phenomenological field-theory models (for a review appeared soon after this conference and a list of references, see [21]). For reasons of time, the rest of this talk will focus on some recent results obtained by the author in collaboration with Bagger, Biggio, Feruglio and Wulzer [22–24]. Concentrating, for simplicity, on some 5D effective field theories compactified on the orbifold S^1/Z_2, we will discuss the following two topics: the general structure of the 5D mass terms induced by coordinate-dependent compactifications [24], and the possibility of localized mass terms for bulk fields at the orbifold fixed points [22]; the superHiggs effect in 5D supergravity, the phenomenon of brane-induced SUSY breaking [23], and the possible relation between the Scherk–Schwarz mechanism and gaugino condensation in M-theory. In conclusion, we will briefly comment on some prospects for future work.

3. General structure of 5D mass terms

To illustrate the general structure of the 5D mass terms generated by the Scherk–Schwarz mechanism, we consider a 5D field theory compactified on the orbifold S^1/Z_2. On the real axis, we identify points connected by a $2\pi R$ translation and by a reflection about the origin:

$$T: \quad y \longrightarrow y + 2\,\pi\,R\,, \quad Z_2: \quad y \longrightarrow -y\,. \tag{18}$$

For definiteness, we focus on a 5D massless spinor $\Psi(x^m, y)$, and ignore the gravitational degrees of freedom associated with the fluctuations of the 5D metric around the flat Minkowski background. Working in terms of 5D fields with 4D spinor indices, we define the Z_2 transformation properties of our spinor by

$$\Psi(-y) = Z\,\Psi(y)\,, \tag{19}$$

where

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \overline{\Psi} = \begin{pmatrix} \overline{\psi_1} \\ \overline{\psi_2} \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \widehat{\sigma}^3\,, \tag{20}$$

with ψ_1 (even) and ψ_2 (odd) 4D Weyl spinors.

The system under consideration is described by the 5D Lagrangian:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm int}\,, \tag{21}$$

where $\mathcal{L}_0$ is the free massless Lagrangian for Ψ,

$$\mathcal{L}_0 = i\overline{\Psi}^T \overline{\sigma}^m \partial_m \Psi - \frac{1}{2}\left(i\,\Psi^T \widehat{\sigma}^2 \partial_y \Psi + \text{h.c.}\right)\,, \tag{22}$$

whilst $\mathcal{L}_{\rm int}$ contains possible interaction terms for the field Ψ, and in general may depend on additional 5D fields. For the consistency of the orbifold construction, both $\mathcal{L}_0$ and $\mathcal{L}_{\rm int}$ must be invariant under Z_2. The free Lagrangian $\mathcal{L}_0$ is also invariant under

$$\Psi'(y) = U\,\Psi(y)\,, \tag{23}$$

where U is a global $SU(2)$ transformation. We require that $\mathcal{L}_{\rm int}$ is $SU(2)$ invariant also.

In this framework, the field $\Psi(y)$ does not need to be periodic in y. It can be periodic up to a global $SU(2)$ transformation, with twisted periodicity conditions:

$$\Psi(y + 2\pi R) = U_{\vec{\beta}} \Psi(y)\,. \tag{24}$$

For the present purposes, it is not restrictive to take

$$U_{\vec{\beta}} \equiv \mathrm{e}^{i\beta\hat{\sigma}^2}\,, \quad (0 < \beta < \pi)\,. \tag{25}$$

At this point, all the physics is completely determined by the Lagrangian $\mathcal{L}(\Psi, \partial\Psi)$ and the twist β. For example (assuming for simplicity that all fields in $\mathcal{L}_{\rm int}$ have trivial background values), the 4D modes have a classical spectrum characterized by a universal shift of the Kaluza–Klein levels, with respect to the mass eigenvalues n/R of the periodic case:

$$m = \frac{n}{R} - \frac{\beta}{2\pi R}\,, \quad (n \in \mathbf{Z})\,. \tag{26}$$

We can now move to a basis of periodic fields by performing the local field redefinition

$$\Psi(y) = V(y)\,\widetilde{\Psi}(y)\,, \quad \widetilde{\Psi}(y+2\pi R) = \widetilde{\Psi}(y)\,, \tag{27}$$

where $V(y)$ must then be a 2×2 matrix satisfying

$$V(y+2\pi R) = U_{\vec{\beta}}\,V(y)\,, \tag{28}$$

as can be immediately checked from eqs (24) and (27). Besides condition (28), we will impose for our convenience two additional constraints on the matrix $V(y)$. One is

$$V(y) \in SU(2)\,, \tag{29}$$

to preserve canonical kinetic terms, and the other is

$$\begin{cases} V_{ij}(-y) = +V_{ij}(y) & (ij = 11, 22) \\ V_{ij}(-y) = -V_{ij}(y) & (ij = 12, 21) \end{cases}\,, \tag{30}$$

to preserve the Z_2 parities of the fields.

It is important to observe that the solution to the above conditions is by no means unique. For each allowed choice of $V(y)$, we obtain an equivalent 5D theory with periodic fields:

$$\mathcal{L}(\Psi,\partial\Psi) = \mathcal{L}(\widetilde{\Psi},\partial\widetilde{\Psi}) + \Big\{-\frac{i}{2}\,[m_1(y) + i\,m_2(y)]\,\widetilde{\psi}_1\widetilde{\psi}_1 + \frac{i}{2}\,[m_1(y) - i\,m_2(y)]\,\widetilde{\psi}_2\widetilde{\psi}_2 + i\,m_3(y)\,\widetilde{\psi}_1\widetilde{\psi}_2 + \text{h.c.}\Big\}\,, \tag{31}$$

where the mass terms $m_a(y)$ ($a = 1,2,3$) are the coefficients of the Maurer–Cartan form

$$m(y) \equiv m_a(y)\,\widehat{\sigma}^a = -i\,V^\dagger(y)\partial_y V(y)\,, \tag{32}$$

and satisfy

$$m_a(y+2\pi R) = m_a(y)\,, \quad m_a(y) \in \mathbf{R}\,, \tag{33}$$

$$m_{1,2}(-y) = +m_{1,2}(y)\,, \quad m_3(-y) = -m_3(y)\,. \tag{34}$$

Where is the physically relevant information on the twist β, in the formulation of the theory on the right-hand side of eq. (31)? It is easy to check that it is contained in the Wilson loop:

$$\cos\beta = \frac{1}{2}\mathrm{tr}\,P\left[\exp\left(i\int_y^{y+2\pi R} \mathrm{d}y' m(y')\right)\right]\,, \tag{35}$$

where P denotes a suitably defined path-ordering [24]. Notice that the mass terms in eq. (31) are of three different types, but do not correspond to the most general set of y-dependent mass terms allowed by 4D Lorentz invariance, which would be characterized by three independent complex functions. To summarize, we have obtained a class of interacting theories, with periodic fields and mass terms, which are all equivalent to the original interacting theory with twisted fields and no mass terms.

Also the converse is true. Given a Lagrangian as on the right-hand side of eq. (31), expressed in terms of periodic fields, we can move to an equivalent Lagrangian where all mass terms have been removed, and the fields satisfy the generalized periodicity conditions of eq. (24), by performing the inverse field redefinition of eq. (27). It can be shown that $V(y)$ is given by

$$V(y) = V(0)\, P \left[\exp \left(i \int_0^y \mathrm{d}y' m(y') \right) \right] . \tag{36}$$

From the above discussion, it should be clear that mass 'profiles' $m(y)$ for periodic fields do not have an absolute physical meaning, what matters is just the twist β. We can make use of this freedom to show that $m_{1,2}(y)$ can be localized at the fixed points $y = 0$ and/or $y = \pi R$, without affecting the physical properties of the theory. We recall first the standard choice for $V(y)$:

$$V^O(y) = \exp \left(i\, \beta \widehat{\sigma}^2 \frac{y}{2\pi R} \right) , \tag{37}$$

the symbol 'O' standing for 'ordinary'. Applying eq. (32) to $V^O(y)$, we find the constant mass profile:

$$m_1^O(y) = m_3^O(y) = 0\, , \quad m_2^O(y) = \frac{\beta}{2\pi R} . \tag{38}$$

However, generalized choices are possible, for example all those giving

$$m_1^G(y) = m_3^G(y) = 0\, , \quad m_2^G(y) \neq 0\, , \tag{39}$$

where $m_2^G(y)$ is an otherwise arbitrary real, periodic, even function of y, as long as it has the property that

$$\int_y^{y+2\pi R} \mathrm{d}y' m_2^G(y') = \int_y^{y+2\pi R} \mathrm{d}y' m_2^O(y') = \beta\, . \tag{40}$$

Two representative and equivalent choices of $m_2^G(y)$ are illustrated in figure 1. The dashed line shows a mild (Gaussian) localization around the orbifold fixed points and the solid line shows a strong localization.

A possible special choice for $m_2(y)$ is the singular limit:

$$m_2^S(y) = \sum_{q=-\infty}^{+\infty} \left[\delta_0\, \delta(y - 2q\pi R) + \delta_\pi\, \delta(y - (2q+1)\pi R) \right] , \tag{41}$$

where $\delta_0 + \delta_\pi = \beta$ and what we actually mean is, as discussed in detail in [22,23], a suitably regularized version of the distribution in eq. (41). This description is

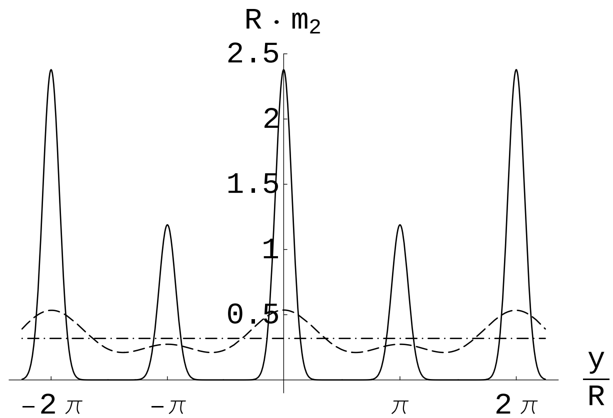

Figure 1. Two representative and equivalent choices for $m_2^G(y)$, corresponding to $\beta = 2$. The dash–dotted line shows the equivalent profile $m_2^O(y) = 1/(\pi R)$.

apparently quite remote from the 'ordinary' one. The mass terms vanish everywhere but at the orbifold fixed points, where there are localized contributions to $m_2(y)$. The matrix bringing from the massive periodic fields to the corresponding massless twisted 5D fields is now

$$V^S(y) = \exp\left[i\alpha(y)\widehat{\sigma}^2\right], \tag{42}$$

where

$$\alpha(y) = \frac{\delta_0 - \delta_\pi}{4}\,\epsilon(y) + \frac{\delta_0 + \delta_\pi}{4}\,\eta(y), \tag{43}$$

the function $\epsilon(y)$ is the periodic sign function, and $\eta(y)$ is the 'staircase' function

$$\eta(y) = 2q + 1, \quad q\pi R < y < (q+1)\pi R, \quad (q \in \mathbf{Z}). \tag{44}$$

Another simple but instructive example corresponds to a Lagrangian for periodic fields of the form in eq. (31), where now

$$m_1(y) = m_2(y) = 0, \quad m_3(y) \neq 0, \tag{45}$$

and $m_3(y)$ is an otherwise arbitrary real, odd, periodic function of y. Notice that, for any such function, eq. (35) gives always $\beta = 0$, since

$$\int_y^{y+2\pi R} \mathrm{d}y' m_3(y') = 0. \tag{46}$$

In other words, real, periodic, odd mass profiles can be completely removed by a field redefinition without introducing a non-trivial twist.

4. Application to 5D supergravity: The super-Higgs effect

The discussion of the previous section can be suitably generalized to the spontaneous breaking of pure 5D Poincaré supergravity compactified on the S^1/Z_2 orbifold [23,25]. For simplicity, we begin with pure $N=1$, $D=5$ Poincaré supergravity [26]. The 5D spinor of the previous section is then replaced by the 5D gravitino:

$$\Psi_M = \begin{pmatrix} \psi_{1M} \\ \psi_{2M} \end{pmatrix}, \quad (M = m, 5). \tag{47}$$

The bosonic superpartners of the gravitino are the fünfbein E_M^A and the graviphoton B_M, singlets under the global $SU(2)$ (R-)symmetry under which Ψ_M transforms as in eq. (23), with the same constant matrix U for every value of the index M. The Z_2 parity assignments to the fermionic fields are now $Z = \widehat{\sigma}^3$ for Ψ_m, $Z = -\widehat{\sigma}^3$ for Ψ_5, and can be consistently completed by assigning even parity to (E_m^a, E_5^5, B_5) and odd parity to (E_m^5, E_5^a, B_m). Notice that, besides the 4D graviton, there are also dilaton and axion zero modes, associated with the even E_5^5 and B_5 fields, often ignored in recent phenomenological studies. We expand the theory around a flat background, solution of the 5D equations of motion:

$$\langle E_M^A \rangle = \delta_M^A, \quad \langle \Psi_M \rangle = \langle B_M \rangle = 0. \tag{48}$$

The implementation of the Scherk–Schwarz twist on S^1/Z_2 and the derivation of the corresponding effective 5D theory for periodic fields can be discussed exactly as in the previous section and so we do not repeat all the details. Here we just comment on the new features and on the resulting structure of 5D gravitino mass terms (details can be found in [23–25]). The twisted boundary conditions on the gravitino can be written as

$$\Psi_M(y + 2\pi R) = U_{\vec{\beta}} \, \Psi_M(y), \quad (M = m, 5), \tag{49}$$

with $U_{\vec{\beta}}$ as in eq. (25). Similarly, the field redefinition bringing to the basis of periodic fields reads:

$$\Psi_M(y) = V(y) \, \widetilde{\Psi}_M(y), \quad (M = m, 5), \tag{50}$$

where $V(y)$ satisfies the same conditions as before.

The only term of the 5D Lagrangian involving derivatives of the gravitino is its generalized kinetic term. Since we are interested here in the structure of the gravitino mass terms, we set all the bosonic fields to their background values of eq. (48) and focus on the fermion bilinears. Up to a universal normalization factor, the Lagrangian is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}\epsilon^{mnpq}\overline{\Psi}_m^T \overline{\sigma}_n \partial_p \Psi_q \\ & + \left(-\frac{i}{2}\Psi_m^T \sigma^{mn} \widehat{\sigma}^2 \partial_y \Psi_n + i \Psi_5^T \sigma^{mn} \widehat{\sigma}^2 \partial_m \Psi_n + \text{h.c.} \right) + \ldots . \end{aligned} \tag{51}$$

After moving to the basis of periodic fields via the field redefinition of eq. (50), we get

$$\begin{aligned}\mathcal{L}(\Psi_M,\partial\Psi_M) &= \mathcal{L}(\widetilde{\Psi}_M,\partial\widetilde{\Psi}_M)\\ &\quad+\Big\{-\frac{i}{2}\,[m_1(y)+i\,m_2(y)]\,\widetilde{\psi}_{m1}\sigma^{mn}\widetilde{\psi}_{n1}\\ &\quad+\frac{i}{2}\,[m_1(y)-im_2(y)]\,\widetilde{\psi}_{2m}\sigma^{mn}\widetilde{\psi}_{2n}\\ &\quad+i\,m_3(y)\,\widetilde{\psi}_{1m}\sigma^{mn}\widetilde{\psi}_{2n}+\text{h.c.}\Big\}. \end{aligned}\tag{52}$$

From here, the discussion can proceed as in the simpler case of the previous section. The only important difference is that gravitino masses occur via the super-Higgs effect, with the goldstino components provided by $\widetilde{\Psi}_5$. To discuss the spectrum in the case of a non-trivial Scherk–Schwarz twist, $\beta \neq 0$, it is convenient to go to the unitary gauge, where $\widetilde{\Psi}_5$ completely disappears from the Lagrangian on the right-hand side of eq. (52). We could now repeat the whole discussion of the previous section. In particular, the singular case corresponds to gravitino mass terms entirely localized at the orbifold fixed points,

$$\begin{aligned}\mathcal{L}_{\text{mass}}(\widetilde{\Psi}_M,\partial\widetilde{\Psi}_M) &= \frac{1}{2}\,[\delta_0\,\delta(y)+\delta_\pi\delta(y-\pi R)]\\ &\quad\times\left(\widetilde{\psi}_{m1}\sigma^{mn}\widetilde{\psi}_{n1}+\widetilde{\psi}_{m2}\sigma^{mn}\widetilde{\psi}_{n2}+\text{h.c.}\right),\end{aligned}\tag{53}$$

where we can interpret the constants δ_0 and δ_π as the remnants of some non-perturbative localized brane dynamics, which may include gaugino condensation: this situation was called brane-induced SUSY breaking in [23].

It would be interesting to extend the above discussion, given for pure 5D supergravity, to the effective 5D supergravity corresponding to M-theory [27], compactified on a small Calabi–Yau manifold times a large orbifold S^1/Z_2. In particular, there are intriguing analogies [28,29] between non-perturbative SUSY breaking via gaugino condensation at the orbifold fixed points [28,30] and the previous description of brane-induced SUSY breaking as a singular generalization of the Scherk–Schwarz mechanism: there are localized gravitino mass terms at the orbifold fixed points, characterized by two independent constants δ_0 and δ_π; the effective 4D theory is of no-scale type, with the classical 4D vacuum energy identically vanishing and the compactification radius R a classical flat direction; the order parameter for SUSY breaking is the non-local quantity $\delta_0+\delta_\pi$, thus we can have one unbroken supersymmetry with $\delta_0 = -\delta_\pi \neq 0$; the goldstinos, absorbed by the massive gravitinos in the super-Higgs effect, are associated with the fifth components of the gravitinos. For gaugino condensation in M-theory, localized gravitino mass terms are induced into the effective 5D supergravity Lagrangian by the non-zero VEV of G_{ABCD}, the four-form of 11-dimensional supergravity. The VEV related with the gaugino condensate, because of a perfect square structure that appears in the Lagrangian, is $\langle G_{11abc}\rangle$, where $a,b,c = 1,2,3$ are holomorphic indices associated with the six-dimensional Calabi–Yau manifold. $\langle G_{11abc}\rangle$ is even under the Z_2 parity, and generates gravitino mass terms of the type of $m_2(y)$ in eq. (52). This is a strong hint for an equivalence: to prove it rigorously, however, some technical complications due to the presence of additional moduli fields with a non-trivial y-dependence should be addressed.

5. Conclusions and outlook

In this talk we have reviewed coordinate-dependent compactifications as a mechanism for SUSY breaking in higher-dimensional effective field theories (and also in superstring theories and in M-theory).

We have discussed on simple examples how seemingly different mass profiles can indeed describe the same physics, being related by suitable field redefinitions. In particular, Scherk–Schwarz compactifications on orbifolds can find a generalized description, involving localized mass terms for bulk fields at the orbifold fixed points. In the simple case of pure 5D supergravity, we have shown that brane-induced SUSY breaking admits a generalized Scherk–Schwarz interpretation.

There are several other aspects that would deserve further investigations: the possible extension of our analysis of brane-induced SUSY breaking to gaugino condensation in M-theory; the study of the quantum consistency of the different models, especially in connection with localized anomalies and last but not the least, the possible extension of the Scherk–Schwarz mechanism to warped compactifications.

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