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Lattice matrix elements and CP violation in B and K physics: Status and outlook

AMARJIT SONI
Physics Department, High Energy Theory Group, Brookhaven National Laboratory, Upton, NY 11973, USA
Email: soni@quark.phy.bnl.gov; soni@bnl.gov

Abstract. Status of lattice calculations of hadron matrix elements along with CP violation in B and in K systems is reviewed. Lattice has provided useful input which, in conjunction with experimental data, leads to the conclusion that CP-odd phase in the CKM matrix plays the dominant role in the observed asymmetry in $B \to \psi K_s$. It is now quite likely that any beyond the SM, CP-odd, phase will cause only small deviations in B-physics. Search for the effects of the new phase(s) will consequently require very large data samples as well as very precise theoretical predictions. Clean determination of *all* the angles of the unitarity triangle therefore becomes essential. In this regard $B \to KD^0$ processes play a unique role. Regarding K-decays, remarkable progress made by theory with regard to maintenance of chiral symmetry on the lattice is briefly discussed. First application already provide quantitative information on B_K and the $\Delta I = 1/2$ rule. In the lattice calculation, the enhancement in Re A_0 appears to arise solely from tree operators, esp. Q_2; penguin contribution to $\mathrm{Re}\, A_0$ appears to be very small. However, improved calculations are necessary for ϵ'/ϵ as the contributions of QCD penguins and electroweak penguins largely seem to cancel. There are good reasons, though, to believe that these cancellations will not survive improvements that are now underway. Importance of determining the unitarity triangle purely from K-decays is also emphasized.

Keywords. CP; lattice; B and K-unitarity triangles; chiral symmetry; $\Delta I = 1/2$ rule; ϵ'/ϵ.

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1. Introduction

With important input from the lattice along with the classic results of indirect CP violation in $K_L \to \pi\pi$, the asymmetric B-factories with measurements of CP asymmetry in $B \to \psi K_s$ are providing valuable support to the CKM paradigm of CP violation [1]. It is now clear that the CP-odd phase in the CKM matrix is the dominant source of CP violation in $B \to \psi K_s$. However, as is well-known essentially compelling theoretical arguments suggest that new CP-odd phase(s) should exist due to physics beyond the SM (BSM). At the same time there is no good reason to think that their effects in B-physics would be particularly large. Indeed, SM teaches a valuable lesson in this regard: even though the CKM phase causes a huge asymmetry (i.e. $O(1)$) in $B \to \psi K_s$, its effects in CP violation in K-decays is miniscule $\approx 10^{-3}$. Clearly this realization should motivate us to prepare for small deviations from the predictions of the SM in B-physics even if the new CP-odd phase is large. For this reason, not only we need very large data samples of Bs giving impetus to super-B factories along with BTeV and LHCB, we also need extremely precise tests of the SM. Residual theory errors are a serious cause of concern as they can easily thwart experimental efforts for search of BSM CP-odd phase(s) [2].

With this perspective in mind, a brief discussion of the lattice method and results for the hadronic matrix elements that are important for weak interaction phenomenology are given in §§1 to 4. Therein I also discuss some of the anticipated experimental input that could help attain greater precision in constraining CKM parameters. Section 5 emphasizes concern about residual errors in theory and the importance of clean determinations, i.e. without theoretical assumptions, of all the angles of the unitarity triangle. In this regard the special role of $B \to KD^0$ processes is also emphasized there.

Progress made in the past few years with regard to maintenance of exact chiral symmetry on the lattice is outlined in §6 along with application of this development to B_K.

Section 7 gives a brief report on the results from the first application of domain wall fermion method, which exhibit excellent chiral behavior, to $K \to \pi\pi$, $\Delta I = 1/2$ rule and ϵ'/ϵ. These first applications give good insight to the $\Delta I = 1/2$ rule; in particular, contribution of penguin operators to Re A_0 in our lattice calculations appears to be extremely small and most of the enhancement seen in Re A_0 is originating from the tree operator, Q_2. Unlike in the case of the $\Delta I = 1/2$ rule, the approximation currently used though appear too crude to give reliable information on ϵ'/ϵ. This difficulty arises as contributions of QCD penguins and electroweak penguins substantially cancel. However, there are very good reasons to suspect that this cancellation is not 'natural' and is unlikely to survive as calculations are improved.

For the purpose of stringent tests of the CKM model of CP violation a separate determination of the unitarity triangle purely from K-decays, to be compared to that obtained from B-physics, is highly desirable and this is finally emphasized in §8.

2. Lattice methodology: A very brief recapitulation

Recall, Green's functions are calculated by numerical evaluation of the Feynman path integral

$$\langle 0|Q|0\rangle = \frac{\int DU \; W(Q)_M \; \det \; M(U) \exp[-S_g(U)]}{\int DU \; \det \; M(U) \exp[-S_g(U)]}. \tag{1}$$

As it stands, dependence of the quark matrix M in this expression on the link variables renders its evaluation extremely difficult. To facilitate the numerical calculation, one often uses the quench approximation (QA) and sets det $M = 1$. Physically, this approximation corresponds to neglect of the $q\bar{q}$ vacuum polarization loops in the propagation of the gauge field. The hint that this may be a reasonable approximation originally came from deep inelastic scattering experiments wherein the effect of $q\bar{q}$ pairs in the 'sea' is accurate to about 15% [3]. There are, though, very good reasons that tell us that the accuracy of the QA in lattice computations is process dependent.

In the past several years more and more 'unquenched' simulations, i.e. those not using the QA so that dynamical $q\bar{q}$ pairs are included, have been underway. These studies show that QA seems to be valid to about 5–10% accuracy in non-singlet hadron spectrum [4].

On the other hand, dynamical quarks seem to increase the B-meson pseudoscalar decay constant quite appreciably, (at least when m_ρ is used to set the scale) [5]

$$f_B^{N_f=3}/f_B^{N_f=0} = 1.23 \pm 0.04 \pm 0.06. \tag{2}$$

In addition to the QA there are several other sources of systematic errors in a typical lattice gauge calculation. Chief among these are finite (box) size and finite lattice spacing (a) errors. Also most lattice simulations are done with rather large values of masses of light (uds) quarks and rather low values for the b quark mass, compared to their physical values. Painstaking and elaborate efforts become necessary to accurately extract from the data information relevant to the physical case. This may, for example, require extrapolation of the data (at a fixed gauge coupling, or lattice spacing) as a function of quark mass to the chiral limit and also extrapolation of the data as a function of the lattice spacing to the continuum limit (i.e. lattice spacing goes to zero). Furthermore, simulations for a fixed gauge coupling at two or more volumes are often needed for extrapolation to the infinite volume limit.

2.1 *Some examples of brute-force*

Relevant to this talk there are three works which serve to illustrate computational brute force used in bringing them to fruition; these are B_K, f_B and $K \to 2\pi$.

1. B_K: A major accomplishment of the lattice gauge effort, and in particular of the JLQCD group is their result [6,7], $\hat{B}_K = 0.860 \pm 0.058$ in the QA. During the first 6–7 years ('84–'91), several exploratory attempts were made [8]. Methodology was in place around '91 [9] which was followed by several years of intensive computations leading to the final result obtained around '98. In the past few years this important result has been the focus of further checks and confirmation using other fermion discretizations, i.e. Wilson [10] as well as the newer discretizations: domain wall fermions [11–13] and the overlap fermions [14,15]. The results of these methods are in rough agreement with the JLQCD result; however, with domain wall quarks (DWQ) method the central value of B_K tends to be 10–15% below the JLQCD result which may amount to a discrepancy of around 1–2 σ (see figure 1). More precise calculations with these newer discretizations including a study with dynamical domain walls [16] is now underway.

2. f_B: Another example is provided by f_B, the B-meson pseudoscalar decay constant wherein the 'heavy' b-quark mass $\approx$4.5 GeV represents an additional technical problem. After the initial 5–6 years of exploratory works the computational strategy became quite well-known around '92 [17]. Indeed the result in the QA has been quite stable and withstood checks with the use of different techniques [18,19]. In the past few years there has been some weak indication from experiment that heavy–light decay constant are somewhat smaller in the QA compared to experiment [20] (i.e. full QCD). Indeed after years of persistent study the MILC collaboration has now finished the calculation of f_B in quenched as well as in full QCD (i.e. with three light flavors of dynamical quarks) [5,21] and finds, the ratio given in eq. (2), giving $f_B \simeq 207 \pm 35$ MeV [22]. Clearly many independent checks and confirmations will take place in the next few years.
3. $K \to \pi\pi$ and ϵ'/ϵ. Our third example is the calculation of the matrix elements of $K \to 2\pi$ and ϵ'/ϵ using domain wall quarks (DWQ) by the CP-PACS [23] and RBC collaborations [13]. Unlike B_K and f_B this is a first attempt to address $K \to 2\pi$ by both the collaborations in which not only QA but also a few other key approximations are made (see below). Nevertheless, given the complexity of the problem it must be considered an important accomplishment which even at this early stage is providing very useful information on the long standing issue of the $\Delta I = 1/2$ rule. However, its repercussions on ϵ'/ϵ require careful study of systematic errors and improved calculations, which could take another few years.

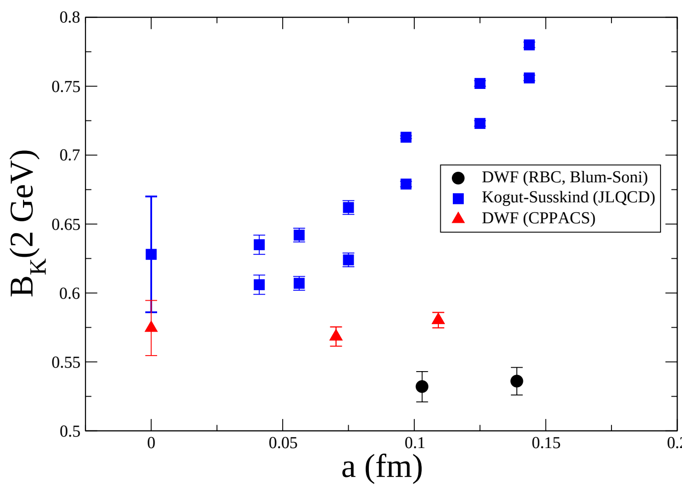

Figure 1. B_K vs. a, the lattice spacing. Only the two data points (circles) from DWF use non-perturbative renormalization of lattice operators, all others use one-loop perturbation theory. Of the circled DWF two points, the coarser spacing (right) corresponds to data from NERSC whereas the finer one (left) belongs to RBC collaboration [13].

3. Lattice matrix elements and CKM constraints

In the Wolfenstein representation, the CKM matrix can be parameterized in terms of the four parameters, $\lambda, A, \bar{\rho}$ and $\bar{\eta}$ [24]. Of these $\lambda = \sin\theta_c = 0.221 \pm 0.002$, is the best known, A is known with modest accuracy, $A = 0.847 \pm 0.041$ and $\bar{\rho}$ and $\bar{\eta}$ are poorly known. An important objective where lattice can help is in the determination of $\bar{\eta}$ and $\bar{\rho}$ accurately. $\bar{\eta}$ is intimately related to the CKM phase δ_{13} [25]; indeed SM cannot accommodate any CP violation if $\bar{\eta} = 0$.

The basic strategy is very simple. Assuming the SM is correct, and using the necessary theoretical input one translates experimental results on to an allowed domain on the $\bar{\eta} - \bar{\rho}$ plane. If a (new) experimental result requires value of $\bar{\rho}$ and/or $\bar{\eta}$ that are inconsistent with those extracted from existing experiments then that could mean a failure of the SM.

3.1 *Theoretical background and brief comments*

For the past several years, the following four experimental measurements have been used for the extraction of $\bar{\eta}$ and $\bar{\rho}$:

1. The indirect CP violation parameter, $\epsilon = (2.274 \pm 0.017) \times 10^{-3}$.
2. The $B_d - \bar{B}_d$ mass difference, $\Delta m_{B_d} = 0.487 \pm 0.014$ ps^{-1}.
3. The $B_s - \bar{B}_s$ mass difference, for which at the moment only a lower bound exists, $\Delta m_{B_s} \geq 15.0$ ps^{-1} at 95% CL.

   This important bound is provided by experiments at LEP and SLD [26]. It is widely anticipated that an actual measurement of Δm_{B_s} (rather than just a bound) will be accomplished at the Tevatron in the next few years. This will be very important for CKM determinations as the ratio $\Delta m_{B_d}/\Delta m_{B_s}$ can give V_{td}/V_{ts} if the $SU(3)$ breaking ratio of hadronic mixing matrix element could be determined [27].
4. $R_{uc} \equiv b \to ul\nu / b \to cl\nu = 0.085 \pm 0.017$.

Recall [24,28]

$$|\epsilon| = \tilde{M}_K C_K A^2 \lambda^6 \bar{\eta} \{-\eta_1 S_0(x_c) + \eta_2 S_0(x_t) A^2 \lambda^4 (1 - \bar{\rho}) + \eta_3 S_0(x_c, x_t)\}. \tag{3}$$

Here $x_q = m_q^2/M_W^2$, where $q = u, c, t$ i.e. the virtual quarks in the box graph for $K^0 - \bar{K}^0$ oscillations, and $S_0(x_q)$ are the so-called Inami–Lin functions [29]. η_{1-3} are QCD corrections; to NLO these are estimated to be [24] $1.38 \pm 0.20, 0.57 \pm 0.01, 0.47 \pm 0.04$, respectively. Also,

$$C_K = \frac{G_{\mathrm{F}}^2 M_W^2 m_K}{6\sqrt{2}\pi^2 \Delta m_K}, \tag{4}$$

$$\tilde{M}_K = \frac{3}{8} \langle K^0 | [\bar{s}\gamma_\mu (1 - \gamma_5) d]^2 | \bar{K}^0 \rangle / m_K^2 \tag{5}$$

is essentially the hadronic matrix element. Once the $\tilde{M}_K$ is known $\bar{\eta}, \bar{\rho}$ can be constrained through the use of eq. (3). This matrix element is often parametrized in terms of B_K,

$$B_K = \langle K^0 | [\bar{s}\gamma_\mu(1-\gamma_5)d]^2 | \bar{K}^0 \rangle / [8 f_K^2 m_K^2/3] \tag{6}$$

which should equal 1 if vacuum saturation approximation (VSA) holds. Since f_K, which in our normalization is about 160 MeV and m_K are known quite precisely from experiment, evaluation of the matrix element is completely equivalent here to that of B_K.

Similarly, we note that for $B_d - \bar{B}_d$ oscillation

$$x_{Bd} = \tilde{M}_{Bd}\ C_{Bd}[(1-\bar{\rho})^{\ 2} + \bar{\eta}^{\ 2}]\eta_B S_0(x_t)\ A^2\lambda^6\tau_{Bd}, \tag{7}$$

where $x_{Bd} \equiv (\Delta\ m_{Bd}/\Gamma_{Bd})$ and $C_{Bd} = (G_F^2 m_W^2 m_{Bd}/6\pi^2)$ and $\eta_B = 0.55 \pm 0.01$ [24] is the NLO QCD corrections factor. Again once the hadronic matrix element, $\tilde{M}_{bd}$, is known eq. (7) can be used to constrain $\bar{\rho}$, $\bar{\eta}$.

This matrix element is a 3-point function, which is directly calculable on the lattice. More often than not, though, in analogy with the kaon case, M_{bd} is parametrized in terms of a 'B-parameter' defined as

$$B_{Bd} = \frac{\langle B_d | [\bar{b}\gamma_\mu(1-\gamma_5)d]^2 | \bar{B}_d \rangle}{\frac{8}{3} m_{Bd}^2 f_{Bd}^2}. \tag{8}$$

Then the physical quantity x_{Bd} requires both B_{Bd} and f_{Bd} since the latter is not yet known from experiment. Besides since $\tilde{M}_{Bd}$ seems to scale roughly as f_{Bd}^2 one needs to know f_{Bd} rather accurately. Also, in practice in most calculations of f_{Bd} one tries to fit the light quark mass dependence through some linear function; such a fit, though is unlikely to give precisely the dependence on light quark mass for f_{Bd}^2.

Once $B_s - \bar{B}_s$ oscillation are experimentally detected and Δm_{B_s} becomes known, then the ratio

$$\frac{\Delta m_{B_d}}{\Delta m_{B_s}} = \frac{|V_{td}|^2}{|V_{ts}|^2} \frac{\langle B_d | [\bar{b}\gamma_\mu(1-\gamma_5)d]^2 | \bar{B}_d \rangle}{\langle B_s | [\bar{b}\gamma_\mu(1-\gamma_5)s]^2 | \bar{B}_s \rangle} \frac{m_{Bs}}{m_{Bd}} \tag{9}$$

can be used to determine $|\ V_{td}|$ if the ratio of hadronic matrix elements could be determined from the lattice.

Since this ratio of matrix elements is completely dependent on $SU(3)$ breaking effects ($s \leftrightarrow d$) it is expected to be close to unity. The objective of lattice calculations should be a precise evaluation of this $SU(3)$ breaking and this necessitates an accurate treatment of light quarks.

Again, introducing 'B-parameters' for B_d and B_s mesons we can rewrite

$$\frac{x_{Bd}}{x_{Bs}} = \frac{\tau_{Bd}}{\tau_{Bs}} \frac{m_{Bd}}{m_{Bs}} \frac{1}{\xi^2}\ \lambda^2 \frac{[(1-\bar{\rho})^2 + \bar{\eta}^2]}{[1-(\lambda^2/2)]^2}, \tag{10}$$

where ξ is the $SU(3)$ breaking ratio,

$$\xi = \frac{f_{Bs}}{f_{Bd}}\sqrt{\frac{B_{Bs}}{B_{Bd}}}. \tag{11}$$

Finally, the semi-leptonic branching ratio $b \to ul\nu/b \to cl\nu$ is another important way of constraining $\bar{\rho}, \bar{\eta}$ as it is a function of V_{ub}/V_{cb},

$$\frac{|\ V_{ub}|}{|\ V_{cb}|} = \lambda\sqrt{\bar{\rho}^2 + \bar{\eta}^2}\bigg/\left(1 - \frac{\lambda^2}{2}\right). \tag{12}$$

To deduce V_{ub}/V_{cb}, from the experimental measurement of the branching ratios requires corresponding form factors for exclusive reactions wherein lattice methods can be useful [30]. In the interest of brevity, we will not disuss this here.

4. Lattice input for CKM fits

Table 1 shows the input from the lattice, experiment and elsewhere used by us [31] and compare it with the works of Ciuchini *et al* [32] and Hocker *et al* [33]. The corresponding determination of the CKM parameter $\bar{\rho}, \bar{\eta}$ and unitarity angles α, β, γ as well as several other quantities of interest are also shown. Note that our error on $f_{Bd}\sqrt{B}_{Bd}$ and on ξ are appreciably bigger than used in the other studies. This is especially so for ξ, where for quite sometime we have been cautioning that the error of ≈ 0.05 that was commonly taken was a serious underestimate [34]. Recently, Kronfeld and Ryan [35] and Yamada [18] have also argued for a reassessment of errors on ξ due to the presence of chiral logs. Following this development as of LAT'02 larger error on ξ is now being widely advocated; for example, Lellouch [36] in his review at ICHEP02 summarized $\xi = 1.18 \pm 0.04^{+12}_{-0}$.

The SM fits now give $(\sin 2\beta)^{\mathrm{SM}} = 0.70 \pm 0.10$ as well as allowed ranges for $\gamma, \bar{\eta}, \bar{\rho}$ etc (see table 1). While these fits provide fairly restrictive range for β and γ, α is constrained rather poorly. Note also that B_s–$\bar{B}_s$ mass difference $\Delta m_{B_s} = 19.8 \pm 3.5$ ps^{-1} is now constrained with a one sigma accuracy of about 15%; measurements at the Tevatron and later at the LHC should be able to test this important prediction of the SM. Meanwhile measurements of the CP asymmetry in $B \to \psi K_s$ is already providing quite an impressive determination, $\sin 2\beta = 0.734 \pm 0.054$ [37] in good agreement with the theoretical prediction. It is also important to note that just in the past year B-factory experiments have improved the determination of $\sin 2\beta$ from an error of $\pm$ 0.10 down to $\pm$ 0.05. With the anticipated increase in luminosities of the B-factories, along with results from the Tevatron, this error should go down further to ≈ 0.02 in another year or two. (Recall that the intrinsic theory error in the determination of $\sin 2\beta$ is expected to be about $\lesssim$0.01) [38].

It is instructive to reflect on the pace of theoretical progress in constraining $\sin 2\beta^{\mathrm{SM}}$. For this purpose we may compare the inputs used in fits of $\approx$1995 [39], with that of $\approx$ 2001 [31]. Indeed the 2001 fit has reduced the error on $\sin 2\beta$ from 0.20 to 0.10; correspondingly the error on $\bar{\eta}$ and on Δm_{B_s} is also appreciably reduced. However, only some of these improvements can be related directly to lattice computations. In fact it seems that a large portion of the improvement is due actually to the reduction especially in the error on V_{cb} and to some degree on V_{ub}/V_{cb} wherein the role of the lattice is less clear.

Table 1. Comparison of some fits.

	Atwood and Soni [31]	Ciuchini *et al* [32]	Hocker *et al* [33]
Input quantity			
$R_{uc} \equiv \lvert V_{ub}/V_{cb}\rvert$	0.085 ± 0.017	0.089 ± 0.009	$0.087 \pm 0.006 \pm 0.014$
$F_{B_d}\sqrt{\hat{B}_{B_d}}$ MeV	230 ± 50	$230 \pm 25 \pm 20$	$230 \pm 28 \pm 28$
ξ	1.16 ± 0.08	$1.14 \pm 0.04 \pm 0.05$	$1.16 \pm 0.03 \pm 0.05$
$\hat{B}_K$	0.86 ± 0.15	$0.87 \pm 0.06 \pm 0.13$	$0.87 \pm 0.06 \pm 0.13$
Output quantity			
$\sin 2\beta$	0.70 ± 0.10	0.695 ± 0.065	0.68 ± 0.18
$\sin 2\alpha$	-0.50 ± 0.32	-0.425 ± 0.220	
γ	$46.2^\circ \pm 9.1^\circ$	54.85 ± 6.0	56 ± 19
$\bar{\eta}$	0.30 ± 0.05	0.316 ± 0.040	0.34 ± 0.12
$\bar{\rho}$	0.25 ± 0.07	0.22 ± 0.038	0.22 ± 0.14
$\lvert V_{td}/V_{ts}\rvert$	0.185 ± 0.015		0.19 ± 0.04
$\Delta m_{B_s}(ps^{-1})$	19.8 ± 3.5	$17.3^{+1.5}_{-0.7}$	24.6 ± 9.1
$J_{\rm CP}$	$(2.55 \pm .35) \times 10^{-5}$		$(2.8 \pm .8) \times 10^{-5}$
$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})$	$(0.67 \pm 0.10) \times 10^{-10}$		$(0.74 \pm 0.23) \times 10^{-10}$
$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})$	$(0.225 \pm 0.065) \times 10^{-10}$		$(0.27 \pm 0.14) \times 10^{-10}$

What should be clear is that it will be extremely difficult to reduce the theory error on $(\sin 2\beta)^{\rm SM}$ from the current level of ± 0.10 down to the level of 0.02 that the experiment is anticipated to reach in the very near future. Thus to test the SM more precisely will require clean determination of the other angles α and γ directly from experiment. We will come back to this point later on.

4.1 *Important input from experiment on the horizon for CKM determination*

1. $B^\pm \to \tau^\pm + \nu_\tau(\bar{\nu}_\tau)$: With 10^8 or more $B - \bar{B}$ pairs that BELLE and BABAR each will soon have access to, an experimental determination of f_B (actually $f_B \times V_{ub}$) may be feasible. Using from the lattice $f_B \approx 207 \pm 35$ MeV [22] and $V_{ub}/V_{cb} \approx 0.085 \pm 0.017$ one gets an estimate, $\mathrm{Br}(B \to \tau + \nu_\tau \approx (7.8 \pm 2.0) \times 10^{-5})$. Decays of τ into final states with $(\nu' s)+\ \mu\ (e, \rho$ or $\pi)$ have a total branching ratio of around 50%. So with a few per cent detection efficiency there should be a few hundreds of events for $B^\pm \to \tau^\pm + \nu_\tau(\bar{\nu}_\tau)$, a respectable sample to provide a reasonable determination of $f_B \times V_{ub}$ and an important check on the lattice calculation.
2. $B \to l\nu\gamma$: Unlike $B \to l\nu$, $l\nu\gamma$ $(l = e, \mu)$ does not suffer from helicity suppression although it is suppressed by α. Emission of the photon from the light quark also tends to enhance the process although precise calculation of the Br is difficult to make [40]; estimates [41] are in the range of 1–6 $\times 10^{-6}$, i.e. about an order of magnitude more than the 2-body helicity suppressed modes, $B \to l\nu$. The constituent quark model, although too simple to pro-

vide reliable details, perhaps does give a valid qualitative picture indicating a 'hard' photon spectrum [40]:

$$\frac{\mathrm{d}N}{\mathrm{d}\lambda_\gamma} = \frac{m_B}{\Gamma_{l\nu\gamma}} \frac{\mathrm{d}\Gamma_{l\nu\gamma}}{\mathrm{d}E_\gamma} = 24\lambda_\gamma(1-2\lambda_\gamma), \tag{13}$$

where $\lambda_\gamma = E_\gamma/m_B$, and yields a total Br $\approx 5\times10^{-6}$ with a constituent light quark mass of about 350 MeV and $f_B = 200$ MeV. Predictions from several other estimates are given in table 2. These radiative modes should be accessible accompanied by μ or e with the data samples currently available. In making contact with the phenomenological models, energy spectra of the photon and of the neutrino (i.e. the invariant mass of ($\gamma+$ the charged lepton)) would be especially useful. Detailed experimental studies of these radiative decays would also give another handle on the approximate value of f_B.

3. $B^0 \to \rho+\gamma$: Another important input from experiment that could aid in the determination of the CKM-parameters (esp. $|V_{td}|/|V_{ts}|$) is the reaction $B^0 \to \rho+\gamma$ since

$$\frac{\mathrm{Br}(B^\circ \to \rho^\circ\gamma)}{\mathrm{Br}(B \to K^*\gamma)} = \frac{[1-m_\rho^2/m_B^2]}{[1-m_{K^*}^2/m_B^2]} \left[\frac{T_1^{B\to\rho}(0)}{T_1^{B\to K^*}(0)}\right]^2 \frac{|V_{td}|^2}{|V_{ts}|^2}. \tag{14}$$

The expected $\mathrm{Br}(B^0 \to \rho+\gamma) \approx 1\times10^{-6}$ seems within reach of experiment. An accurate calculation of the ($SU(3)$) breaking ratio of form factors $[T_1^{B\to\rho}(0)]/[T_1^{B\to K^*}(0)]$, for example by lattice methods, the feasibility of which was demonstrated long time already [42,43], along with the anticipated experimental measurement of $B\to\rho+\gamma$ could lead to another determination of V_{td}/V_{ts} [44].

This method of extracting V_{td}/V_{ts} has some advantages and some disadvantages compared to $B_s-\bar{B}_s$ oscillations. For $B\to\rho\gamma$ the relevant operator is a bilinear one, whereas for $B_s-\bar{B}_s$ oscillations a 4-quark operator enters; the renormalization of a lattice 4-quark operator can be more complicated compared to a bilinear one. On the other hand, $B\to\rho+\gamma$ involves a large recoil thereby extracting the form factor at or near $q^2=0$ (where q is the 4-momentum of the photon) is numerically difficult on the lattice.

From the experimental side in the numerator of eq. (14) only neutral Bs, $B^0, \bar{B}^0 \to \rho^0\gamma$ should be used as charged $B^\pm \to \rho^\pm\gamma$ provides a non-negligible long-distance contribution [45] which is proportional to V_{ub} (and independent of V_{td}) coming from the annihilation graph and cannot be estimated accurately.

Due to these anticipated input from experiments along with developments in theory, especially with the expected improvement in computational resources because of the scientific discovery through advanced computing (SCIDAC) initiative [46], it is fairly safe to say that errors on SM parameters such as $\sin 2\beta$ will go down by a factor of about 2–3. However, with larger pool of B-samples that are expected

Table 2. Sample of rough estimates for $B^{\pm} \to l\nu\gamma$.

Method	Reference	Br
Const. quark model	Atwood *et al* [40]	$\approx 5 \times 10^{-6}$
Light cone + HQET	Korchemsky *et al* [41]	$\approx 5 \times 10^{-6}$
Light-front	Geng *et al* [41]	$\approx 4 \times 10^{-6}$
Rel. potential	Colangelo *et al* [41]	$\approx 1 \times 10^{-6}$
QCD-fact, SCET	Descotes-Genon and Sachrajda [41]	$\approx$ a few $\times 10^{-6}$

from B-factories and the hadron facilities, experiments should be able to directly determine $\sin 2\beta$ from CP asymmetry measurements in $B \to \psi K_s$ to an accuracy of 0.02; so experiment is likely to stay ahead of theory. Uncovering new sources of CP violation in B-physics though may well require more precise tests, as we will emphasize in the next few paras.

5. Theory errors and the hunt for new sources of CP violation in *B*-physics

Theory errors should be a concern as they can thwart experimental efforts to search for beyond the standard model (BSM)-CP odd new phase(s) which we will collectively denote as χ. The main point is that χ may well cause only small deviations from the SM in B-physics. Indeed the emerging understanding of the CKM-paradigm serves as an important lesson in this regard. The CP-odd phase δ_{13}, in the standard notation, ($\delta_{13} \approx \gamma \approx 50 \pm 10°$) is not small and although it causes $O(1)$ CP-asymmetry in $B \to \psi K_s$ its effects in $K_L \to \pi\pi$, ϵ or ϵ' are very small, $O(10^{-3}$–$10^{-6})$ respectively [25]. Analogously it is clearly not inconceivable that even though a BSM-CP-odd phase χ is not small its effect on B-physics will be small. As an example, this may happen if χ arises in models with extra Higgs bosons; then its effects may be much larger in top physics and quite small in B-physics [47].

For one thing this means we may need very large data samples of Bs. Indeed for an asymmetry of $O(10^{-3})$ (as in K_L), since the relevant Br is unlikely to be larger than $\approx 10^{-3}$, which is about the branching ratio for $B \to \eta' X_s$, detection may require $O(10^{10})$ Bs. Higher luminosity super BELLE/BABAR B-factories as well as efforts at hadron B-facilities BTEV and LHCB may well be needed in the hunt for χ.

In the search for χ the ability to detect small deviations from the SM also requires that we develop tests of the SM that use little or no theory assumptions and are as free of theory errors as possible. Note in this regard that for detection of deviations from the SM at the level of $\approx 10^{-3}$ means that even isospin approximation, widely used in many methods for extracting angles of the unitarity triangle can mask χ and thereby defeat the experimental effort for detection of new physics.

Motivated by these considerations we now discuss methods for getting unitarity angles with very little theory error, i.e. to $O(< 1\%)$.

5.1 *Pristine determination of the unitarity triangle via $B \to KD^0$.*

Angles of the unitarity triangle can be obtained very 'cleanly' i.e. without any theoretical assumptions from analysis of final states containing $D^0, \bar{D}^0$ in charged or neutral B-decays [2].

γ can be extracted from a study of direct CP violation in charged B-decays, $B^\pm \to K^\pm D^0, \bar{D}^0$ [48–50]. $\delta \equiv (\beta - \alpha + \pi) = 2\beta + \gamma$ [51–54] as well as β can be obtained from time-dependent CP-asymmetry measurements in $B^0, \bar{B}^0 \to K^0 D^0, \bar{D}^0$. In both cases common final states of $D^0, \bar{D}^0$ have to be used, as flavor tagging of $D^0, \bar{D}^0$ is very difficult [49]. There are 3 types of such common final states:

1. $D^0, \bar{D}^0$ decays to CP-non-eigenstates that are doubly Cabibbo suppressed [49], for example, $K^+\pi^-$, $K^+\rho^-$, $K^{*+}\pi^-$ etc.
2. $D^0, \bar{D}^0$ decays to CP-eigenstates [48], for example, K^+K^-, $\pi^+\pi^-$, $K^0_S\pi^0$.
3. $D^0, \bar{D}^0$ decays to CP-non-eigenstates that are singly Cabibbo suppressed [50] for example, $K^{*+}K^-$, $K^{*-}K^+$, $\rho^+\pi^-$, $\rho^-\pi^+$ etc.

It turns out that CP asymmetry are expected to be small ($\leq 10\%$) for CPES and for singly Cabibbo suppressed modes (i.e. second and third types) whereas the interference and CP asymmetry is maximal for CPNES (first type). On the other hand, the branching ratios are expected to be largest for CPES and smallest for doubly Cabibbo suppressed CPNES modes. The general expectations are that for extraction of γ, doubly Cabibbo suppressed modes should be most efficient among the three types. However, this is not guaranteed and all three methods should, for sure, be used. What is important, for the long run is to note that only two common decay modes of $D^0\bar{D}^0$ are needed to give enough observables to algebraically solve the CP-odd weak phase γ, the strong final states phase(s) as well as the suppressed $\mathrm{Br}(B^- \to K^-\bar{D}^0)$ that is very difficult to measure experimentally; indeed perhaps a dozen or so such modes are available. This should greatly help the analysis in extracting a precise value for γ without discrete ambiguities.

For time-dependent CP asymmetry [54] in $B^0, \bar{B}^0 \to K^0 D^0, \bar{D}^0$, the discussion is analogous to the above. Again for extraction of δ (as well as β) one needs only two common final states of D^0, $\bar{D}^0$ from the many; whether they be CPNES, CPES doubly or singly Cabibbo suppressed modes, that are available; however, both modes cannot be CPES.

Especially noteworthy is the fact that for clean extraction of the angles, final states containing $D^0, \bar{D}^0$ in the decays of $B^\pm, B^0, \bar{B}^0$ are involved and furthermore common final states of $D^0, \bar{D}^0$ decay play a critical role and should aid in increasing the efficiency of the analysis.

In passing we briefly note of the analogous methods involving B_s decays to D_s $K^\pm$ [55] (or their vector counterparts [56]) that can also give γ very cleanly [57].

6. Exact chiral symmetry on the lattice

6.1 *Introduction*

In the past few years a significant development for lattice gauge computations has taken place. For the first time, we have practically viable discretization methods that exhibit exact chiral symmetry on the lattice even at a finite lattice spacing, i.e. even before the continuum limit is taken. By now, not only the viability of these methods has been convincingly demonstrated, large scale simulations, with some success, have already been using them to address some outstanding problem in weak interaction phenomenology pertaining to $K \to 2\pi$ that were very difficult to address heretofore, as briefly reminded below.

6.2 *Difficulties of calculating weak matrix elements on the lattice with conventional discretizations*

Recall that conventionally there are two fermion discretizations: Wilson and staggered (or Kogut–Susskind). Wilson fermion explicitly break chiral symmetry whereas conventional staggered fermions while possessing some residual chiral symmetry break flavor symmetry. The difficulties of maintaining chiral symmetry on the lattice is enunciated in the form of a no-go (Nielsen-Ninomiya [58]) theorem.

Although conventional wisdom says that these symmetries get restored on the lattice in the continuum limit, in practice, in the study of hadronic weak decays, lack of chiral symmetry imposes an extremely serious if not an insurmountable limitation.

Lack of chiral symmetry leads to two types of significant difficulties:

1) Precise renormalization of 4-quark operators can become a difficult fine-tuning problem. The point is that, in the absence of chiral symmetry operators such as $O_{LL} \equiv [\bar{s}\gamma_\mu(1-\gamma_5)d]^2$, that are relevant to $K-\bar{K}$ oscillations and B_K computation, mix under renormalization with wrong chirality operators [59], for example, with $O_{PP} \equiv (\bar{s}\gamma_5 d)^2$.

The problem is that whereas $\langle K|O_{LL}|\bar{K}\rangle$ is proportional to the quark mass and therefore vanishes in the chiral limit, $\langle K|O_{PP}|\bar{K}\rangle$ goes to a constant in the chiral limit. Thus even if the mixing coefficients of the wrong chiral operators are small you need to know them very accurately in order to precisely extract the matrix element of physical interest.

2) Mixing with lower dimensional operators is even a worse problem. This happens, for instance, when one considers the operators of the $\Delta S = 1$ Hamiltonian (e.g. $\bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)d$) relevant to $K \to 2\pi$. Now such a dim-6 operation mixes with lower dimensional operations, for example, $\bar{s}d$, $\bar{s}\gamma_5 d$, $\bar{s}\sigma_{\mu\nu}dG^{\mu\nu}$ [60]. The mixing coefficients are now power divergent, for example $\sim a^{-n}$ ($n = 3$ for $\bar{s}d$ and $\bar{s}\gamma_5 d$ and $n = 1$ for $\bar{s}\sigma_{\mu\nu}dG^{\mu\nu}$). So they become increasingly important in the continuum limit. Non-perturbative methods (that respect chiral symmetry) are essential for handling them. This was the main reason that early efforts [61] to calculate $\Delta I = \frac{1}{2}$, $K \to 2\pi$ amplitudes on the lattice did not make much progress.

6.3 *Domain wall fermions*

In 1992, Kaplan [62] in a celebrated paper showed a simple method to attain exact chiral symmetry on the lattice even at finite lattice spacing. This remarkable feat is accomplished by embedding the 4-dim. theory on to 5-dim. with a fermion mass-term that has the shape of a domain wall across the 4-dimensional boundary and switches sign. The low lying mode bound to the walls then possesses exact chiral symmetry on the lattice in the limit that the length (L_S) of the fifth dimension has an infinite extent [63]. Nielsen–Ninomiya theorem [58] is evaded as in Kaplan's construction the number of fermionic degrees of freedom per (4-dim.) site is no longer finite as assumed in the theorem, and in fact becomes infinite as $L_S \to \infty$. Narayanan and Neuberger [64] gave an elegant flavor interpretation of this fifth dimension.

In 1997 first (quenched) lattice QCD simulation done to test the practical viability of this approach showed very encouraging results [11]. In those numerical simulations for QCD actually the domain wall formulation of Shamir [63] was used. These early results showed that even with a modest extent of the fifth dimension, domain wall fermion possesses very good chiral behavior setting the stage for their use in large scale simulations.

Since DWF are continuum like their renormalization (perturbative and non-perturbative) properties are fairly simple. Also discretization errors tend to go as $O(a^2)$ rendering them with very good scaling properties which tends to offset the cost of the extra-dimension.

Since in practice the extent of the fifth dimension is finite, the coupling between the two walls separated by L_S causes a coupling between the light modes and gives them a residual mass, m_{res}. This mass can be measured quite precisely [65,66]. In low energy applications one can systematically include the effect of m_{res} in the context of an effective chiral Lagrangian [67].

6.4 *Application to B_K*

CP-PACS [12] and RBC [13] collaborations have made considerable progress towards a precise calculation of B_K with DWF. Both results are in the range of 1–2σ below the old result from JLQCD [6], $B_K(2\ \text{GeV}) = 0.628 \pm 0.042$. CP-PACS and RBC central values for B_K differ by about 5–10%; most likely this difference is due to the fact that CP-PACS uses 1-loop lattice perturbation [68,69] theory for renormalization of the $\Delta S = 2$ operators whereas RBC is using non-perturbative renormalization [67]. Efforts are now underway to repeat this calculation at weaker (quenched) coupling [70] as well as with dynamical domain wall quarks [16].

7. $\Delta I = 1/2$ rule and ϵ'/ϵ: Progress and outlook

7.1 *Introduction*

There have been two recent attempts by the CP-PACS [23] and the RBC [13] collaborations, at attacking this old problem on the lattice using the relatively new discretization method of domain wall fermions (DWF) [62–64].

First lattice studies of $K \to \pi\pi$ by both CP-PACS [23] and RBC [13] with DWF used the lowest order chiral perturbation theory (ChPT) approach suggested by Bernard *et al* [71]. The method then calls for using the lattice to compute matrix element of 4-quark operators between $K \to \pi$ and $K \to$ vacuum which are used to obtain the corresponding desired $K \to 2\pi$ matrix elements [72]. While this is a simple method which avoids technical (Maiani–Testa theorem [73]) and also practical, computational limitations, it is nevertheless a severe approximation. In particular, at the leading order in ChPT being used, final state interactions, which in reality are very likely important [74] are necessarily absent. Note also that in this approximation the good chiral behavior of domain wall fermions becomes crucial. For one thing in the absence of chiral symmetry, the unphysical (cubically divergent) contribution from mixing with lower dimensional operators to $\langle\pi|\theta^{8,1}|K\rangle$ cannot be subtracted away in a relatively simple way by using $\langle 0|\theta^{8,1}|K^0\rangle$. Furthermore, the renormalization of 4-quark operators also becomes vastly more complicated due to the mixing of operators with the wrong chirality ones [59]. Since the $K \to \pi$ matrix elements of some of the operators go to a constant in the chiral limit, whereas those of the right chirality tend to vanish, subtraction of the unwanted contribution needs to be done at a very high precision, i.e. it becomes a fine tuning problem. For these reasons, as mentioned above, earlier efforts [61] for computation of $K \to 2\pi$ and ϵ'/ϵ to the LO in ChPT by the use of $K \to \pi$ and $K \to 0$ on the lattice with Wilson fermions, which explicitly break chiral symmetry, were able to make little headway.

Since chiral symmetry is so critical in the calculation of matrix elements for $K \to 2\pi$ and since, due to the finite extent of the fifth dimension, rigorously speaking, domain wall quarks do not possess exact chiral symmetry, it is important to be able to take into account residual symmetry breaking effects in a systematic fashion. For matrix element dominated by long-distance physics this can be accomplished by shifting the bare masses in ChPT by $m_{\rm res}$, where $m_{\rm res}$ is the residual quark mass which the massless quarks on the lattice possess due to the coupling between the walls of the fifth dimension [67]. Of course, for this simple picture to hold, L_s (length in the fifth dimension) must be sufficiently large so that $m_{\rm res}$ is a lot smaller than the input (light) quark masses in the lattice calculation. On the lattice we can calculate $m_{\rm res}$ quite precisely [65,66] and the chiral limit is then taken by setting $(m_{\rm quark} + m_{\rm res}) \to 0$.

For operators such as Q_6 which receive power divergent (i.e. short distance) contributions that are not physical and have to be subtracted away, the symmetry breaking effect cannot be precisely described by $m_{\rm res}$ [13]. Fortunately, the LEC can still be computed accurately by taking the slope of the matrix elements with the $m_{\rm quark}$ so long as $m_{\rm res}$ is independent of $m_{\rm quark}$ to a good approximation [13]; again, it should be emphasized that this does require that in actual simulations the length of the fifth dimension is sufficiently long that $m_{\rm res} \ll m_{\rm quark}$.

Note also that for power divergent subtractions ChPT is taken into account to all orders [13].

Table 3 shows the low energy chiral perturbation theory constants that give the (subtracted) $K \to \pi$ matrix element of each of the 4-quark operators of interest for the $I = 1/2$ channel as well as for the $I = 3/2$ channel. These directly give the corresponding $K \to \pi\pi$ matrix elements of each operator to LO in ChPT using formuals given in [71]. One can thus obtain the corresponding $K \to 2\pi$ amplitudes via eq. (201) in [13].

Table 4 gives the full results for $\mathrm{Re}\, A_0$, $\mathrm{Re}\, A_2$, $\omega^{-1} \equiv \mathrm{Re}\, A_0/\mathrm{Re}\, A_2$ and ϵ'/ϵ of RBC [13].

Table 3. The lattice values for the low energy, chiral perturbation theory constants decomposed by isospin for Q_1 to Q_{10} (taken from [13]).

i	$a^{(1/2)}_{i,\mathrm{lat}}$	$a^{(3/2)}_{i,\mathrm{lat}}$
1	$-1.19(31) \times 10^{-5}$	$-1.38(6) \times 10^{-6}$
2	$2.22(16) \times 10^{-5}$	$-1.38(6) \times 10^{-6}$
3	$0.15(113) \times 10^{-5}$	0.0
4	$3.55(96) \times 10^{-5}$	0.0
5	$-2.97(100) \times 10^{-5}$	0.0
6	$-8.12(98) \times 10^{-5}$	0.0
7	$-3.22(16) \times 10^{-6}$	$-1.61(8) \times 10^{-6}$
8	$-9.92(54) \times 10^{-6}$	$-4.96(27) \times 10^{-6}$
9	$-1.85(16) \times 10^{-5}$	$-2.07(9) \times 10^{-6}$
10	$1.55(31) \times 10^{-5}$	$-2.07(9) \times 10^{-6}$

Table 4. Final values for physical quantities using 1-loop full QCD extrapolations to the physical kaon mass and a value of $\mu = 2.13$ GeV for the matching between the lattice and continuum; taken from [13] (statistical errors only).

Quantity	Experiment	This calculation (statistical errors only)
$\mathrm{Re}A_0$(GeV)	3.33×10^{-7}	$(2.96 \pm 0.17) \times 10^{-7}$
$\mathrm{Re}A_2$(GeV	1.50×10^{-8}	$(1.172 \pm 0.053) \times 10^{-8}$
ω^{-1}	22.2	(25.3 ± 1.8)
$\mathrm{Re}(\epsilon'/\epsilon)$	$(15.3 \pm 2.6) \times 10^{-4}$ (NA48) $(20.7 \pm 2.8) \times 10^{-4}$ (KTEV)	$(-4.0 \pm 2.3) \times 10^{-4}$

While both the groups [23,13] use L0ChPT, DWF and the quenched approximation, there are some important differences in these two calculations as well. For one thing, RBC [13] used the standard Wilson gauge actions whereas CP-PACS [23] used renormalization group improved (Iwasaki) gauge action [75]. Also in their extractions of $\mathrm{Re}\,A_2$ (and B_K), RBC used the 1-loop quenched chiral perturbation theory [76] to fit $\langle\pi|Q^{3/2}_{1,2}|K\rangle$ whereas CP-PACS used a phenomenological fit.

7.2 *When the dust settles*

I. Regarding Re ϵ'/ϵ

a) Key contributions.

Listed below are the key contributions to ϵ'/ϵ from $I = 0$ and 2 final states, all given in units of 10^{-4} resulting from [13]. Recall, experiment finds (in this unit) [77,78] Re $\epsilon'/\epsilon = 17 \pm 2$. (Note contributions not shown are negligible in comparison.)

Operator	$I=0$	$I=2$
Q_4	-4.8 ± 1.1	
Q_6	14.2 ± 1.9	
Q_8	1.48 ± 0.12	-16.97 ± 0.84
Q_9		1.56 ± 0.00

b) Q_6 and Q_8 are not the only ones that matter.

Although, as widely expected [79–82], Q_6 and Q_8 are the dominant players, due to the cancellations between these two contributions, other operators (e.g. Q_4) seem to be making an appreciable difference to the final result.

c) Buras approximate formula [81].

In this context recall Buras' approximate formula

$$\epsilon'/\epsilon \simeq \epsilon'/\epsilon|_{6+8} \equiv \epsilon'/\epsilon|_6 + \epsilon'/\epsilon|_8. \tag{15}$$

From our lattice data one can see that the contribution of operators other than Q_6 and Q_8 is about 60% of $\epsilon'/\epsilon|_{6+8}$.

d) Cancellation *not* between large numbers.

While there is a cancellation between Q_6 and Q_8, in magnitude each of this contribution is comparable to the experimental number for ϵ'/ϵ. Had it been that

$$\epsilon'/\epsilon|_6, \quad \epsilon'/\epsilon|_8 >> \epsilon'/\epsilon|_{\rm expt} \tag{16}$$

then the cancellation would have been between 'large numbers' (compared to the final result that one is seeking) and the prognosis for future improvements would have been even harder.

e) Unnatural cancellations.

The substantial cancellation (∼85%) between contributions of Q_6 and Q_8 to ϵ'/ϵ, in all likelihood, is not natural, i.e. not stable to perturbations. Recall that these numbers emerge after using at least three key (uncontrolled) approximations: lowest order chiral perturbation theory, quench approximation and heavy charm quark. It is virtually impossible that these approximations affect Q_6 and Q_8 in the same way. Indeed there are good reasons to think that both chiral perturbation theory and quench approximation are having a bigger effect on Q_6 than on Q_8. It seems reasonable therefore to expect that in improved calculations of ϵ'/ϵ these cancellations between Q_6 and Q_8 will not remain.

f) Phenomenological bound.

To the extent that $\epsilon'/\epsilon|_{I=2} < 0$, there is a useful pheomenological bound,

$$\epsilon'/\epsilon|_{I=0} > \epsilon'/\epsilon|_{\rm expt} \tag{17}$$

with which one can test the SM. For the purpose of this test the cancellations between the $I = 0$ and $I = 2$ contributions are not quite relevant. Note that our current data gives left hand side of eq. (16) of $\sim 11 \pm 2$ whereas the RHS (from experiment) is $\sim 17\pm2$. Clearly then $\epsilon'/\epsilon|_{I=0}$ must increase appreciably as improvements in our lattice calculation are made if the SM's description of CP is to continue to hold.

These considerations suggest that tests of the SM with improvements in accuracy appear feasible.

II. Repercussions for the origin of the $\Delta I = 1/2$ rule.

The octet enhancement, i.e. $\mathrm{Re}\, A_0/\mathrm{Re}\, A_2 \sim 20 >> 1$, has been a longstanding puzzle in particle physics. The lattice calculation with domain wall quarks, although not having sufficient control over all the systematic errors cannot at present give a reliable result for ϵ'/ϵ, they do provide with a useful and unambiguous information for the $\Delta I = 1/2$ enhancement. The lattice result leads to an important and remarkable conclusion regarding the $\Delta I = 1/2$ rule as can be seen from table 3. The contribution of $\mathrm{Re}\, A_2$ and especially of $\mathrm{Re}\, A_0$ originate almost entirely from the aboriginal 4-Fermi operator Q_2. Indeed, we find

Operator	$\mathrm{Re}\, A_0$ (GeV)	$\mathrm{Re}\, A_2$ (GeV)
Q_1	$(3.48 \pm .77) \times 10^{-8}$	$(-0.363 \pm 0.016) \times 10^{-8}$
Q_2	$(24.5 \pm 1.6) \times 10^{-8}$	$(1.520 \pm 0.068) \times 10^{-8}$
Q_6	$(0.050 \pm 0.006) \times 10^{-8}$	

These numbers should be compared to the experimental ones: $\mathrm{Re}\, A_0^{\mathrm{expt}} = 33.30 \times 10^{-8}$ GeV, $\mathrm{Re}\, A_2^{\mathrm{expt}} = 1.50 \times 10^{-8}$ GeV. Clearly $\mathrm{Re}\, A_0$ is completely dominated by Q_2, making about ~80–85% contribution and Q_1 makes the remaining ~15% contribution to $\mathrm{Re}\, A_0$. In particular, in our lattice calculation, the contribution of Q_6 to $\mathrm{Re}\, A_0$ is completely negligible, being ~0.2%. This is in sharp contrast to some model calculations for ϵ'/ϵ in which contribution of Q_6 to $\mathrm{Re}\, A_0$ is typically almost 20–30% [79,80]. Note that while numbers given here for contribution of individual operators are based on calculations at $\mu \sim 2$ GeV we have studied the μ dependence from ~1.3–2 GeV and the dependence is quite mild [13].

It must be emphasized that all these quantitative findings, in particular Q_6 vs Q_2 contributions to $K \to 2\pi(I = 0)$, are based on LOChPT calculations and this could change as higher order corrections are included.

It is indeed interesting and ironical that the penguin operators [83] originally invoked to explain the $\Delta I = 1/2$ rule seem to play little role therein, at least in the context of our lattice calculation [13]. However, the subsequent conjecture of their importance to rendering a largish ϵ'/ϵ [84] seems to be substantiated although significant theoretical progress still needs to be made before the repercussions of the precise experimental measurement can be fully assessed.

7.3 *Approximations and concerns*

There were several approximations made in the lattice of calculations [23,13] using DWF that were recently completed.

1. The quenched approximation so that quark antiquark loops are ignored in the propagation of the gauge field (gluon).
2. Lowest order chiral perturbation theory (LOChPT), so that the matrix elements of the 4-quark operators are calculated in the leading order (LO) in this approximation. This means that operators [Q_1–Q_6, Q_9, Q_{10}] that transforms as (8,1) and/or (27,1) under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}$ are calculated to $O(p^2)$ whereas those of Q_7, Q_8 which transform as (8,8) are to $O(p^0 \to 1)$ in the chiral expansion.
3. The charm quark is assumed to be very heavy and integrated out. The effective Hamiltonian [85,86] consequently consists of only three active flavors: u, d, s.

These approximations are uncontrolled, i.e. we do not have a reliable estimate of how inaccurate they are. While the quenched approximation seems to be accurate to 10–15% in many spectrums and decay constant calculations, it may well be a lot worse for some hadronic matrix elements. In particular, comparison of the analytical formulas for $\langle\pi|Q_{7,8}^{3/2}|K\rangle$ to NLO [87] in full ChPT with the lattice data [13] obtained using the quenched approximation shows that the logs of the full ChPT seem to be absent [88]. Also the matrix element of Q_6, which is of crucial importance to ϵ'/ϵ is claimed to be very susceptible to quenching effects [89].

Although, in many low energy applications, LOChPT works fairly well, in $K \to 2\pi$ there are reasons to be suspicious. First of all an important mass scale here is m_K and not just m_π. Furthermore, in the $I = 0$ channel the π–π rescattering effects (FSI), which cannot occur in the LOChPT, are likely to be quite important [74]. Indeed for $\langle\pi\pi|Q_6|K\rangle$ higher order chiral corrections may well be intertwined with a $O^{++}(\sigma)$ resonance in the π–π channel [90,91].

Since the mass of the charm quark is only ~ 1.3 GeV, integrating it out assuming it is very heavy (i.e. $>> \Lambda_{\mathrm{QCD}}$) is very likely not a good approximation. Corrections from higher dimensional operators are likely to be sizeable [92]. Also in the 4-flavor theory, GIM cancellation forbids power-divergent mixing of dim.-6 operators of the $I = 0$, H_{eff} with lower dim. operators, so the 4-flavor theory is preferable over the 3-flavor theory for that reason too, although this advantage is only relevant in computation of $\mathrm{Re}\, A_0$ [93–95]. In the calculation of $\mathrm{Im}\, A_0$ where (8,1) penguin operators, such as Q_6, become important which originate from integrating out the top quark, the top quark is so heavy compared to the lattice cut-off that integrating it out is unavoidable.

7.4 *Future outlook*

While the above approximations used in the current lattice calculations are not controllable, systematic improvements are feasible and efforts at these are well underway.

First, recent studies of renormalization group-improved gauge actions in the context of domain wall fermions appear very promising in significant improvements in chiral symmetry (which was already remarkably good) [96]. Efforts are also underway towards creating a large ensemble of gauge configurations with dynamical

(2-flavor) domain wall quarks [16] with lattice spacing $\sim$ (2 GeV^{-1}). This should allow a first study of the quenching effects in $K \to 2\pi$ matrix elements in another year or so. Also work is being done at finer lattice spacing $\sim$ (3 GeV^{-1}) with the hope that this will allow a better treatment of the charm-quark and a calculation of the matrix elements in the effective theory with four active flavors (u, d, s, c) [70].

Note also that new calculations of the $K \to 2\pi$ matrix elements have begun [97] using another discretization (overlap fermions [98]) possessing excellent chiral symmetry.

Recent works also show how lattice computations of all the matrix elements relevant to $K \to 2\pi$ and ϵ'/ϵ can be obtained beyond the leading order in ChPT. In one method [88] matrix elements of all the relevant operators ($\Delta I = 1/2$ or $3/2$) can be obtained to NLO by using lattice computations of $K - \bar{K}$, $K \to \pi$, $K \to 0$ and $K \to 2\pi$ at the two unphysical kinematics ($m_K = m_\pi$ and $m_K = 2m_\pi$) wherein Maiani–Testa theorem [73] can be evaded. In another construction [99] the $K \to 2\pi$ matrix elements for $\Delta I = \frac{3}{2}$ transitions can be obtained to NLO using lattice computations of $K \to 2\pi$ with momentum insertion on one of the final state pions.

Indeed in a very interesting paper, Lellouch and Luscher have also proposed a method wherein $K \to 2\pi$ matrix elements may be directly calculated without using ChPT by relating them to finite volume correlation functions [100]. There is also a proposed method which makes use of dispersion relations to calculate physical $K \to \pi\pi$ amplitudes to all orders in ChPT [101]. We note that both of these methods make rather stringent demands on unitarity therefore their implementation, especially for the $\Delta I = 1/2$ case, may need full QCD simulations.

8. Unitarity triangle from *K*-decays

While determination of the unitarity triangle (UT) from B decays has been receiving considerable attention and is much in the news, it is useful to note that not only an independent determination of the UT can also be made purely from K-decays but it is important to do so. In principle, there are four physical processes that can be useful here, whereas any three of them would be sufficient.

1. ϵ, the indirect CP-violation parameter characterizing the CP-violation in $K_L \to 2\pi$.
2. The $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, a determination of which has been underway for a long time and a crude measurement now exists thanks to the two candidate events that have been seen, $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = (1.57^{+1.75}_{-.82}) \times 10^{-10}$ [102].
3. There is considerable experimental interest in measuring the $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$. This is a very interesting mode which is CP violating [103] and is theoretically extremely clean, but clearly an extremely difficult experimental challenge.
4. The direct CP-violation parameter ϵ'/ϵ. Although the experimental number is now quite precisely known [77,78], it can only be useful in the context of the UT, if the theory can be brought under-control. Renewed interest on the lattice, in the light of recent progress in maintenance of chiral symmetry on

the lattice (described briefly in preceding pages) gives some encouragement that perhaps a few years down the road we would be able to make use of the experimental result and translate it into the CP violation parameter η of the CKM paradigm. In the $\rho - \eta$ plane, ϵ'/ϵ (when the numerical value of ϵ is taken from experiment) would provide a horizontal line (actually a band due to the error in theory and in experiment).

As is well-known transplanting ϵ to ρ, η plane does require knowledge of the non-perturbative hadronic parameter, B_K. Fortunately as already mentioned, lattice calculations of B_K are now quite mature. In fact, several different discretization methods have been used to determine this important quantity. While the current accuracy is around 15%, efforts with dynamical quarks are underway and in 3–5 years we should expect the accuracy to improve appreciably.

The theory for $K^+ \to \pi^+\nu\bar{\nu}$ is also rather clean [24]. The basic process $s \to d\nu\bar{\nu}$ is dominated by the top quark. Conversion of that to $K^+ \to \pi^+\nu\bar{\nu}$ can be done using isospin by relating it to the thoroughly studied charge current process $K \to \pi e\nu$.

The observed Br, deduced on the basis of the two events seen so far, is consistent with the expectation from the SM; our [31] CKM fits give, $\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = (0.67 \pm 0.10) \times 10^{-10}$. Experimental efforts are underway to improve this measurement in the near future at BNL and further down the road at FNAL [104].

The decay $K_L^0 \to \pi^0\nu\bar{\nu}$ is fascinating as it is CP violating. The theory [24] in this case is even clearer then for the charged counterpart and $\mathrm{BR}(K_L^0 \to K^0\nu\bar{\nu}) = 1.5 \times 10^{-3}A^4\lambda^{10}\eta^2$. Our [31] fit value is $\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = (0.23 \pm 0.07) \times 10^{-10}$. So an experimental measurement would give a clean determination of the CKM phase η. Note though that ($A \approx V_{cb}$) the current accuracy in $A(\sim 7\%)$ should be improved otherwise it introduces significant error on the η determination. The $K_L \to \pi^0\nu\bar{\nu}$ experiment is clearly very challenging and it is receiving attention at KEK (E391), at BNL (KOPIO) and at FNAL (CKM) [104].

Given the intrinsic difficulties of this experiment and those of an accurate theoretical calculation of ϵ'/ϵ it would be interesting to see which of these is brought under control first.

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