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Tachyon dynamics in string theory

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Abstract. We summarize the recent developments in the study of time dependent solutions describing the rolling of a tachyon on a non-BPS D-brane system.

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Our understanding of the role of open string tachyons living on unstable D-brane systems in string theory has increased considerably over the last few years. In this talk I plan to give a general overview of the subject, with particular focus on the recent developments in the study of time dependent solutions. I shall use the convention $\hbar = c = \alpha' = 1$. In these units the tension of the fundamental string is $1/(2\pi)$.

We begin our discussion by reviewing the spectrum of D-branes in type-IIA and IIB superstring theories. Dp-branes are by definition p-dimensional extended objects on which fundamental open strings can end. It is well-known that type-IIA/IIB string theory contains stable BPS Dp-branes for even/odd p, and that these D-branes carry Ramond–Ramond charges. These D-branes are oriented, and have definite mass per unit p-volume known as tension. The tension of a BPS Dp-brane in type-IIA/IIB string theory is given by

$$\mathcal{T}_p = (2\pi)^{-p}\, g_{\rm s}^{-1}\,, \tag{1}$$

where $g_{\rm s}$ is the closed string coupling constant. The BPS D-branes are stable, and all the open string modes living on such a brane have mass$^2 \geq 0$. Since these branes are oriented, given a specific BPS Dp-brane, we shall call a Dp-brane with opposite orientation an anti-Dp-brane, or a $\bar{\rm D}p$-brane.

Besides these stable BPS Dp-branes, type-II string theories also contain in their spectrum unstable, non-BPS D-branes [1–4]. These branes have precisely those dimensions which BPS D-branes do not have. Thus type-IIA string theory has non-BPS Dp-branes for odd p and type-IIB string theory has non-BPS Dp-branes for even p. These branes are unoriented and also carry a given mass per unit p-volume, given by

$$\widetilde{\mathcal{T}}_p = \sqrt{2}\,(2\pi)^{-p}\, g_{\rm s}^{-1}\,. \tag{2}$$

The most important feature that distinguishes the non-BPS D-branes from BPS D-branes is that the spectrum of open strings on a non-BPS D-brane contains a single mode of negative mass^2 besides infinite number of other modes of mass$^2 \geq 0$. This mode, known as the tachyon, has mass^2 given by

$$m^2 = -\frac{1}{2}\,. \tag{3}$$

Another important feature that distinguishes a BPS Dp-brane from a non-BPS Dp-brane is that unlike a BPS Dp-brane which is charged under the Ramond–Ramond (RR) $(p+1)$-form gauge field of string theory, a non-BPS D-brane is neutral under these gauge fields. Various properties of non-BPS D-branes have been reviewed in [5,6]. We shall use the convention that a D-brane will generically refer to a BPS D-brane, and when we want to refer to a non-BPS D-brane we shall explicitly use the adjective non-BPS.

Although a BPS Dp-brane does not have a tachyonic mode, if we consider a coincident BPS Dp-brane–$\bar{\text{D}}p$-brane pair, then the open string stretched from the brane to the anti-brane (or vice-versa) also has a tachyonic mode. This gives rise to two tachyonic modes on a coincident brane–anti-brane pair. The mass^2 of each of these tachyonic modes is given by the same expression as (3).

Our main goal will be to study the decay of (a system of) branes via classical dynamics of these tachyonic modes. There are several reasons why this problem is of interest. First of all, the very existence of such systems in string theory make them interesting objects to study. They also provide examples of solvable time-dependent solutions in string theory. Such examples are few in number. Furthermore, this study may have cosmological significance since such unstable D-brane systems are likely to have been present in the early Universe.

The dynamics of open strings living on a Dp-brane is described by a $(p+1)$-dimensional (string) field theory, defined such that the free field quantization of the field theory reproduces the spectrum of open strings on the Dp-brane, and the S-matrix elements computed from this field theory reproduce the S-matrix elements of open string theory on the D-brane. On a non-BPS D-brane, the existence of a single scalar tachyonic mode shows that the corresponding open string field theory must contain a real scalar field T with mass$^2 = -1/2$, whereas the same reasoning shows that open string field theory associated with a coincident brane–anti-brane system must contain two real scalar fields, or equivalently one complex scalar field T, of mass$^2 = -1/2$. However, these fields have non-trivial coupling to all the infinite number of other fields in open string field theory, and hence one cannot study the dynamics of these tachyonic modes in isolation. Furthermore, since the |mass^2| of the tachyonic modes is of the same order of magnitude as that of the other heavy modes of the string, one cannot work with a simple low energy effective action obtained by integrating out the other heavy modes of the string. This is what makes the analysis of the tachyon dynamics non-trivial. Nevertheless, it is convenient to state the results of the analysis in terms of an effective action $S_{\text{eff}}(T, \ldots)$ obtained by formally integrating out all the positive mass^2 fields. This is what we shall do [6a]. Here $\ldots$ stands for all the massless bosonic fields, which in the case of non-BPS Dp-branes include one gauge field and $(9-p)$ scalar fields associated with the transverse coordinates. For Dp–$\bar{\text{D}}p$ brane pair the massless fields consist of two $U(1)$ gauge fields and $2(9-p)$ transverse scalar fields.

First we shall state two properties of $S_{\rm eff}(T,\ldots)$ which are trivially derived from the analysis of the tree-level S-matrix:

(1) For a non-BPS D-brane the tachyon effective action has a Z_2 symmetry under which $T \to -T$, wheras for a brane–anti-brane system the tachyon effective action has a phase symmetry under which $T \to \mathrm{e}^{i\alpha} T$.

(2) Let $V(T)$ denote the tachyon effective potential, defined such that for space-time independent field configuration, and with all the massless fields set to zero, the tachyon effective action $S_{\rm eff}$ has the form:

$$-\int \mathrm{d}^{p+1}x\, V(T)\,. \tag{4}$$

In that case $V(T)$ has a maximum at $T=0$. This is a trivial consequence of the fact that the mass^2 of the field T is given by $V''(T=0)$, and this is known to be negative.

The question that we shall be most interested in is whether $V(T)$ has a (local) minimum, and if it does, then how does the theory behave around this minimum? The answer to this question is summarized in the following three 'conjectures' [1,3,7–11] [11a]:

(1) $V(T)$ does have a pair of global minima at $T=\pm T_0$ for the non-BPS D-brane, and a one-parameter (α) family of global minima at $T=T_0\mathrm{e}^{i\alpha}$ for the brane–anti-brane system. At this minimum the tension of the original D-brane configuration is exactly canceled by the negative contribution of the potential $V(T)$. Thus

$$V(T_0)+\mathcal{E}_p = 0\,, \tag{5}$$

where

$$\mathcal{E}_p = \begin{cases} \widetilde{\mathcal{T}}_p & \text{for non-BPS D}p\text{-brane} \\ 2\mathcal{T}_p & \text{for D}p\text{–}\bar{\text{D}}p\text{ brane pair} \end{cases}\,. \tag{6}$$

Thus the total energy density vanishes at the minimum of the tachyon potential. This has been illustrated in figure 1. Since $V(T)+\mathcal{E}_p$ denotes the total energy density of the system, it is more natural to call this the tachyon potential. From now on we shall adopt this convention and denote this sum by $V(T)$.

(2) Since the total energy density vanishes at $T=T_0$, and furthermore, neither the non-BPS D-brane nor the brane–anti-brane system carries any RR charge, it is natural to identify the configuration $T=T_0$ as the vacuum without any D-brane. This in turn implies that there are no physical perturbative open string states around the minimum of the potential, since open string states live only on D-branes. This is counterintuitive, since in conventional field theories the number of perturbative physical states do not change as we go from one extremum of the potential to another extremum.

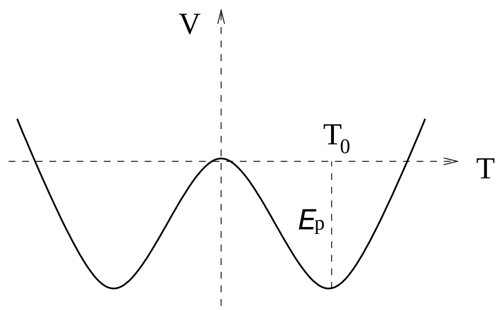

Figure 1. The tachyon potential on an unstable D-brane or brane–anti-brane system in superstring theories.

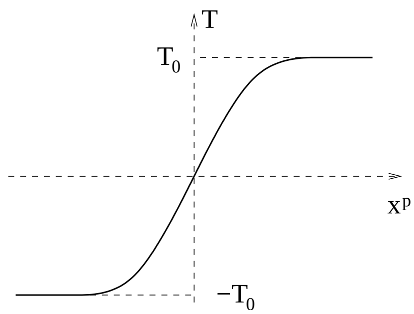

Figure 2. The kink solution on a non-BPS D-brane.

(3) Although there are no perturbative physical states around the minimum of the potential, the equations of motion derived from the tachyon effective action $S_{\text{eff}}(T, \ldots)$ does have non-trivial time-independent classical solutions. These solutions represent lower-dimensional D-branes. Some examples are given below:

(a) The tachyon effective action on a non-BPS Dp-brane admits a classical kink solution as shown in figure 2. This solution depends on only one of the spatial coordinates, labeled by x^p in the figure, such that T approaches T_0 as $x^p \to \infty$ and $-T_0$ as $x^p \to -\infty$, and interpolates between these two values around $x^p = 0$. Since the total energy density vanishes for $T = \pm T_0$, we see that for the above configuration the energy density is concentrated around a $(p-1)$-dimensional subspace $x^p = 0$. This can be identified with a BPS D-$(p-1)$-brane in the same theory [9,11].

(b) There is a similar solution on a brane–anti-brane system, where the imaginary part of the tachyon field is set to zero, and the real part takes the form given in figure 2. This is not a stable solution, but describes a non-BPS D-$(p-1)$-brane in the same theory [1,3].

(c) Since the tachyon field T on a Dp–$\bar{\text{D}}p$-brane system is a complex field, one can also construct a vortex solution where T is a function of two of the spatial coordinates (say x^{p-1} and x^p) and takes the form:

$$T = T_0\, f(\rho) e^{i\theta}\,, \tag{7}$$

where

$$\rho = \sqrt{(x^{p-1})^2 + (x^p)^2}, \quad \theta = \tan^{-1}(x^p/x^{p-1})\,, \tag{8}$$

are the polar coordinates and the function $f(\rho)$ has the property:

$$f(\infty) = 1, \quad f(0) = 0\,. \tag{9}$$

Thus the potential energy associated with the solution vanishes as $\rho \to \infty$. Besides the tachyon the solution also contains an accompanying background gauge field which makes the covariant derivative of the tachyon fall off sufficiently fast for large ρ so that the net energy density is concentrated around the $\rho = 0$ region. This gives a codimension two-soliton solution which can be identified as a BPS D-($p-2$)-brane in the same theory [3,12].

(d) If we take a pair of non-BPS D-branes, then the D-brane effective field theory around $T = 0$ contains a $U(2)$ gauge field, and the tachyon transforms in the adjoint representation of this gauge field. At the minimum of the tachyon potential the $SU(2)$ part of the gauge group is broken to $U(1)$ by the vacuum expectation value of the tachyon. As a result the theory contains 't Hooft–Polyakov monopole solution which depends on three of the spatial coordinates. This describes a codimension 3-brane and can be identified as a BPS D-($p-3$)-brane in the same theory [11,12].

(e) If we consider a system of two Dp-branes and two $\bar{\text{D}}p$-branes, all along the same plane, then the D-brane world-volume theory has a $U(2)\times U(2)$ gauge field, and a 2×2 matrix values complex tachyon field, transforming in the (2,2) representation of the gauge group. Let $A^{(1)}_\mu$ and $A^{(2)}_\mu$ denote the gauge fields in the two $SU(2)$ gauge groups. Then we can construct a codimension 4-brane solution where the fields depend on four of the spatial coordinates, and have the asymptotic behaviour:

$$T \simeq T_0 U, \quad A^{(1)}_\mu \simeq i\partial_\mu U U^{-1}, \quad A^{(2)}_\mu = 0, \tag{10}$$

where U is an $SU(2)$ matrix valued function, corresponding to the identity map (winding number one map) from the surface S^3 at spatial infinity to the $SU(2)$ group manifold. This describes a BPS D-($p-4$)-brane in the same theory [3,12].

Quite generally if we begin with sufficient number of non-BPS D9-branes in type-IIA string theory, or D9–$\bar{\text{D}}$9-branes in type-IIB string theory, we can describe any lower-dimensional D-brane as classical solution in this open string field theory [10–12]. This has led to a classification of D-branes using a branch on mathematics known as K-theory [10,11]. This has also led to the suggestion that perhaps we can give a non-perturbative formulation of string theory in terms of open string field theory on space-filling D-brane system. For this to work, we need to also find ways of describing closed strings and Neveu–Schwarz (NS) 5-branes in this open string field theory. This has not yet been achieved to complete satisfaction.

So far we have only discussed time-independent solutions of the tachyon equations of motion. But one could also ask questions about time-dependent solutions. In particular, given that the tachyon potential on a non-BPS Dp-brane or a Dp–$\bar{\text{D}}p$ pair has the form given in figure 1, one could ask: what happens if we displace the tachyon from the maximum of the potential and let it roll down towards its minimum? If T had been an ordinary scalar field then the answer is simple: the tachyon field T will simply oscillate about the minimum T of the potential, and in the absence of any dissipative force (as is the case at the classical level) the oscillation will continue for ever. The energy density T_{00} will remain constant during this oscillation, but other components of the energy–momentum tensor, e.g., the pressure $p(x^0)$, defined through $T_{ij} = p\,\delta_{ij}$ for $1 \le i, j \le p$, will oscillate about their average value [12a]. However, for the case of the string theory tachyon the answer is different and somewhat surprising [13,14]. It turns out that in this case the energy density remains constant as in the case of a usual scalar field, but the pressure, instead of oscillating about an average value, goes to zero asymptotically. The evolution of the pressure follows the curve:

$$p(x^0) = -\mathcal{E}_p\,\widetilde{f}(x^0)\,, \tag{11}$$

where $\mathcal{E}_p$ is given by (6), and the function $\widetilde{f}(x^0)$ depends on the initial energy density T_{00} of the system. In order to specify the form of $f(x^0)$ we need to consider two different cases:

(1) $T_{00} \le \mathcal{E}_p$: In this case we can parametrize the solution by a parameter $\widetilde{\lambda}$ defined through the relation

$$T_{00} = \frac{\mathcal{E}_p}{2}(1 + \cos(2\pi\widetilde{\lambda}))\,. \tag{12}$$

T_{00} includes the contribution from the tension of the D-brane(s) as well as the tachyon kinetic and potential energy. Since the total energy density available to the system is less than $\mathcal{E}_p$, the energy density at the maximum of the tachyon potential describing the original brane configuration, at some instant of time during its motion the tachyon is expected to come to rest at some point away from the maximum of the potential. We can choose this instant of time as $x^0 = 0$. The function $\widetilde{f}(x^0)$ in this case takes the form:

$$\widetilde{f}(x^0) = \frac{1}{1 + \mathrm{e}^{\sqrt{2}x^0}\sin^2(\widetilde{\lambda}\pi)} + \frac{1}{1 + \mathrm{e}^{-\sqrt{2}x^0}\sin^2(\widetilde{\lambda}\pi)} - 1\,. \tag{13}$$

From this we see that as $x^0 \to \infty$, $\widetilde{f}(x^0) \to 0$. Thus the pressure vanishes asymptotically.

Note that for $\widetilde{\lambda} = \frac{1}{2}$, both T_{00} and $p(x^0)$ vanish identically. Thus this solution has the natural interpretation as the tachyon being placed at the minimum of its potential. The solution for $\widetilde{\lambda} = \frac{1}{2} + \epsilon$ is identical to the one at $\widetilde{\lambda} = \frac{1}{2} - \epsilon$, thus the inequivalent set of solutions are obtained by restricting $\widetilde{\lambda}$ to the range $[-\frac{1}{2}, \frac{1}{2}]$.

(2) $T_{00} \geq \mathcal{E}_p$: In this case we can parametrize the solution by a parameter $\widetilde{\lambda}$ defined through the relation

$$T_{00} = \frac{\mathcal{E}_p}{2}(1 + \cosh(2\pi\widetilde{\lambda}))\,. \tag{14}$$

Since the total energy density available to the system is larger than $\mathcal{E}_p$, at some instant of time during its motion the tachyon is expected to pass the point $T = 0$ where the potential has a maximum. We can choose our initial condition such that at $x^0 = 0$ the tachyon is at the maximum of the potential and has a non-zero velocity. The function $\widetilde{f}(x^0)$ in this case takes the form

$$\widetilde{f}(x^0) = \frac{1}{1 + \mathrm{e}^{\sqrt{2}x^0}\sinh^2(\widetilde{\lambda}\pi)} + \frac{1}{1 + \mathrm{e}^{-\sqrt{2}x^0}\sinh^2(\widetilde{\lambda}\pi)} - 1\,. \tag{15}$$

As $x^0 \to \infty$, $\widetilde{f}(x^0) \to 0$, the pressure vanishes asymptotically. This result can be trusted for $|\widetilde{\lambda}| \leq \sinh^{-1} 1$.

The assertion that around the tachyon vacuum there are no physical open string states, implies that there is no small oscillation of finite frequency around the minimum of the tachyon potential. The lack of oscillation in the pressure is consistent with this result. However, the existence of classical solutions with arbitrarily small energy density (which can be achieved by taking $\widetilde{\lambda}$ close to $1/2$ in (12)) still poses a puzzle, since it indicates that quantization of open string field theory around the tachyon vacuum does give rise to non-trivial quantum states which in the semi-classical limit are described by the solutions that we have found. These states are either new states in string theory, or provide an alternative description of closed string states. At present the precise interpretation of these solutions is not known.

If we are considering the rolling of the tachyon on a non-BPS Dp-brane (and not on a Dp–$\bar{\text{D}}p$-brane pair), then, besides producing the energy–momentum tensor, the rolling tachyon solutions described above also act as source of the massless RR p-form field $C_{1\ldots p}$. In the first case this source term is proportional to [15]

$$\sin(\widetilde{\lambda}\pi)\left[\frac{\mathrm{e}^{x^0/\sqrt{2}}}{1 + \sin^2(\widetilde{\lambda}\pi)\mathrm{e}^{\sqrt{2}x^0}} - \frac{\mathrm{e}^{-x^0/\sqrt{2}}}{1 + \sin^2(\widetilde{\lambda}\pi)\mathrm{e}^{-\sqrt{2}x^0}}\right], \tag{16}$$

whereas in the second case it is proportional to

$$\sinh(\widetilde{\lambda}\pi)\left[\frac{\mathrm{e}^{x^0/\sqrt{2}}}{1 + \sinh^2(\widetilde{\lambda}\pi)\mathrm{e}^{\sqrt{2}x^0}} - \frac{\mathrm{e}^{-x^0/\sqrt{2}}}{1 + \sinh^2(\widetilde{\lambda}\pi)\mathrm{e}^{-\sqrt{2}x^0}}\right]. \tag{17}$$

All the results stated above are derived using conformal field theory methods and are exact at the open string tree-level. However, it is natural to ask: is there an effective action involving the tachyon field which reproduces some of these results, at least qualitatively? It turns out that there is such an effective action, given by [15–20]

$$S = \int \mathrm{d}^{p+1} x\, \mathcal{L}\,,$$
$$\mathcal{L} = -V(T)\,\sqrt{-\det \mathbf{A}}\,, \tag{18}$$

where

$$\mathbf{A}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T\,, \tag{19}$$

and

$$V(T) \simeq \mathrm{e}^{-T/\sqrt{2}} \quad \text{for large } T\,. \tag{20}$$

$V(T)$ denotes the tachyon potential including the brane tension. It has a maximum at $T = 0$, and in this parametrization has its minimum at infinity. The energy–momentum tensor computed from the action (18) is given by [20a]

$$T_{\mu\nu} = \frac{V(T)\,\partial_\mu T \partial_\nu T}{\sqrt{1 + \eta^{\rho\sigma}\partial_\sigma T \partial_\rho T}} - V(T)\,\eta_{\mu\nu}\,\sqrt{1 + \eta^{\rho\sigma}\partial_\rho T \partial_\sigma T}\,. \tag{21}$$

We shall first verify that the action (18) produces the correct large x^0 behaviour of the pressure for spatially homogeneous, time-dependent field configurations. For such configurations the conserved energy density for large T is given by

$$T_{00} = V(T)(1 - (\partial_0 T)^2)^{-1/2} \simeq \mathrm{e}^{-T/\sqrt{2}}(1 - (\partial_0 T)^2)^{-1/2}\,. \tag{22}$$

Since T_{00} is conserved, we see that for any given T_{00}, as $T \to \infty$, $\partial_0 T \to 1$. In particular, for large x^0 the solution has the form

$$T = x^0 + C\mathrm{e}^{-\sqrt{2}x^0} + \mathcal{O}(\mathrm{e}^{-2\sqrt{2}x^0})\,. \tag{23}$$

In order to see that (23) gives the correct form of the solution we simply need to note that the leading contribution to T_{00} computed from this configuration remains constant in time:

$$T_{00} \simeq \frac{1}{\sqrt{2\sqrt{2}C}}\,. \tag{24}$$

The pressure associated with this configuration is given by

$$p = T_{11} = -V(T)\,(1 - (\partial_0 T)^2)^{1/2} \simeq -\sqrt{2\sqrt{2}C}\mathrm{e}^{-\sqrt{2}x^0}\,. \tag{25}$$

This is in precise agreement with (13) for large x^0.

Note that at late time $\partial_\mu\partial_\nu T \to 0$ and $T \to \infty$. Since it is at late time that the results of the effective field theory agree with those of string theory, the natural guess would be that the effective action given above is a valid description of the system for large T and small $\partial_\mu\partial_\nu T$.

Next we need to verify if this action reproduces the conjectured static properties of the tachyon effective action. First of all, note that $V(T)$ vanishes at the minimum by construction, thus the first conjecture is automatically satisfied. We shall soon analyse the spectrum of perturbative states around the tachyon vacuum and verify the second conjecture. However, the verification of the third conjecture involves construction of the kink and other topological soliton solutions. Since such solutions contain regions in which T is finite and $\partial_\mu\partial_\nu T$ is finite or large, the effective action given in (18) is not a valid approximation in this case. Thus, we cannot verify the third conjecture using this effective action.

Let us now demonstrate the absence of perturbative states upon quantization of the theory around the tachyon vacuum. Since *a priori* it is not clear how to quantize a non-linear theory of this type, we shall use a pragmatic definition of the absence of perturbative states. Since in conventional field theory perturbative states are associated with plane wave solutions, we shall assume that absence of perturbative quantum states implies absence of plane-wave solutions (which are not pure gauge) and vice versa. Thus we need to show the absence of plane-wave solutions around the tachyon vacuum in this theory.

This leads us to the analysis of classical solutions in this theory. Since around the tachyon vacuum $V(T) = 0$ and hence the action (18) vanishes, it is more convenient to work in the Hamiltonian formalism [15,21–25]. Defining the momentum conjugate to T as

$$\Pi(x) = \frac{\delta S}{\delta(\partial_0 T(x))} = \frac{V(T)\partial_0 T}{\sqrt{1-(\partial_0 T)^2 + (\vec{\nabla}T)^2}}\,, \tag{26}$$

we can construct the Hamiltonian H:

$$\begin{aligned} H &= \int \mathrm{d}^p x\,(\Pi\partial_0 T - \mathcal{L}) \equiv \int \mathrm{d}^p x \mathcal{H}, \\ \mathcal{H} &= T_{00} = \sqrt{\Pi^2 + (V(T))^2}\,\sqrt{1+(\vec{\nabla}T)^2}\,. \end{aligned} \tag{27}$$

The equations of motion derived from this Hamiltonian take the form

$$\begin{aligned} \partial_0\Pi(x) = -\frac{\delta H}{\delta T(x)} &= \partial_j\left(\sqrt{\Pi^2+V^2}\,\frac{\partial_j T}{\sqrt{1+(\vec{\nabla}T)^2}}\right) \\ &\quad -\frac{V(T)V'(T)}{\sqrt{\Pi^2+V^2}}\,\sqrt{1+(\vec{\nabla}T)^2}\,, \end{aligned} \tag{28}$$

$$\partial_0 T(x) = \frac{\delta H}{\delta\Pi(x)} = \frac{\Pi}{\sqrt{\Pi^2+V^2}}\,\sqrt{1+(\vec{\nabla}T)^2}\,. \tag{29}$$

In the limit of large T (i.e., near the tachyon vacuum) at fixed Π, we can ignore the $V^2 \simeq \mathrm{e}^{-\sqrt{2}T}$ term, and the Hamiltonian and the equations of motion take the form

$$H = \int \mathrm{d}^p x\,|\Pi|\sqrt{1+(\vec{\nabla}T)^2}\,, \tag{30}$$

$$\partial_0\Pi(x) = \partial_j\left(|\Pi|\,\frac{\partial_j T}{\sqrt{1+(\vec{\nabla}T)^2}}\right), \tag{31}$$

$$\partial_0 T(x) = \frac{\Pi}{|\Pi|}\sqrt{1+(\vec{\nabla}T)^2}\,. \tag{32}$$

From (32), we see that in this limit we have $(\partial_0 T)^2 - (\vec{\nabla}T)^2 = 1$.

These equations can be rewritten in a suggestive form by defining [15]

$$u_\mu \equiv -\partial_\mu T, \quad \epsilon(x) \equiv |\Pi(x)|/\sqrt{1+(\vec{\nabla}T)^2}\,. \tag{33}$$

Equations (31) and (32) then take the form

$$\eta^{\mu\nu}u_\mu u_\nu = -1, \quad \partial_\mu(\epsilon(x)u^\mu) = 0\,. \tag{34}$$

Expressed in terms of these new variables, $T_{\mu\nu}$ given in (21) take the form:

$$T_{\mu\nu} = \epsilon(x)u_\mu u_\nu\,, \tag{35}$$

where we have used the small $V(T)$ approximation and used the equations of motion (31), (32). These are precisely the equations governing the motion of non-rotating, non-interacting dust, with u_μ interpreted as the local $(p+1)$-velocity vector [15], and $\epsilon(x)$ interpreted as the local rest mass density. Conversely, any configuration describing flow of non-rotating, non-interacting dust can be interpreted as a solution of the equations of motion (31), (32).

It is now clear that there are no plane-wave solutions in this classical theory. A system of non-interacting dust, if compressed, remains in that compressed state without responding back. Thus if we begin with an initial static configuration with an inhomogeneous distribution of energy, this disturbance will not propagate. On the other hand a plane-wave solution always propagates. Thus the particular field theory described here does not have any plane-wave solution, and is not expected to have any perturbative physical state upon quantization.

During last year there has been many attempts to use the tachyon effective field theory for describing various aspects of cosmology. Clearly one needs a much better understanding of the effect of quantum corrections before one can have a complete understanding of tachyon cosmology. However, I would like to conclude by saying that brane–anti-brane annihilation process is likely to have taken place in the early Universe, and it will be interesting to see if we can find any signature of such processes in the present day cosmology.

References

[1] A Sen, *J. High Energy Phys.* **9808**, 010 (1998); arXiv:hep-th/9805019

[2] O Bergman and M R Gaberdiel, *Phys. Lett.* **B441**, 133 (1998); arXiv:hep-th/9806155

[3] A Sen, *J. High Energy Phys.* **9809**, 023 (1998); arXiv:hep-th/9808141

[4] A Sen, *J. High Energy Phys.* **9810**, 021 (1998); arXiv:hep-th/9809111

[5] A Sen, arXiv:hep-th/9904207

[6] A Lerda and R Russo, *Int. J. Mod. Phys.* **A15**, 771 (2000); arXiv:hep-th/9905006

[6a] At this stage we would like to remind the reader that our analysis will be only at the level of classical open string field theory, and hence integrating out the heavy fields simply amounts to eliminating them by their equations of motion

[7] A Sen, *J. High Energy Phys.* **9808**, 012 (1998); arXiv:hep-th/9805170
[8] A Sen, *J. High Energy Phys.* **9912**, 027 (1999); arXiv:hep-th/9911116
[9] A Sen, *J. High Energy Phys.* **9812**, 021 (1998); arXiv:hep-th/9812031
[10] E Witten, *J. High Energy Phys.* **9812**, 019 (1998); arXiv:hep-th/9810188
[11] P Horava, *Adv. Theor. Math. Phys.* **2**, 1373 (1999); arXiv:hep-th/9812135
[11a] Although initially these properties were conjectured, by now there is sufficient evidence for these conjectures so that one can refer to them as results rather than conjectures
[12] J Majumder and A Sen, deformation, *J. High Energy Phys.* **0006**, 010 (2000); arXiv:hep-th/0003124
[12a] The energy–momentum tensor $T_{\mu\nu}$ is confined to the plane of the original D-brane, and hence all expressions for $T_{\mu\nu}$ are accompanied by a δ-function in the transverse coordinates which will often be omitted for brevity. Also, only the components of the stress tensor along the world-volume of the brane are non-zero, i.e., $T_{\mu\nu} \neq 0$ only for $0 \leq \mu, \nu \leq p$
[13] A Sen, *J. High Energy Phys.* **0204**, 048 (2002); arXiv:hep-th/0203211
[14] A Sen, *J. High Energy Phys.* **0207**, 065 (2002); arXiv:hep-th/0203265
[15] A Sen, *Mod. Phys. Lett.* **A17**, 1797 (2002); arXiv:hep-th/0204143
[16] A Sen, *J. High Energy Phys.* **9910**, 008 (1999); arXiv:hep-th/9909062
[17] M R Garousi, *Nucl. Phys.* **B584**, 284 (2000); arXiv:hep-th/0003122; *Nucl. Phys.* **B647**, 117 (2002); arXiv:hep-th/0209068
[18] E A Bergshoeff, M de Roo, T C de Wit, E Eyras and S Panda, *J. High Energy Phys.* **0005**, 009 (2000); arXiv:hep-th/0003221
[19] J Kluson, *Phys. Rev.* **D62**, 126003 (2000); arXiv:hep-th/0004106
[20] A Sen, arXiv:hep-th/0209122.
[20a] In writing down the expression for the energy–momentum tensor, it will be understood that these are localized on the plane of the brane by a position space delta function in the transverse coordinates. Also only the components of the energy–momentum tensor along the world-volume of the brane are non-zero
[21] G W Gibbons, K Hori and P Yi, *Nucl. Phys.* **B596**, 136 (2001); arXiv:hep-th/0009061
[22] P Yi, *Nucl. Phys.* **550**, 214 (1999); arXiv:hep-th/9901159
[23] U Lindstrom and R von Unge, *Phys. Lett.* **B403**, 233 (1997); hep-th/9704051
H Gustafsson and U Lindstrom, *Phys. Lett.* **B440**, 43 (1998); hep-th/9807064
U Lindstrom, M Zabzine and A Zheltukhin, *J. High Energy Phys.* **9912**, 016 (1999); hep-th/9910159
[24] A Sen, *J. Math. Phys.* **42**, 2844 (2001); arXiv:hep-th/0010240
[25] G Gibbons, K Hashimoto and P Yi, *J. High Energy Phys.* **0209**, 061 (2002); arXiv:hep-th/0209034