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Transplanckian collisions in TeV scale gravity

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Abstract. Collisions at transplanckian energies offer model independent tests of TeV scale gravity. One spectacular signal is given by black-hole production, though a full calculation of the corresponding cross-section is not yet available. Another signal is given by gravitational elastic scattering, which may be less spectacular but which can be nicely computed in the forward region using the eikonal approximation. In this talk I discuss the distinctive signatures of eikonalized scattering at future accelerators.

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One of the puzzles of Nature is the extreme weakness of gravity compared to all other interactions. In particular the small ratio $G_{\mathrm{N}}/G_{\mathrm{F}} \sim 10^{-33}$ between the Newton and Fermi constants represents what is normally called the 'hierarchy problem'.

Until a few years ago, the scenario in which to formulate the hierarchy problem assumed that the Planck energy $M_{\mathrm{P}} = G_{\mathrm{N}}^{-1/2} = 10^{19}$ GeV is a truly fundamental scale, at which quantum gravity phenomena become important. In this formulation the hierarchy is associated to a 'small' vacuum expectation value (VEV) $\langle H \rangle \sim G_{\mathrm{F}}^{-1/2} \sim 100$ GeV for the Higgs field. A natural explanation of the hierarchy requires stability of this small VEV against quantum corrections. Supersymmetric models realize this stability through non-trivial cancellations between fermionic and bosonic quantum corrections. Technicolor models explain the hierarchy by simply having a composite Higgs boson, whose VEV is automatically insensitive to ultraviolet quantum corrections. Arkani-Hamed, Dimopoulos and Dvali (ADD) [1] have instead suggested to formulate (and may be solve) the problem in a new scenario, where $M_{\mathrm{P}} \sim 10^{19}$ GeV is far from being the fundamental scale of quantum gravity. Their assumption is that there exists a number n of extra space-like dimensions compactified at a radius R. Their basic remark is that the relation between the microscopic Planck scale M_D, describing genuine quantum gravity effects in $4+n$ dimensions, and the macroscopic $M_{\mathrm{P}} = G_{\mathrm{N}}^{-1/2}$, describing gravity at distances larger than the compactification radius R, is

$$M_{\mathrm{P}}^2 = M_D^{2+n} R^n. \tag{1}$$

For $R \gg 1/M_D$, the macroscopic Planck mass M_{P} can be much larger than the true scale of quantum gravity M_D. In particular ADD have made the bold proposal that M_D coincides essentially with the weak scale, i.e., $M_D \sim 1$ TeV. In this case the right value of $M_{\rm P}$ is reproduced for rather large compactification radii ranging from $R \sim 1$ fm for $n = 6$ to $R \sim 1$ mm for $n = 2$. Such large values of R are not in contradiction with the present gravity experiments, as Newton's law has been tested only down to distances just below a millimeter. On the other hand, gravity excluded, all the observed particles and interactions are very well-described by a $3+1$-dimensional quantum field theory, the standard model, down to length scales smaller than the Z boson Compton wavelength $\lambda_Z \sim 10^{-3}$ fm. In order to account for this fact, ADD assume that the SM degrees of freedom are localized on a defect extending over the three ordinary non-compact directions in space, a 3-brane. The possibility to localize particles on defects, or submanifolds, was remarked a while back in field theory [2] and is also naturally realized in string theory by Dirichlet-branes [3]. A possible string realization of the ADD proposal was first given in ref. [4]. Therefore, as long as the size of the brane is somewhat smaller than the weak scale, SM particles behave as ordinary $3+1$-dimensional degrees of freedom up to the energies explored so far.

As it stands, the ADD proposal is a reformulation of the hierarchy problem and not yet a solution [5]. Instead of the small Higgs VEV of the old formulation, we now need to explain why the compactification volume R^n is much bigger that its most natural scale $1/M_D^n$:

$$R^n M_D^n \sim 10^{33}. \tag{2}$$

As we are dealing with gravity, R is a dynamical degree of freedom, a scalar from the point of view of four dimensions. Since we want a large $\langle R\rangle$ the scalar potential $V(R)$ will have to be much flatter than naively expected at large values of R. As far as we know, the most natural way to achieve such flat potentials is by invoking supersymmetry. So, if the ADD scenario is realized in Nature it is likely to be so together with supersymmetry at some stage. Notice that in the conventional formulation of the hierarchy problem supersymmetry is invoked to ensure a flat potential (small mass) at small values of the Higgs field. As a matter of fact, the ADD proposal maps a small VEV problem into a basically equivalent large VEV problem. However in the new scenario the hierarchy problem has become a sort of cosmological constant problem. Indeed a vacuum energy density Λ^{4+n} would add to the radius potential a term $\sim \Lambda^{4+n}R^n$. This grows very fast at large R and so we expect [5] that Λ^{4+n} should be much smaller than its natural value $(\mathrm{TeV})^{4+n}$.

There are two classes of laboratory tests of this scenario. The first is given by the search for deviations from Newton's law at short but macroscopic distances. This is done in table top experiments. These deviations could be determined by the light moduli, like the radius R [5], or by the lowest Kaluza–Klein (KK) $J = 2$ modes. Another source of deviation could be the lowest KK mode of a bulk vector field gauging baryon number [6]. At present, $O(1)$ deviations from Newton's law have been excluded down to a length ~ 200 μm [7], while forces that have a strength $>10^4$ of gravity are bounded to have a range smaller than $20\,\mu$m [8,9]. Notice that this class of effects crucially depends on the features of the compactification manifold at large 'lengths', as they determine the masses of the lightest modes. For instance the presence of an extra scale $L \ll R$ associated to the curvature of the compactification manifold, generically leads to a mass gap $1/L$, instead of $1/R$, in the KK spectrum (see for instance ref. [10]).

The second class of tests is given by high-energy collisions [11]. In this case we deal with either gravitons at virtuality $Q \gg 1/R$ or with real gravitons measured with too poor an energy resolution to distinguish individual KK levels [11a]. Therefore we can take the limit $R \to \infty$ and work as if our brane were embedded in $(4+n)$-dimensional Minkowski space. (If the compactification manifold had curvature length $L \ll R$, then the same reasoning would apply for energies $E \gg 1/L$). Gravity couples with a strength $(E/M_D)^{n+2}$, and so we can distinguish three energy regimes. At $E \ll M_D$ we are in the cisplanckian regime. Here gravity is weak, and the signals involve the emission of a few graviton quanta, which escape undetected into the extra dimensions. Interesting examples are $e^+e^- \to \gamma + \text{graviton} = \gamma + \not{E}$ or $pp \to \text{jet} + \not{E}$ or even the invisible decay of the Higgs into just one graviton [12]. The differential cross-section for real graviton emission can be predicted in a model independent way in terms of just one parameter M_D (only in the case of Higgs decay another parameter enters). On the other hand, virtual graviton exchange is dominated already at tree level by uncalculable UV effects. These amplitudes are therefore associated to a new class of contact interactions whose coefficients depend on the details of the fundamental theory of quantum gravity. The second regime is the planckian one where $E \sim M_D$, which would give experimental access to the theory of quantum gravity. The signals here are highly model dependent, meaning that this is the regime where the most relevant experimental information will be extracted. If string theory is at the core, then one characteristic signal is given by the observation of Regge excitations [13]. Finally there is the transplanckian regime $E \gg M_D$, which is the subject of this talk. Though very naively one would expect things to be completely out of control, in the transplanckian regime gravity becomes rather simple: it is basically described by classical general relativity. As such, the transplanckian regime, like the cisplanckian, can offer model independent tests of the large extra dimension scenario.

To better understand the transplanckian regime it is useful to do some dimensional analysis working in units where $c = 1$ but keeping $\hbar \neq 1$. Using the $4+n$-dimensional Newton constant G_D ($[G_D] = \text{length}^{n+1} E^{-1}$) and the center of mass (c.m.) energy $\sqrt{s}$ we have

$$M_D^{n+2} = \hbar^{n+1}/G_D, \quad \lambda_{\mathrm{P}}^{n+2} = \hbar G_D, \quad \lambda_{\mathrm{B}} = \hbar/\sqrt{s}, \tag{3}$$

where the Planck length λ_{P} represents the length below which quantum gravity fluctuations of the geometry are important, while λ_{B} is the de Broglie wavelength of the scattering quanta in the c.m. Combining G_D and $\sqrt{s}$, we can form the Schwarzschild radius of a system with c.m. energy $\sqrt{s}$ [14]

$$R_S = \frac{1}{\sqrt{\pi}} \left[\frac{8\Gamma((n+3)/2)}{(n+2)} \right]^{1/(n+1)} (G_D \sqrt{s})^{1/(n+1)}. \tag{4}$$

This is the length at which curvature effects become significant. In the limit $\hbar \to 0$, with G_D and $\sqrt{s}$ fixed, M_D vanishes, showing that classical physics correspond to transplanckian (macroscopically large) energies. Moreover, in the same limit, R_{S} remains finite, while the two length scales λ_{P} and λ_{B} go to zero. Therefore, the transplanckian regime corresponds to a classical limit in which the length scale $R_{\rm S}$ characterizes the dynamics,

$$\sqrt{s} \gg M_D \quad \Rightarrow \quad R_{\rm S} \gg \lambda_{\rm P} \gg \lambda_{\rm B}. \tag{5}$$

We can see this more explicitly by considering the classical scattering angle for a collision with impact parameter b. By simple dimensional arguments it is $\theta \sim G_D\sqrt{s}/b^{n+1} = (R_{\rm S}/b)^{n+1}$. This shows that by increasing $\sqrt{s}$ we can sill obtain a finite θ by going to large b, where short distance quantum gravity effects are suppressed. More precisely, in order to describe the collision classically, two conditions must be met: (i) $b \gg \lambda_{\rm P}$ in order to suppress quantum gravity fluctuations; (ii) $\theta L/\hbar = \theta b\sqrt{s}/\hbar \gg 1$ [15] in order to suppress ordinary quantum mechanical effects due to the undulatory nature of the colliding particles. This second requirement corresponds to $b^n \ll G_D s/\hbar \equiv b_c^n$. In eq. (3) we have knowingly disregarded b_c as it is related to ordinary quantum mechanical effects. It corresponds to the critical impact parameter above which the classical scattering angle becomes smaller than its quantum indetermination. (The presence of a b_c in potential scattering is a known property of potentials vanishing faster than $1/r$ at infinity: notice indeed that in our case b_c is only defined for $n > 0$ corresponding to a $1/r^{1+n}$ potential.) Now, for $\sqrt{s} \gg M_D$ we have $\lambda_{\rm P} \ll R_{\rm S} \ll b_c$, so that there is a range of impact parameters where the motion is well-described by classical physics.

This property of gravity should be contrasted to what happens in the case of gauge interactions mediated by vector bosons. In gravity the role of charge is played by energy. So with just one incoming quantum we can have a macroscopic source of gravity if $E = \hbar\nu \gg M_D$. In the case of gauge interactions the charge of one fundamental quantum is $\hbar$ times a number which we can conventionally set to 1 by rescaling the gauge field. Then in order to have a macroscopic source we need an object like a nucleus or a soliton involving many charged quanta. Consider the angle for the scattering between two objects carrying Z units of charge $\theta = g^2\hbar^2 Z^2/\sqrt{s}b^{n+1}$, where the gauge coupling has dimension $[g^2] = \text{length}^{n-1}E^{-1}$ and $Q = \hbar Z$ is the charge. The conditions for classical motion are as before. (i) is replaced by $b \gg \lambda_g = (\hbar g^2)^{1/n}$: λ_g is the typical length scale where a gauge theory in $4+n$ dimensions becomes strongly coupled. Simultaneous satisfaction of (i) and and (ii) implies $Z \gg 1$, which excludes the scattering of elementary quanta. The classical limit corresponds indeed to $\hbar \to 0$ with Q fixed, so that $Z \to \infty$.

The physics of transplanckian collisions was studied in a series of papers more than ten years ago [16–19]. String theory corrections to the classical result were even considered [17]. The basic picture is that, for impact parameter $b \gg R_{\rm S}$, the particles scatter by a small angle $\theta \sim (R_{\rm S}/b)^{n+1}$ while for even larger $b > b_c$ the classical angle is so small that ordinary quantum mechanical effects come into play. In the $b \gg R_{\rm S}$ regime non-linear effects due to the superposition of the gravitational fields of the two scatterers are small and so one can work with linearized gravity. Moreover, since the scattering is at small angle the amplitude can be calculated by using the eikonal approximation. The eikonal amplitude can be obtained in two equivalent ways [20]. In one approach what is calculated is the phase shift of the wave function of one particle when crossing the gravitational shock wave field created by the other particle [16]. The other approach [17,18] consists of the direct resummation of the series of graviton exchange ladder diagrams [21].

The eikonal approximation breaks down for impact parameters $b \sim R_{\rm S}$, where the scattering angle becomes $O(1)$. No full calculation in this regime is available right now. A reasonable expectation is that for $b \lesssim R_{\rm S}$ the two particles, with most of their energy, collapse to form a black-hole (BH). Heuristically, this is because at the moment the particles cross, there is an amount of energy $\sqrt{s}$ localized within a radius $b < R_{\rm S}$. By a variant of Thorne's hoop conjecture, gravitational collapse should then follow. More rigorously, following original unpublished work by Penrose on head-on collisions ($b = 0$) in 4-D, it has been recently proven [22] that for small enough impact parameter a marginally trapped surface forms at the overlap between the two gravitational shock waves. Then by the singularity theorems [23] a horizon should form. By this rigorous treatment one finds a lower bound on the cross-section. For $D = 4$ it is $\sigma_{\rm BH} \geq 0.65\pi R_{\rm S}^2$, of the order of the naive geometrical estimate ($R_{\rm S}$ is given in eq. (4)). Similarly, one gets an interesting upper bound on the energy radiated in gravitational waves: for $D = 4$, less than about 50% of the initial energy is shown to be radiated away. At $b = 0$ a perturbative analysis [24] sets an even stronger bound $\sim$16% on the total radiated energy. It is thus expected that for $b \neq 0$ also the radiated energy is somewhat less than 50%. Similar results are reasonably expected to hold in $D > 4$, although no quantitative analysis has been done. Phenomenological studies so far have just taken $\sigma_{\rm BH} = \pi R_{\rm S}^2$ also neglecting gravitational radiation. In view of the above results and expectations this seems a reasonable approach in first approximation.

The view that black holes are copiously produced in transplanckian collisions was seriously criticized in ref. [25]. In those references it was argued that black-hole production in the collision of two particles is a classically forbidden process, which can take place only through quantum tunnelling. The resulting amplitude is then expected to be suppressed by a factor $\exp -S_{\rm GH}$, where $S_{\rm GH} \sim (M_{\rm BH}/M_D)^{(n+2)/(n+1)}$ is the Gibbons–Hawking Euclidean action for a black-hole of mass $M_{\rm BH}$. However, in view of the later rigorous results in ref. [22], black-hole production is not classically forbidden. Semiclassically the amplitude is therefore dominated by paths in real (instead of imaginary) time. Notice (see discussion below) that for elastic scattering at $b \sim R_{\rm S}$, close to the critical black hole production radius, the amplitude goes roughly like $\exp iS_{\rm GH}$, indicating that ref. [25] captured the right order of magnitude for the absolute value of the semiclassical action, but the wrong phase. Notice also that for scattering at $b \sim R_{\rm S}$, and expectedly for black-hole production, the number of virtual exchanged gravitons is very large $\sim S_{\rm GH}$ [17]. If back-hole production were pictured as the collapse of these many virtual equivalent quanta, the associated combinatoric factor ($\sim S_{\rm GH}$) could undo the exponential suppression of ref. [25]!

If the large extra dimension scenario is realized in nature with $M_D \sim 1$ TeV, then (may be optimistically) LHC with its 14 TeV c.m. energy may start probing physics in the transplanckian regime [26–30]. Of course a machine with $O(100)$ TeV c.m. energy like VHLC [31] would require less optimism. In the remaining part of the talk I will outline the signatures of gravitational elastic scattering and also, briefly, black-hole production at the LHC. The results can easily be generalized to higher energy machines.

Consider elastic scattering first. Since the dynamical regime we are focusing on, overlaps with the classical limit where the action $S/\hbar \gg 1$, the amplitude is not given by a perturbative calculation. In other words, the classical limit implies exchange or emission of a large (infinite) number of graviton quanta. So we cannot do with a finite number of Feynman diagrams. In the forward region, however, this infinite set is consistently given by the series of ladder and crossed ladder diagrams. The result does not depend on the spin of the particles, since in the eikonal limit the particle line is taken on shell and the coupling to the graviton is simply given by $\langle T_{\mu\nu}\rangle = p_\mu p_\nu$, where p is the incoming four-momentum. In the forward region the momentum transfer q is basically given by the two-dimensional tranverse momentum $q_\perp$: $t = q^2 \simeq -q_\perp^2$. The series of ladder diagrams nicely exponentiates. The resulting amplitude is more conveniently written by trading $q_\perp$ for its Fourier conjugate variable, the impact parameter 2-vector b,

$$\mathcal{A}_{\rm eik} = \mathcal{A}_{\rm Born}(q_\perp^2) + \mathcal{A}_{1-\rm loop}(q_\perp^2) + \cdots = -2is\int \mathrm{d}^2 b\, \mathrm{e}^{iq_\perp b}(\mathrm{e}^{i\chi} - 1), \quad (6)$$

$$\chi(b_\perp) = \frac{1}{2s}\int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2}\mathrm{e}^{-iq_\perp b}\mathcal{A}_{\rm Born}(q_\perp^2). \quad (7)$$

$\chi(b)$ is called the eikonal phase and $\mathrm{e}^{i\chi}$ represents the amplitude in impact parameter space. Unitarity at small angle is thus manifestly satisfied. Notice that we work with a two-dimensional transferred momentum since the scattered particles live on a 3-brane. On the other hand the exchanged gravitons are D-dimensional. So the Born amplitude involves a sum over the n-momentum $k_\perp$ tranverse to the brane [11]. This sum is known to be UV divergent for $n \geq 2$. These divergences can be parameterized at low energy by a set of contact interactions. Naively, they would give δ-function contributions localized at $b = 0$ to $\chi(b)$. On physical grounds, however we expect these local divergences to be softened by the fundamental theory of gravity at some finite but small, impact parameter $b \sim \lambda_{\rm P}$ (or, more likely, at b of the order of the string length $\lambda_{\rm s}$). These short-distance effects should plausibly give rise to $\mathcal{O}(\lambda_{\rm P}^2)$ corrections to the cross-section, while, as we will see shortly, the long-distance eikonal amplitude gives a cross-section that grows with a power of $\sqrt{s}$, thus dominating at large energies [31a].

Then we only need to focus on the non-local calculable piece in $A_{\rm Born}$. For this purpose it is convenient to use dimensional regularization

$$\begin{aligned}\mathcal{A}_{\rm Born}(-t) &= \frac{s^2}{M_D^{n+2}}\int \frac{\mathrm{d}^n k_{\rm T}}{t - k_{\rm T}^2} \\ &= \pi^{n/2}\Gamma(1-n/2)\left(\frac{-t}{M_D^2}\right)^{(n/2)-1}\left(\frac{s}{M_D^2}\right)^2, \end{aligned} \quad (8)$$

from which we get the eikonal phase

$$\chi = \left(\frac{b_c}{b}\right)^n, \quad b_c \equiv \left[\frac{(4\pi)^{(n/2)-1}s\Gamma(n/2)}{2M_D^{n+2}}\right]^{1/n}. \quad (9)$$

Notice that by inserting this result for χ in the integral in eq. (6) we obtain an ultraviolet finite result. This is so, even though the contributions to eq. (6) from the individual terms in the expansion $e^{i\chi} = 1 + i\chi + \cdots$ are ultraviolet divergent, corresponding to the fact that each individual Feynman diagram in the ladder expansion is ultraviolet divergent but the complete sum is finite. Moreover, since $\chi \propto b^{-n}$, the integrand in eq. (6) oscillates very rapidly as $b \to 0$, showing that the ultraviolet region gives but a small contribution to the amplitude. Replacing eq. (9) into eq. (6), the momentum space amplitude [29] is written in terms of Meijer's G-functions.

The length scale b_c, as expected from the general discussion at the beginning, controls ordinary quantum mechanical effects. For $b \ll b_c$ the eikonal phase is large and rapidly oscillating, corresponding to the classical limit. For $b \gtrsim b_c$ the phase is small and quantum mechanics sets in. These two regimes are realized in momentum space as follows: For semi-hard momenta $\sqrt{s} \gg q \gg b_c^{-1}$ the integral in eq. (6) is dominated by the stationary-phase value of the impact parameter $b_{\rm s} \equiv b_c(n/qb_c)^{1/n+1} < b_c$. This is precisely the classical region. Here the concept of trajectory makes sense and the scattering angle is

$$\theta_{\rm cl} = -\frac{\partial\chi}{\partial L} = \frac{2n\Gamma(n/2)}{\pi^{n/2}}\ \frac{G_D\sqrt{s}}{b^{n+1}}. \tag{10}$$

In the limit $n \to 0$, we recover the Einstein angle $\theta_{\rm cl} = 4G_D\sqrt{s}/b$ while, for $n > 0$, eq. (10) gives its higher-dimensional generalization (for the case in which also the Sun moves at ultrarelativistic speed in the c.m.!).

In the soft region $q \lesssim b_c^{-1}$, the integral in eq. (6) is dominated by b of the order of (or slightly smaller than) b_c. This means that the eikonal phase $\chi = (b_c/b)^n$ is of order one and the quantum nature of the scattering particles is important (although quantum gravity effects are negligible and the exchanged graviton is treated as a classical field). Moreover, notice that the relevant χ never becomes much smaller than 1, and therefore we never enter the perturbative regime in which a loop expansion for the amplitude applies. Even though the interaction vanishes at $b \to \infty$ (where $\chi \to 0$), we never reach the Born limit. Even for $q = 0$, the scattering is dominated by $b \sim b_c$ and not by $b = \infty$, as opposed to the Coulomb case. This result follows from the different dimensionalities of the spaces on which the scattered particles and exchanged graviton live. It does not hold for the scattering of bulk particles. In that case the eikonal phase is unchanged, but the impact parameter vector b becomes $(2+n)$-dimensional. In particular $d^2b \to d^{2+n}b$ in eq. (6) so that for $q = 0$, the integral is infrared dominated by large values of b and quadratically divergent (for any n). Indeed one finds $\mathcal{A}_{\rm eik}(q \to 0) = A_{\rm Born} \sim b_c^n/q^2$, encountering the Coulomb singularity characteristic of long-range forces.

At the LHC, the observable of interest is jet–jet production at small angle (close to beam) with large center-of-mass collision energy [30]. The scattering amplitude is the same for any two partons. The total jet–jet cross-section is then obtained by summing over all possible permutations of initial state quarks and gluons, using the appropriate parton distribution weights and enforcing kinematic cuts applicable for the eikonal approximation.

Defining $\hat{s}$ and $\hat{t}$ as Mandelstam variables of the parton–parton collision, we are interested in events that have $\sqrt{\hat{s}}/M_D \gg 1$ and $-\hat{t}/\hat{s} \ll 1$. We can extract $\sqrt{\hat{s}}$ from the jet–jet invariant mass $M_{jj} = \sqrt{\hat{s}}$, and $\hat{t}$ from the rapidity separation of the two jets $-\hat{t}/\hat{s} = 1/(1+e^{\Delta\eta})$, where $\Delta\eta \equiv \eta_1 - \eta_2 = \ln(1+\cos\hat{\theta})/(1-\cos\hat{\theta})$

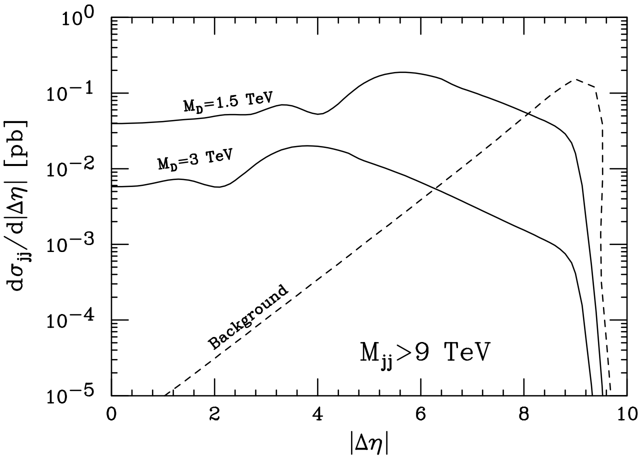

Figure 1. The di-jet differential cross-section $d\sigma_{jj}/d|\Delta\eta|$ from eikonal gravity for $n = 6$, $M_{jj} > 9$ TeV, when both jets have $|\eta| < 5$ and $p_T > 100$ GeV, and for $M_D = 1.5$ TeV and 3 TeV. The dashed line is the expected rate from QCD.

and θ is the c.m. scattering angle. The kinematical region of interest is defined by the equivalent statements

$$\Delta\eta \to \infty \leftrightarrow \hat{\theta} \to 0 \leftrightarrow \frac{-\hat{t}}{\hat{s}} \to 0. \tag{11}$$

Since the partonic scattering probes a region of size b inside the protons, it is reasonable to evaluate the parton distribution functions at the scale $Q^2 = b_s^{-2}$ if $q > b_c^{-1}$ and $Q^2 = q^2$ otherwise ($q^2 \equiv -\hat{t}$) [29]. In the computations the CTEQ5 [32] parton distribution functions have been used. The SM di-jet cross-section has been computed using Pythia [33], ignoring higher-order QCD corrections. For simplicity the background is defined as the jet–jet cross-section from QCD with gravity couplings turned off, and the signal as the jet–jet cross-section from the eikonal gravity computation with QCD turned off. In reality, SM and gravity contributions would be simultaneosly present. However in the interesting kinematic region gravity dominates, so this simple approach is adequate.

The di-jet differential cross-section $d\sigma_{jj}/d|\Delta\eta|$ is plotted in figure 1 for $n = 6$, $M_{jj} > 9$ TeV and $M_D = 1.5$ TeV and 3 TeV. Similar plots for different n can be found in ref. [30]. Since the parton distribution functions decrease rapidly at higher M_{jj}, the plot is dominated by events with $M_{jj} \sim 9$ TeV. Notice the peak structure at intermediate values of Δy, corresponding to impact parameters of order b_c (evaluated at $\sqrt{s} = 9$ TeV), i.e., to the transition region between classical and quantum mechanical scattering. These peaks represent the diffraction of waves scattered around $b \sim b_c$. They are a characteristic feature of the higher-dimensional gravitational field which, while being of infinite range, behaves somewhat like a potential well of size b_c. In the Coulomb case ($n = 0$), such a length scale does not exist and therefore no diffractive pattern is produced.

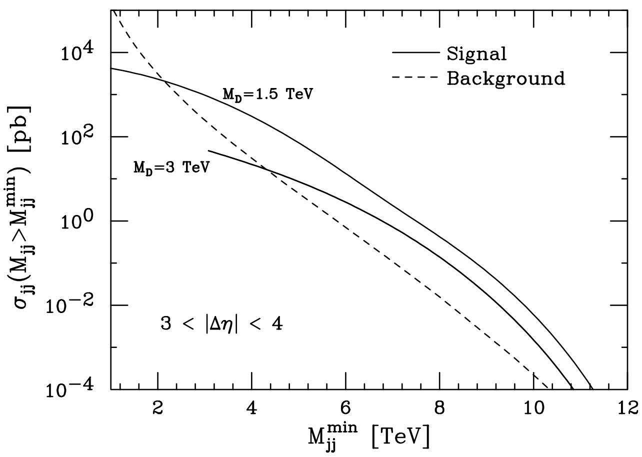

Figure 2. Total integrated di-jet cross-section for $3 < |\Delta\eta| < 4$, $n = 6$, and $M_{jj} > M_{jj}^{\rm min}$, when both jets have $|\eta| < 5$ and $p_{\rm T} > 100$ GeV. Lines are plotted for $M_D = 1.5$ and 3 TeV. The eikonal approximation is reliable only where $M_{jj}/M_D \gg 1$. The expected QCD rate is given by the dashed line.

Since the two jets are experimentally indistinguishable, I have used $|\Delta\eta|$, instead of $\Delta\eta$, as the appropriate kinematical variable to plot. This means that the experimental signal considered here contains also contributions from scattering with large and negative $\Delta\eta$, which corresponds to partons colliding with large momentum transfer and retracing their path backwards. For the background, these effects are calculable and taken into account. However, the theoretical estimate of the signal at negative $\Delta\eta$ lies outside the range of validity of the eikonal approximation. Indeed the region of $\theta \sim \pi$ corresponds to impact parameters $\sim R_{\rm S}$. Its contribution to the differential cross-section will be $\mathrm{d}\sigma/\mathrm{d}t \sim \pi R_{\rm S}^2/s$, i.e., parametrically smaller than the forward one $\mathrm{d}\sigma/\mathrm{d}t \sim \pi b_c^4$. Therefore it is expected to be negligible and can be safely ignored.

As is evident from the figure the gravitational cross-section is harder than the QCD one. This is because the latter is dominated by the forward Coulomb singularity, while the forward eikonal amplitude is finite. In the semi-hard region $q \gtrsim b_c^{-1}$ the gravitational cross-section is also much bigger than the QCD one: as we discussed before, large cross-sections at large energy and finite angle are a clear signal of gravitational interactions. This is because energy itself plays the role of charge in gravity. It is difficult to imagine some other physics that mimics this result.

To get an idea of the sensitivity at LHC one can study the total integrated cross-section using some illustrative cuts. To stay in the small-angle region while beating the QCD background a reasonable choice is $3 < \Delta\eta < 4$. The integrated cross-section as a function of minimum jet–jet invariant mass is shown in figure 2. This plot shows the important feature that the signal cross-section is flatter in M_{jj} than the background. This enables better signal to background for larger M_{jj} cuts, which is the preferred direction to go for the validity of the transplankian limit. Therefore, one should make the largest possible M_{jj} cut that still has a countable signal rate for a given luminosity. For an integrated luminosity of $30\,\mathrm{fb}^{-1}$, corresponding to expectations for one year of running, the plot of figure 2 shows that several hundreds to thousands of events can plausibly be expected.

One can also study (see ref. [30]) the sensitivity reach at LHC considering the total integrated cross-section as a function of M_D for $M_{jj} > 3M_D$ (optimistic) and $M_{jj} > 6M_D$ (more conservative). One finds that in the two cases the reach for M_D is respectively 3.5 and 1.8 TeV almost independent of n.

Similar sensitivities are obtained for the more spectacular events with black-hole production [27,30]. As we said, here a full calculation is not avaliable, but the order of magnitude estimate leads to a cross-section for BH production at LHC which, for $M_D \lesssim 3$ TeV, can range from 10^{-2} to 10^{2} pb. With 30 fb^{-1} per year, LHC could then see several hundreds (or even millions, if one is optimistic) of BH events. These objects happily decay very fast by Hawking radiation with a temperature $T_{\mathrm{H}} = (n+1)/(4\pi R_{\mathrm{S}})$. Their lifetime is $< 10^{-24}$ s. So one should not worry about their growing by eating up the detector! A simple estimate [33a] shows that, for n extra dimensions, in order to produce such environmentally dangerous BHs a c.m. energy in excess of 10^{10+7n} GeV is needed. The black holes decay with comparable probability to any particle living either on the brane or in the bulk [34]. This is because the BH decays mostly to quanta with wavelength λ of the order of its size R_{S}. The space phase for each dimension is $\mathrm{d}p\mathrm{d}q \sim R_{\mathrm{S}}/\lambda \sim 1$ and there is no gain in decaying to more dimensions. If only gravity propagates in the bulk, then the BHs will essentially decay on the brane as it hosts a large number (~ 100) of degrees of freedom. The lifetime of a BH with mass M is $\tau \sim M_D^{-1}(M/M_D)^{(3+n)/(1+n)}$ while the multiplicity of the decay products (mostly quarks and gluons giving rise to jets) scales like $(M/M_D)^{(2+n)/(1+n)}$. So at the LHC the multiplicity could be of order $10 \div 100$. As the parton distribution functions decrease rapidly at large x, BHs of a given mass are most often produced with a small boost. So the characteristic of BH events is high multiplicity with high spherocity [27], a signal which has practically no background.

Elastic scattering and BH production may also affect the physics of cosmic rays [35,29] (see also ref. [36]). This is because they can lead to a significative enhancement of the cross-section of cosmic ultra-high energy neutrini with nucleons in the atmosphere. It is known that there should exist a cosmogenic neutrino flux, originated by the inelastic scattering of primary protons on the cosmic microwave background. The c.m. energy of the neutrino nucleon system is $\sim\sqrt{(E_\nu/10^6\ \mathrm{GeV})}$ TeV, which could well be in the transplanckian regime for the ultra-energetic neutrini with $E_\nu \sim 10^{10}$ GeV. The standard model cross-section dominated by W-boson exchange is roughly given by [37] $\sigma_{\mathrm{SM}} = 10^{-5}(E_\nu/10^{10}\,\mathrm{GeV})^{0.363}$ mb. The production of BH and elastic gravitational scattering could lead to an enchancement of about 10^2 of the cross-section. The origin of this enhancement is that gluons, which at the relevant x have a parton density much larger than quarks, do not contribute to the SM cross-section but they do contribute to the gravitational one. As a result one expects a similar enhancement in the rate of deeply penetrating horizontal showers. Future detectors with improved sensitivity may be able to detect such events [38,39]. The eikonalized neutrino nucleon cross-section at small angle could even become of the order of 1 mb. Such a large cross-section starts being interesting if one wants to explain the vertical ultra GZK events as due to cosmic neutrini [40]. However, the eikonal cross-section is soft, corresponding to a very small energy transfer to the shower. So it cannot explain the ultra GZK events [29].

The above discussion neglects quantum gravity effects. On general grounds and by analyticity in the transferred momentum we expect the corrections to elastic scattering in the interesting region $b \sim b_c$ to be of order $(\lambda_{\rm P}/b_c)^2$. Then by selecting events with $M_{jj} > 6M_D$ we expect $O(5\%)$ effects which is fairly good, while for $M_{jj} > 3M_D$ the effect can go up to 20% [40a]. However, it is possible that at LHC the effects are bigger. First of all, transplanckianity at LHC requires $M_D \lesssim 2-3$ TeV. Such a low value is consistent with present direct bounds on M_D from direct graviton production. On the other hand one may in general expect other quantum gravity effects, and in particular contact four-fermion interactions of dimension six. The presence of these operators, given the bounds from LEP2, can push the lower bound [42] on M_D to above ∼4 TeV, in which case LHC could not be considered a transplanckian machine. So our LHC study is truly based on the assumption that such dimension six terms are mildly suppressed. Indeed there are examples in this direction in string realizations of the braneworld [13]. A second remark precisely concerns the case in which string theory is the theory of quantum gravity. Then $\lambda_{\rm s} = 1/M_{\rm s}$ and not $\lambda_{\rm P}$ controls the onset of quantum gravity effects. In perturbative string theory (for instance [4] type I) we have $(\lambda_{\rm P}/\lambda_{\rm s})^{2+n} = \pi g_{\rm s}^2 < 1$. Here $g_{\rm s}$ is the string coupling which is related to the gauge coupling by $g_{\rm s} \sim 2\alpha$. We can assume such a relation to work qualitatively in the realistic case with α taken to be some average between α_2 and α_3. Then we expect a separation $\lambda_{\rm s} > \lambda_{\rm P}$ which means less separation between b_c and $\lambda_{\rm s}$ and bigger quantum gravity effects. The previous naive estimate of quantum gravity effects is enhanced by a factor $(1/4\pi\alpha^2)^{2/n}$. Taking $\alpha \sim 0.05$ we find that for $n = 2$ string effects could be 100% while for $n = 6$ they may conceivably be less than 20%. Of course we do not want to take these estimates too seriously, as we truly need a full braneworld model to calculate them. Otherwise we should wait LHC and see, first of all, if there is a signal and then decide how well it is explained by transplanckian scattering. Physically we expect string effects to suppress scattering at large angles [17], i.e., angles θ corresponding to impact parameters $b \lesssim \lambda_{\rm s}$. So in the plot of figure 1 the cross-section at lower rapidity would be depleted. Anyway, even if quantum effects at LHC will be large, it is important to have a clear idea about what the features of the asymptotic transplanckian regime are as they provide a benchmark with which to compare the data. Notice, in this respect that elastic scattering is better off than black-hole production, since $b_c \propto s^{1/n}$ is parametrically bigger (grows faster with s) than $R_{\rm s} \sim s^{1/(2n+2)}$. If TeV gravity is truly realized in nature then VLHC [31], whose center-of-mass energy for proton–proton collisions is envisaged between 50 TeV and 200 TeV, would probably be a better place to study transplanckian effects. For instance, by assuming $\sqrt{s} = 100$ TeV, $M_{\rm s} = 3$ TeV (which is may be more realistic than $M_{\rm s} = 1$ TeV) and choosing an 'average' value $\alpha = 0.05$ one finds that string effects parameterized by $\lambda_{\rm s}^2/b_c^2$ are of order 5% for $n = 2$ to 6. On the other hand, for the same choice of parameters, one finds that the parameter $\lambda_{\rm s}^2/R_{\rm S}^2$, controlling string corrections to black-hole production, is still ∼1/3 for $n = 2$ and ∼0.1 for $n = 6$. So, for small n, it is possible that even at VLHC the production of black holes is more appropriately replaced by the production of string balls [43].

In summary, transplanckian scattering offers a model independent test of theories with a low gravity scale. The main processes are black-hole production and elastic scattering. The elastic cross-section is parametrically larger than the one for black holes and moreover can be nicely calculated in the forward region. At the moment the black-hole production cross-section can only be estimated by dimensional analysis. The observation of a cross-section at finite angle growing with a power of s would be a clean signal that the high-energy dynamics of gravity has been detected. If we are lucky and M_D is low enough, then these signals may already show up at the LHC. Otherwise the discovery modes at LHC are graviton emission, showing up as jet plus missing energy (roughly [11] for $4\,\mathrm{TeV} < M_D < 8\,\mathrm{TeV}$), or the production of Regge excitations [13]. Anyway if low scale gravity is discovered at LHC, transplanckian scattering will very likely be studied at the VLHC.

References

[1] N Arkani-Hamed, S Dimopoulos and G R Dvali, *Phys. Lett.* **B429**, 263 (1998)
[2] V A Rubakov and M E Shaposhnikov, *Phys. Lett.* **B125**, 136 (1983)
[3] J Polchinski, arXiv:hep-th/9611050
[4] I Antoniadis, N Arkani-Hamed, S Dimopoulos and G R Dvali, *Phys. Lett.* **B436**, 257 (1998)
[5] N Arkani-Hamed, S Dimopoulos and J March-Russell, *Phys. Rev.* **D63**, 064020 (2001)
[6] N Arkani-Hamed, S Dimopoulos and G R Dvali, *Phys. Rev.* **D59**, 086004 (1999)
[7] EOT-WASH Group Collaboration: E G Adelberger, arXiv:hep-ex/0202008
[8] J Chiaverini, S J Smullin, A A Geraci, D M Weld and A Kapitulnik, *Phys. Rev. Lett.* **90**, 151101 (2003)
[9] J C Long, H W Chan, A B Churnside, E A Gulbis, M C Varney and J C Price, *Nature* **421**, 922 (2003)
[10] N Kaloper, J March-Russell, G D Starkman and M Trodden, *Phys. Rev. Lett.* **85**, 928 (2000)
[11] G F Giudice, R Rattazzi and J D Wells, *Nucl. Phys.* **B544**, 3 (1999)
E A Mirabelli, M Perelstein and M E Peskin, *Phys. Rev. Lett.* **82**, 2236 (1999)
T Han, J D Lykken and R J Zhang, *Phys. Rev.* **D59**, 105006 (1999)
J L Hewett, *Phys. Rev. Lett.* **82**, 4765 (1999)
[11a] For instance, if the extra dimensions are compactified on the n-sphere S_n, the mass levels are given by $m_k^2 = k(k+n-1)/R^2$ with integer k. Since S_n has maximal symmetry the levels have a large degeneracy growing like k^{n-1}. Of course at each level only one linear combination couples to the brane. As far as the brane observer is concerned, the KK tower consists of levels with roughly constant separation $m_{k+1} - m_k \sim 1/R$. For spaces with lower symmetry, like for instance the n-torus, the degeneracy is lifted. For a generic manifold the average level separation around mass m is $\Delta m \sim 1/(R^n m^{n-1})$. Therefore, at high energy, much better energy resolution than $1/R$ is in general needed in order to distinguish the individual levels
[12] G F Giudice, R Rattazzi and J D Wells, *Nucl. Phys.* **B595**, 250 (2001)
[13] S Cullen, M Perelstein and M E Peskin, *Phys. Rev.* **D62**, 055012 (2000)
E Dudas and J Mourad, *Nucl. Phys.* **B575**, 3 (2000)
E Accomando, I Antoniadis and K Benakli, *Nucl. Phys.* **B579**, 3 (2000)
[14] R C Myers and M J Perry, *Ann. Phys.* **172**, 304 (1986)
[15] L D Landau and E M Lifshitz, *Quantum mechanics* vol. 3 of *Course of Theoretical Physics*

[16] G 't Hooft, *Phys. Lett.* **B198**, 61 (1987)
[17] D Amati, M Ciafaloni and G Veneziano, *Phys. Lett.* **B197**, 81 (1987); *Int. J. Mod. Phys.* **A3**, 1615 (1988); *Nucl. Phys.* **B347**, 550 (1990); *Phys. Lett.* **B289**, 87 (1992); *Nucl. Phys.* **B403**, 707 (1993)
[18] I J Muzinich and M Soldate, *Phys. Rev.* **D37**, 359 (1988)
[19] H Verlinde and E Verlinde, *Nucl. Phys.* **B371**, 246 (1992)
[20] D Kabat and M Ortiz, *Nucl. Phys.* **B388**, 570 (1992)
D Kabat, *Comm. Nucl. Part. Phys.* **20**, 325 (1992)
[21] H Cheng and T T Wu, *Phys. Rev. Lett.* **22**, 666 (1969)
H Abarbanel and C Itzykson, *Phys. Rev. Lett.* **23**, 53 (1969)
M Levy and J Sucher, *Phys. Rev.* **186**, 1656 (1969)
[22] D M Eardley and S B Giddings, *Phys. Rev.* **D66**, 044011 (2002)
E Kohlprath and G Veneziano, *J. High Energy Phys.* **0206**, 057 (2002)
[23] S W Hawking and G R F Ellis, *The large scale structure of space-time* (Cambridge University Press, 1973)
[24] P D D'Eath and P N Payne, *Phys. Rev.* **D46**, 658, 675, 694 (1992)
[25] M B Voloshin, *Phys. Lett.* **B518**, 137 (2001); **524**, 376 (2002)
[26] T Banks and W Fischler, hep-th/9906038.
[27] S B Giddings and S Thomas, *Phys. Rev.* **D65**, 056010 (2002)
S Dimopoulos and G Landsberg, *Phys. Rev. Lett.* **87**, 161602 (2001)
S B Giddings, hep-ph/0110127
[28] R Emparan, *Phys. Rev.* **D64**, 024025 (2001)
[29] R Emparan, M Masip and R Rattazzi, *Phys. Rev.* **D65**, 064023 (2002)
[30] G F Giudice, R Rattazzi and J D Wells, *Nucl. Phys.* **B630**, 293 (2002)
[31] See http://www-ap.fnal.gov/VLHC/
[31a] In fact, in the fully transplanckian regime where $R_{\rm S} \gg \lambda_{\rm s}$ these short distance effects are screened by the black-hole horizon. For instance the leading corrections to the leading eikonal phase go like $\Delta\chi/\chi \sim (R_{\rm S}/b)^{2n+2}$ [17] and are controlled by classical black-hole dynamics, not by quantum gravity
[32] H L Lai *et al*, *Phys. Rev.* **D51**, 4763 (1995)
[33] T Sjostrand, L Lonnblad and S Mrenna, hep-ph/0108264
[33a] The accretion rate is roughly $\dot{M}_{\rm BH} \sim R_{\rm S}^2 v\rho$, where v is the BH velocity and ρ the medium density. The estimate in the text is obtained using $v \sim 1$ and $\rho \sim 10$ GeV/Å^3 $\sim 10^{-16}$ GeV^4
[34] R Emparan, G T Horowitz and R C Myers, *Phys. Rev. Lett.* **85**, 499 (2000)
[35] J L Feng and A D Shapere, *Phys. Rev. Lett.* **88**, 021303 (2002)
L Anchordoqui and H Goldberg, *Phys. Rev.* **D65**, 047502 (2002)
[36] S Nussinov and R Shrock, *Phys. Rev.* **D64**, 047702 (2001)
[37] R Gandhi, C Quigg, M H Reno and I Sarcevic, *Phys. Rev.* **D58**, 093009 (1998)
[38] A Ringwald and H Tu, *Phys. Lett.* **B525**, 135 (2002)
[39] L A Anchordoqui, J L Feng, H Goldberg and A D Shapere, *Phys. Rev.* **D65**, 124027 (2002)
[40] T J Weiler, *AIP Conf. Proc.* **579**, 58 (2001)
[40a] Notice that lead–lead collisions at LHC are a worse probe of the transplanckian regime than *pp*-collisions: the parton luminosity is higher but the energy per nucleon is only about 1/3 of that in *pp*-collisions [41]
[41] A Chamblin and G C Nayak, *Phys. Rev.* **D66**, 091901 (2002)
[42] I Antoniadis, K Benakli and A Laugier, *J. High Energy Phys.* **0105**, 044 (2001)
[43] S Dimopoulos and R Emparan, *Phys. Lett.* **B526**, 393 (2002)