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Electroweak breaking and supersymmetry breaking

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Abstract. We discuss the clash between the absence of fine tuning in the Higgs potential and a sufficient suppression of flavour changing neutral current transitions in supersymmetric extensions of the standard model. It is pointed out that horizontal $U(1)$ symmetry combined with the D-term supersymmetry breaking provides a realistic framework for solving both problems.

Keywords. Supersymmetry; electroweak symmetry breaking; horizontal symmetries; D-term supersymmetry breaking.

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The origin of the Fermi constant $G_{\rm F}$ is an outstanding problem in particle physics. One would like to calculate it in terms of more fundamental parameters. In the standard model (SM), spontaneous breaking of the gauge symmetry is described by two free parameters, the Higgs field mass m_Φ and its self-coupling λ. Not only the vector boson masses must be determined by experiment but, in addition, the Higgs mechanism is extremely sensitive to any physics at much higher energy scales (there are quadratic divergences in quantum corrections to the scalar sector). Thus, the hierarchy $M_Z/M_{\rm GUT,PL} \approx 10^{-16}$ is a big puzzle of the SM.

Supersymmetry could be an attractive explananation of the origin and stability of the electroweak scale. The electroweak symmetry breaking may be triggered by radiative corrections to the Higgs potential and the stability of the electroweak scale is assured by the absence of quadratic divergences in the supersymmetric extension of the SM. Indeed, the quadratic divergences of the SM are cut-off at the soft supersymmetry breaking scale $m_{\rm SUSY}$ (the D-term contribution to the scalar mass is also only logarithmically divergent) and (schematically) the mass m_Φ is calculable:

$$m_\Phi^2 \approx -m_{\rm SUSY}^2 \frac{\ln \Lambda/m_{\rm SUSY}}{30}. \tag{1}$$

The above result is due to the large radiative corrections generated by the top quark Yukawa coupling Y_t [1–4]. Moreover, the scalar self-coupling is given by the gauge couplings. The vector boson masses are then predicted in terms of the Y_t, the gauge couplings and the two dimensional parameters $m_{\rm SUSY}$ and the cut-off Λ. The prediction is consistent with the measured vector boson masses for $m_{\rm SUSY} \approx \mathcal{O}$

(1 TeV) and $\Lambda \approx M_{\rm GUT}, M_{\rm PL}$, the values that nicely fit to the unification of gauge couplings.

Thus, the mechanism of supersymmetry breaking becomes the central issue in this approach to the origin of the Fermi constant. In fact, the original question is now replaced by the question about the origin of $m_{\rm SUSY}$ and instead of explaining the hierarchy $M_W/M_{\rm PL}$ we have to explain now the hierarchy $m_{\rm SUSY}/M_{\rm PL}$ (and $\mu/M_{\rm PL}$, where μ is the supersymmetric higgsino mass, $\mu H_1 H_2$). However, those questions may be more fundamental.

First, taking the minimal supersymmetric standard model (MSSM) as our laboratory and assuming its validity up to the GUT scale, let us be more specific about the parameters $m_{\rm SUSY}$ and μ. The tree level Higgs potential reads:

$$V = (m^2_{H_d} + \mu^2)|H_d|^2 + (m^2_{H_u} + \mu^2)|H_u|^2 \\ -m_3^2(\epsilon_{ab} H_d^a H_u^b + \text{c.c.}) + \text{quartic terms.} \tag{2}$$

Defining

$$\tan\beta = \frac{v_2}{v_1}, \tag{3}$$

we get

$$\sin 2\beta = \frac{2m_3^2}{M_A^2}, \tag{4}$$

where

$$M_A^2 = m^2_{H_d} \ + \ m^2_{H_u} \ + \ 2\mu^2. \tag{5}$$

Moreover, we have

$$\frac{1}{2}M_Z^2 = \frac{m^2_{H_d} - m^2_{H_u}\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \\ \approx -m^2_{H_u} - \mu^2 \ \ (\text{for not too small} \tan\beta). \tag{6}$$

The parameters of the Higgs potential are related by the RG equations to many parameters at the high scale (we denote them with tilde): $\tilde{\mu}, \tilde{m}_{H_u}, \tilde{m}_{H_d}$, squark masses $\tilde{m}_i$ $(i = 1,2,3,L,R)$, $\tilde{m}_{\rm gluino}, \tilde{m}_{\rm gaugino}, \tilde{A}, \ldots$. Let us denote them all as $\tilde{m}_i$. Then

$$\frac{1}{2}M_Z^2 = \Sigma c_{ij}\tilde{m}_i\tilde{m}_j, \tag{7}$$

where

$$c_{ij} = c_{ij}\left(g, Y, \tan\beta, \ln\frac{\Lambda}{M_Z}\right) \tag{8}$$

are determined by the RG equations (see for instance [5–10]). There are two constraints on acceptable solutions to eq. (7):

(a) no large cancellations on the right-hand side of eq. (7)

(b) no large flavour changing neutral current (FCNC) transition generated by sparticle exchange.

The absence of large cancellations in eq. (7) (no fine tuning in the Higgs potential) is a requirement that is difficult to quantify. It is not clear what 'large' really means. Moreover, the soft terms $\tilde{m}_i$s are most likely correlated by the mechanism of supersymmetry breaking and treating them as independent parameters may by misleading. Nevertheless, one expects that the fine tuning argument has some qualitative sense and, therefore, it is worth having a closer look at the magnitude of the coefficients c_{ij}. Their RG evolution and their magnitude in the MSSM are well understood. The dominant effects come from α_{strong} and from Y_t. Generically, they vary from $\mathcal{O}(1)$ to $\mathcal{O}(0.01)$ and, for instance, for cancellations in eq. (7) not larger than 1:100 we have the following limits:

$\tilde{m}_i \lesssim \mathcal{O}(500\text{–}1000)$ GeV, if $c_{ii} \sim \mathcal{O}(1)$,
$\tilde{m}_i \lesssim \mathcal{O}$ (2–3) TeV, if $c_{ij} \sim \mathcal{O}(0.1)$,
$\tilde{m}_i \lesssim \mathcal{O}$ (5–7) TeV if $c_{ij} \sim \mathcal{O}(0.01)$

In fact, $c_{ii} \gtrsim \mathcal{O}(1)$ for $\tilde{\mu}$ and $\tilde{m}_{\text{gluino}}(\tilde{M}_3)$ whereas $c_{ij} \sim \mathcal{O}(0.01)$ for $\tilde{m}_{sq}$ of the first two generations of squarks. The dependence of the Higgs potential on the gaugino masses and A terms is also weak. The dependence of the Higgs potential on the large scale values of $\tilde{m}_{H_d}, \tilde{m}_{H_u}, \tilde{m}_{Q_3}, \tilde{m}_{U_3}, \tilde{m}_{D_3}$ (which we collectively denote by $\tilde{m}_0$) varies with $\tan\beta$:

low $\tan\beta(\lesssim 5) \rightarrow c_{\tilde{m}_0} \sim \mathcal{O}(1)$;
intermediate and large $\tan\beta \rightarrow c_{\tilde{m}_0} \lesssim \mathcal{O}(0.1)$.

Hence, for most of the presently allowed $\tan\beta$ region, we get

$$\frac{1}{2}M_Z^2 \approx -\tilde{\mu}^2 + 3\tilde{M}_3^2. \tag{9}$$

The weak dependence on the scalar masses has recently been termed a focus point [11].

We conclude that the level of cancellations in the Higgs potential depends on the values of $\tilde{\mu}$ and $\tilde{M}_3$ (see eq. (9)) and to a much lesser extent on the values of the other soft mass parameters. At this point it is worth remembering that, on the other hand, the values of $\tilde{M}_3$ play a very important role in the loop corrections to the lightest Higgs boson mass [12–14]. For instance, one has a limit $\tilde{M}_3 \gtrsim 300$ GeV ($m_{\text{gluino}} \gtrsim \mathcal{O}$ (1 TeV)) if $m_{\text{higgs}} \gtrsim 115$ GeV, and the degree of cancellations in the Higgs potential would then necessarily be at least of the order of 1:100. Sfermion masses of the first two generations can be up to several TeV and of the third generation up to 1–2 TeV without further increase in fine tuning.

We now turn our attention to the supersymmetric FCNC problem. In the SM not only there is no tree level contribution to the FCNC transitions but also the generic 1-loop contributions,

$$\mathcal{L}_{\text{eff}} \sim \alpha G_{\text{F}}, \tag{10}$$

are suppressed by the generalized GIM mechanism and

$$\mathcal{L}_{\rm eff} \sim \alpha G_{\rm F} \left(\frac{m_c}{M_W}\right)^2 \sim 10^{-4} \alpha G_{\rm F}. \tag{11}$$

Suppose now that some new physics, with the characteristic scale Λ, contributes at 1-loop to the FCNC transitions with the coupling constant $\tilde{\alpha}$. The 1-loop contribution then reads

$$\mathcal{L}_{\rm eff} \sim \frac{\tilde{\alpha}^2}{\Lambda^2} = \alpha G_{\rm F} \left(\frac{\tilde{\alpha}}{\alpha}\right)^2 \left(\frac{M_W}{\Lambda}\right)^2. \tag{12}$$

To be consistent with the experimentally observed suppression of FCNC processes, we need $(\tilde{\alpha}/\alpha)^2 (M_W/\Lambda)^2 \sim 10^{-4}$ and for $\tilde{\alpha} \sim \alpha_{\rm strong}$ (as in the MSSM) we need $\Lambda \sim 100$ TeV. Clearly, with superpartner masses $\mathcal{O}(100)$ TeV there is a clash with the absence of fine tuning in the Higgs potential. Thus, suppression of FCNC transitions requires either the sfermion masses to be sufficiently universal [15–17] or sufficiently aligned with the fermion masses [18–20], or some combination of both.

The solution to the supersymmetric FCNC problem is linked to the supersymmetry breaking mechanism. Universality is not generic for gravity mediated models of supersymmetry breaking but can be realized in models based on gauge mediation or anomaly mediation. Universality ansatz is an interesting laboratory to study the minimal flavour violation in the sfermion sector.

A very interesting possibility are horizontal symmetries, that correlate the fermion and sfermion masses and could solve the fermion mass and the FCNC problems, simultaneously. This generalized 'alignment' mechanism can be based on non-abelian or abelian spontaneously broken horizontal gauge symmetries. The fermion masses are explained in the framework of the Froggatt–Nielsen approach. We shall focus in this talk on the horizontal $U(1)$ symmetry [19,21], which is the simplest version of the generalized 'alignment' mechanism and has an interesting link to the D-term supersymmtry breaking.

We shall consider $U(1)$ models with single $U(1)$ horizontal symmetry and one Froggatt–Nielsen field Φ which, we assume carries the charge $q_\Phi = -1$ of the horizontal $U(1)$. Furthermore, we assume that all $U(1)$ matter charges are non-negative, with the third generation fermion doublets and the two Higgs doublets (we talk about MSSM) having charge zero, $q_3 = h_1 = h_2 = 0$. Acceptable fermion masses and mixings (including the neutrino sector) can be obtained from a superpotential of the form

$$W \sim y^u_{ij} \Theta(q_i + u_j + h_2) \left(\frac{\Phi}{M}\right)^{q_i+u_j+h_2} Q^i U^j H_2 \tag{13}$$

plus similar terms for the down quarks, charged leptons and the Dirac neutrino mass parameters. The neutrino masses are obtained from the see-saw mechanism. Here $\Phi/M \sim \lambda$ (the Cabibbo angle), M is some fundamental scale and Φ is the vacuum expectation value of the scalar component of a chiral superfield Φ. The constants y_{ij} are arbitrary $\mathcal{O}(1)$ coefficients, not predicted by the model. The Yukawa couplings then read $Y_{ij} = y_{ij} \lambda^{q_i+u_j+h_2}$ etc.

The horizontal charges also control the flavour off-diagonal sfermion masses obtained from the Kaehler potential, e.g.

$$\tilde{m}_{ij}^2 = \mathcal{O}(1) * \frac{F^2}{M^2} \left(\frac{\Phi}{M}\right)^{|q_i - q_j|} Q_i^\dagger Q_j, \tag{14}$$

where F is some generic supersymmetry breaking F-term and the scalar components of the chiral fields are denoted by the same symbols as the chiral fields themselves. However, the diagonal entries in the sfermion masses in eq. (14) are not controlled by the $U(1)$ charges and are $\mathcal{O}(\frac{F^2}{M^2})$ for all the three generations of sfermions. Generically non-universal, they can generate large flavour off-diagonal terms in the CKM basis, where fermions are mass-diagonal and a sufficient suppression of FCNC transitions requires them to be larger than $\mathcal{O}(10)$ TeV. We need a control over the diagonal entries, too.

This is possible since they get additional contribution from the D-term of the $U(1)$ gauge group

$$\tilde{m}_{ii}^2 = \mathcal{O}\left(\frac{F^2}{M^2}\right)_{ii} + g q_i D \tag{15}$$

and, if

$$m_{\mathrm{F}}^2 \equiv \frac{F^2}{M^2} \ll D \equiv m_{\mathrm{D}}^2, \tag{16}$$

the diagonal entries in the sfermion masses are controlled by the $U(1)$ charges, too. Such a mechanism requires the D-term supersymmetry breaking by a superpotential [22–24]. We then get

$$m_{\mathrm{F}}^2 \sim \lambda^2 m_{\mathrm{D}}^2, \tag{17}$$

also if the model is embedded in supergravity.

In that hybrid scenario of supersymmtry breaking, if the third generation of sfermions is uncharged under the $U(1)$, it remains light ($\sim \mathcal{O}(m_{\mathrm{F}}^2)$), whereas the first and second generation can be sufficiently heavy to avoid any conflict with FCNC transitions. A detailed study of realistic models [25] shows that one needs $m_{\mathrm{D}}^2 \gtrsim$ (3–4) TeV.

In conclusion, horizontal $U(1)$ symmetry combined with the D-term supersymmetry breaking provide a realistic framework for understanding fermion and sfermion mass pattern and for solving the problem of a clash between the naturalness of the Higgs potential and a sufficient FCNC suppression in supersymmetric extension of the SM.

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