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Leptogenesis

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Abstract. I present the theoretical basis for leptogenesis and its implications for the structure of the universe. It is suggested that density fluctuations grow during the transition period and remnants of this effect should be sought in the universe. The relation between theories with Majorana neutrinos and low energy phenomena, including oscillations, advanced considerably during the past two years with a consistent picture developed in several models.

Keywords. Leptogenesis; baryon asymmetry; Majorana neutrinos; density fluctuations.

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1. Introduction

The discovery of neutrino oscillations signals the extension of the standard model with the introduction of right-handed neutrinos. The evidence so far shows the mixing of $\nu_e \to \nu_\mu$ (solar) and $\nu_\mu \to \nu_\tau$ (atmospheric) with very small mass differences and large mixing angles. The mixing of $\nu_e \to \nu_\tau$ has not been observed as yet and consequently the mixing angle must be much smaller. The oscillations require right-handed components for the states in order to produce neutrino masses. Introducing the right-handed components of neutrinos leads to several new issues to be investigated.

1) Are the neutrinos Dirac or Majorana particles? In other words, should we introduce only Dirac type terms in the mass matrices or also Majorana mass terms. When we follow the philosophy that all allowed mass terms must be included, then Majorana terms must be present.
2) Is the mixing observed so far related to other phenomena? An attractive possibility is related to the generation of a lepton asymmetry in the early universe, which was subsequently converted into a baryon asymmetry.
3) Is it possible to find evidence for the Majorana nature of neutrinos?
4) Are the low energy phenomena related to the high energy theory we need for leptogenesis?

This field has become very interesting because there are many unanswered questions, some of which can be investigated experimentally. I will address some of these issues in this talk.

2. Majorana neutrinos

The neutrinos participate in their observed interactions as left-handed particles. The right-handed component of the Dirac spinor was intentionally left out believing that neutrinos were massless. The mixing phenomena require the introduction of right-handed components. With them we have the choice of two mass terms

$$\bar{\nu}_{\mathrm{L}} N_{\mathrm{R}} \quad \text{Dirac} \quad \Delta L = 0, \tag{1}$$

$$\bar{N}_{\mathrm{R}}^{C} N_{\mathrm{R}} \quad \text{Majorana} \quad \Delta L = 2. \tag{2}$$

One may also think of introducing the term

$$\bar{\nu}_{\mathrm{L}}^{C} \nu_{\mathrm{L}} \tag{3}$$

but this carries weak isospin of two units and must couple to a Higgs triplet which is absent in the standard model. It is natural to introduce the terms in (1) and (2) and look for solutions of the mass matrix. In general, we obtain solutions

$$\psi = \frac{1}{\sqrt{2}}(N_{\mathrm{R}} + N_{\mathrm{R}}^{C}) \tag{4}$$

which are Majorana states and even under charge conjugation (C). Introducing bare Majorana mass terms, like the one in eq. (2), one obtains physical states which are C-eigenstates. In theories with Majorana mass terms it is possible to introduce interactions with scalar particles, like

$$h_{ij} \bar{\ell}_{\mathrm{L}}^{i} \phi \psi j, \tag{5}$$

where ℓ_{L}^{i} is the left–handed doublet (leptons), ϕ the ordinary Higgs doublet and h_{ij} the coupling constant with i and j referring to various generations. The result is that such interactions together with the mass terms produce physical states which are neither C- nor CP-eigenstates.

This property was emphasized in the article by Fukugida and Yanagida [1] that the decays of heavy Majorana states generate a lepton asymmetry. Later on, it was observed by Flanz, Sarkar and myself [2] that the construction of the physical states contains an additional lepton asymmetry. In the latter case, the situation is analogous to the K^0 and $\bar{K}^0$ states, which mix through the box diagrams and produce K_{L} and K_S as physical states. The mixing of the neutrinos originates from fermionic self-energies which contribute to the mass matrix of the particles. The final result is the creation of an asymmetry in the decay of heavy Majorana particles, which depends on a modified Dirac term $\tilde{m}_{\mathrm{D}}$ of the mass matrix (the modification is discussed in §5). The final result is

$$\begin{aligned} \varepsilon &= \frac{\Gamma(N_{\mathrm{R}_i} \to \ell) - \Gamma(N_{\mathrm{R}_i} \to \ell^c)}{\Gamma(N_{\mathrm{R}_i} \to \ell) + \Gamma(N_{\mathrm{R}_i} \to \ell^c)} \\ &= \frac{1}{8\pi v^2} \frac{1}{(\tilde{m}_{\mathrm{D}}^{+} \tilde{m}_{\mathrm{D}})_{11}} \sum_{j=2,3} \mathrm{Im}\left((\tilde{m}_{\mathrm{D}}^{+} \tilde{m}_{\mathrm{D}})_{1j}^{2}\right) f(x) \end{aligned} \tag{6}$$

with $f(x) = \sqrt{x}\left\{\frac{1}{1-x} + 1 - (1+x)\ln\left(\frac{1+x}{x}\right)t\right\}$, $x = (M_j/M_1)^2$, M_1 the mass of the lightest Majorana neutrino and v the vacuum expectation value of the standard Higgs. The term $(1/1-x)$ comes from the mixing of the states [2] and the rest from the interference of vertex corrections with Born diagrams [1]. The above formula is an approximation for the case when the two masses are far apart. In case they are close together there is an exact formula, showing clearly a resonance phenomenon [3] from the overlap of the two wave functions. The origin of the asymmetry has also been studied in field theories [4] and supersymmetric models [5,6]. The purpose of these articles, especially [4], was to justify the formalism and eliminate some objections.

3. $(B-L)$ asymmetry

According to the scenario described above, the $(B-L)$ quantum number is violated easily through Majorana neutrinos. We assign to particles lepton and baryon quantum numbers, as follows: $n_B = 1/3$ for each baryon, $n_L = +1$ for each lepton and the negative numbers for their antiparticles. Then there is a combined number

$$n_C = 3n_B - n_L = B - L$$

which is conserved. More explicitly we assign

$$\begin{aligned}
n_L &= -1, & n_C &= +1; & &\text{for antimuons } \mu^+ \text{ and } \bar{\nu}_\mu\\
n_L &= +1, & n_C &= -1; & &\text{for muons } \mu^- \text{ and } \nu^\mu\\
n_B &= +1, & n_C &= +1; & &\text{for baryons : protons and neutrons}\\
n_B &= -1, & n_C &= -1; & &\text{for antiprotons and antineutrons.}
\end{aligned} \tag{7}$$

Under laboratory conditions processes involving violation of n_B and n_L play a negligible role, but they were important during the early state of the universe. Violation of n_B will produce proton decays which so far have been shown to be small. It was thought that grand unified theories (GUT) could violate the $(B-L)$ or the B quantum number. However, there is a theorem which says that the decays of heavy gauge bosons into quarks and leptons with conventional quantum numbers involve operators of dimensions higher than six [7]. This suppresses proton decay and the generation of a baryon asymmetry in GUT. The standard model obeys this rule, but it has another property: topological solutions of the theory (sphalerons) conserve to a high degree of accuracy $B-L$ but violate $B+L$ [8]. Thus there is the attractive possibility to generate a net $(B-L)$ in the decays of heavy Majorana particles and subsequently convert a fraction of it into a baryon asymmetry. This scenario was named leptogenesis and presents an attractive possibility, perhaps the only viable one, for baryon generation.

4. Lepton asymmetric universe

As mentioned already, a lepton asymmetry is generated either in the decays or the construction of the physical states. The two possibilities may be distinguished

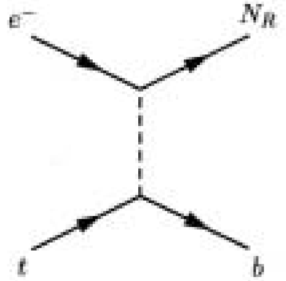

Figure 1. A typical scattering.

by their consequences. The C- and CP-asymmetric states will be physical and propagating states, if during their life-times they interact many times with particles in their surroundings. A typical interaction is shown in figure 1 with the two couplings being present in the theory.

Taking the density of states in the early universe to be $n = (2/\pi^2)T^3$ and calculating the cross section at an energy $E = T$, we obtain

$$n \cdot \sigma \cdot u = \frac{|h_t|^2 |h_\ell|^2}{8\pi^3} T \tag{8}$$

with σ the cross section, u their relative velocity of the particles and h_t, h_ℓ the couplings of the Higgses to quarks and leptons, respectively. At that early time the decay width of the moving leptons with mass M_N is

$$\Gamma_N = \frac{|h_\ell|^2}{16\pi} \frac{M_N^2}{T}. \tag{9}$$

Consequently

$$\frac{n \cdot \sigma \cdot u}{\Gamma_N} = \frac{2}{\pi^2} |h_t|^2 \left(\frac{T}{M_N}\right)^2 . \tag{10}$$

Thus, at an early stage of the universe with $T \gg M_N$, when the mixed states are created, they live long enough so that in one life-time they have many interactions with the surroundings. They are incoherent states. As the universe started deviating from thermal equilibrium, it became lepton asymmetric and the transition period lasted for a relatively long time [9,10]. Estimates for the decays of the lightest Majorana [9] and the development of the lepton asymmetry are shown in figure 2.

The total asymmetry is given by

$$Y = D\epsilon \tag{11}$$

with D a dilution factor and ϵ given in eq. (6). The dilution factor is obtained from solutions of the Boltzmann equations [9], which depend on $\kappa = \Gamma/H$ with Γ the decay width and H the Hubble constant. The asymmetry starts from zero and grows reaching eventually a constant value asymptotically.

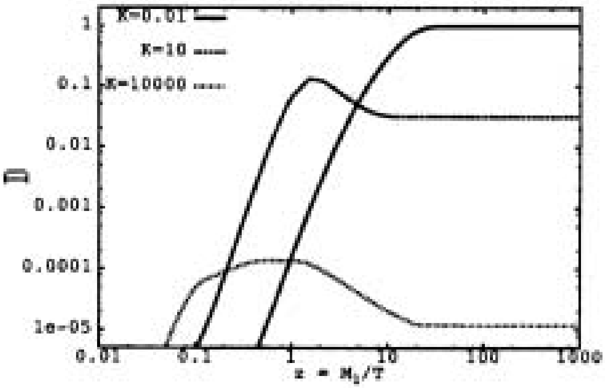

Figure 2. Dilution factor *D*.

The figure indicates that the heavy states are incoherent from a temperature $T_h = 10\,M_1$ down to a temperature $T_l = M_1/10$. During this period the right-handed neutrinos are non-relativistic and will cause fluctuations to grow. Heavy neutrinos, $M_N \sim 10^8$ GeV, have an interaction length and an interaction horizon. Fluctuations within the interaction horizon will be washed out, but fluctuations at larger scales will grow. Matter within the larger region will continue gathering together, forming gravitational wells which attract more matter.

As the temperature of the universe approached the mass of the lightest right-handed neutrinos the recombination of lighter particles to Majorana states ceased. From the decays of the Majorana states comes the generation of a lepton asymmetry. An excess of leptons survives all the way down to the electroweak phase transition. During their journey from the decay of the Majoranas to the electroweak phase transition part of the leptons was transformed into a baryon excess. With the conversion of leptons into quarks, density fluctuations which were formed during the transition period and later on, are transformed into fluctuations of matter.

This scenario remains an attractive possibility for generating the baryon asymmetry in the universe. We now need observables supporting this scenario. Three interesting questions come to mind.

(1) Is the excess of baryons produced in leptogenesis consistent with the observed amount of matter relative to the photons observed in the universe?
(2) Are the neutrino oscillations and the CP asymmetries observed at low energies related to the CP in the early universe?
(3) Are there remnants of this scenario at cosmological scales that verify and support, or perhaps contradict, leptogenesis?

The answer to the first question is positive. Numerical studies have shown that a consistent picture emerges provided that

(i) the dilution factor in the out-of-equilibrium decays is $D \sim 10^{-3}$, and
(ii) the asymmetry ε from individual decays is of order 10^{-4} to 10^{-5} and for $g_* = 100$ degrees of freedom it gives the correct amount of matter relative to photons.

The answer to the second question is again positive. There is now a flurry of activity with many models proposed to relate the laboratory observations with leptogenesis. I will discuss some of them in the next section.

5. Consequences of leptogenesis

Several groups developed models with massive neutrinos which include neutrino oscillations, CP-violation in the leptonic sector and the generation of a lepton asymmetry. A general approach includes the standard model in a larger symmetry group and shows that the parameters of the theory are consistent with both the low energy phenomena and the generation of a lepton-asymmetry [11–26]. Most of these models incorporate the see-saw mechanism

$$m_\nu = m_{\mathrm{D}} \frac{1}{M_{\mathrm{R}}} m_{\mathrm{D}}^T \tag{12}$$

with m_{D} the Dirac mass matrix and M_{R} the mass matrix for the right-handed neutrinos. We note that Dirac matrices that occur here and in eq. (6) are slightly different. As we diagonalize the right-handed mass matrix a unitary matrix U_{R} appears. In the lepton-asymmetry occurs the product

$$\tilde{m}_{\mathrm{D}} = m_{\mathrm{D}} U_{\mathrm{R}} \tag{13}$$

and thus the structure of the right-handed sector influences the asymmetry.

Now many models imbed the standard model into a larger group and determine m_{D} from the low energy structure of the theory, including recent observations, and deduce U_{R} from the structure of the enlarged theory. The effort in this approach is to identify general aspects common in several theories, like $SO(10)$, $SU(5)$, Frogatt–Nielsen or texture models [18,26]. In several models the low energy phase from m_{D} does not appear in the lepton asymmetry [22], which means that the entire phase comes from the right-handed sector. However, there are models where the phases which are responsible for leptogenesis are the ones that generate CP-violation at low energies.

A second approach enlarges the group to be left–right symmetric [21], i.e. $SU(2)_{\mathrm{L}} \otimes SU_{\mathrm{R}}(2)$. In this case a Majorana mass term is also present in the left-handed sector. The mass relation is now modified as

$$m_\nu = m_{\mathrm{LL}} - m_{\mathrm{D}} \frac{1}{M_{\mathrm{R}}} m_{\mathrm{D}}^T, \tag{14}$$

where m_{LL} is a Majorana mass term for the left-handed neutrinos. The introduction of this term is possible with the introduction of Higgs triplets for the left- and right-handed sectors of the theory. The mass matrices are given as

$$m_{\mathrm{L}} = F v_{\mathrm{L}} \qquad \text{and} \qquad m_{\mathrm{R}} = F v_{\mathrm{R}}$$

with F the same matrix and $v_{\mathrm{L,R}}$ the vacuum expectation values of the triplet Higgses in the left- and right-sectors of the theory. The fact that both mass matrices are proportional to the matrix F simplifies the situation, because the unitary matrices which diagonalize the mass matrices for left-handed and right-handed fermions are now the same.

In case that the see-saw term in (14) is important only the top quark contributes to the Dirac matrix and a consistent solution was found. Alternatively, when $m_{\rm D}$ is proportional to the charged lepton mass matrix the see-saw term in eq. (13) is negligible, because $M_{\rm R}$ is very heavy [21]. In this case the out-of-equilibrium condition is automatically fulfilled for typical values of the parameters. In addition, the baryon asymmetry is of the correct magnitude and the large mixing angle solution for the solar problem is preferred.

In the laboratory, lepton number violation produces neutrinoless double beta decay of certain nuclei $(A,Z) \to (A,Z+2)+2e^-$ [21,24]. The decay width in these extremely rare processes is proportional to the square of an effective neutrino mass

$$\langle m_{ee} \rangle = \sum_i U_{ei}^2 m_i$$

originating from the Majorana sector. Since the matrix elements U_{ei} are complex cancellations can take place. It is therefore interesting to ask if the values of the parameters, which produce an acceptable baryon asymmetry, also deliver a sizeable $\langle m_{ee} \rangle$. In the simple model mentioned in the previous paragraph, the effective neutrino mass $\langle m_{ee} \rangle$ can have values which are close to the experimental bound on $\langle m_{ee} \rangle$ so that new experiments will be sensitive to neutrinoless double-beta decay.

6. CP asymmetry in the leptonic sector

In parallel to these activities there are serious plans to measure CP violation in the leptonic sector. The experiments will measure the difference

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

which requires measuring ν_α, ν_β, and $\bar{\nu}_\alpha$, $\bar{\nu}_\beta$ interactions with hadrons at two different places. This demands precise knowledge of the neutrino–hadron cross sections at low energies. Even though neutrino–hadron interactions are more than 30 years old the low energy cross sections are still poorly known. In addition since the targets are very large, detailed instrumentation of the detectors is not always possible. For these reasons there is an increasingly active community of physicists concerned with these matters. They are concerned in measuring the neutrino cross sections carefully and at the same time developing accurate theoretical calculations. The reactions of interest are quasi-elastic scattering, resonance production and the transition region to deep inelastic scattering. There is already an effort in this direction with a conference organized every year (NUINT 01 and 02).

A second aspect deals with the fact that the reactions occur in light and medium-heavy nuclei where additional corrections are present. Figure 3 shows the neutrino nucleon cross section for $E_\nu < 6$ GeV from a Brookhaven experiment [27]. The contributions of quasi-elastic scattering and resonance production are clearly evident in the data up to ~ 2 GeV. They produce the structure which looks like a step. The theoretical curve reproduces the data [28]. More work and cross-checks

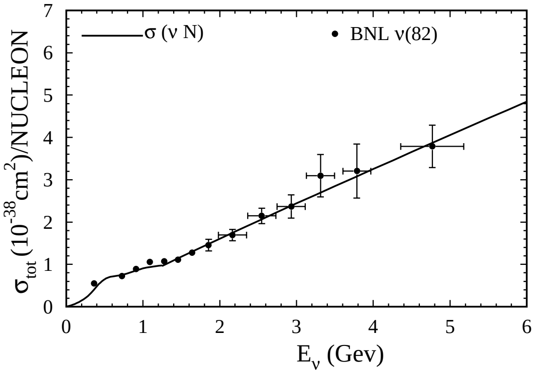

Figure 3. Neutrino nucleon cross section at low energies [27].

will be necessary in order to obtain a precise understanding of the data, required in order to be able to observe CP-violation. I presented only one figure as an example, however, more results are available and several studies are in progress [29].

7. Summary

Leptogenesis presents a very attractive mechanism for triggering the baryon asymmetry in the universe. It has implications for the development of fluctuations and inhomogeneities triggered by Majorana neutrinos and later on by ordinary matter. This topic is very interesting and deserves further investigation [29].

A second development studies the connection between the theoretical framework proposed for the heavy Majorana states and laboratory phenomena including neutrino oscillations. Here there are many models which try to find out common aspects and fulfill the general conditions required by leptogenesis. A consistent picture has been developed in many models and new developments are expected.

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