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Understanding neutrino masses and mixings

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Abstract. We discuss ways to understand large neutrino mixings using new symmetries of quarks and leptons beyond the standard model for the three allowed patterns of neutrino masses: normal, inverted hierarchy and degenerate masses.

Keywords. See-saw; bi-maximal; left–right symmetry; grand unification; horizontal symmetry.

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1. Introduction

There is now strong evidence for neutrino masses and mixings from the solar and atmospheric neutrino observations, where there is a deficit of observed neutrinos compared to theoretical expectations. The simplest way to understand the deficits in neutrino fluxes observed in the above experiments is to assume that the incident neutrinos oscillate into another species which cannot be detected. Laboratory experiments in Japan that use accelerator muon neutrinos as in the K2K experiment and reactor neutrinos as in the Kamland experiment have also shown deficits in their flux compared to expectations providing additional evidence for oscillations. Thus the phenomenon of neutrino oscillations seems to be well-established.

For neutrino oscillations to take place, they must be massive and must mix among themselves, with appropriate mass differences and mixing angles. It seems that all the above data can be understood in terms of oscillations of the three known neutrinos, i.e., ν_e, ν_μ, ν_τ among themselves. Since the standard model predicts that neutrinos are massless, this is evidence for physics beyond the standard model. The hope is that by studying the pattern of neutrino mixings required to understand observations, we will have a roadmap of physics beyond the standard model.

It is worth mentioning that in the domain of accelerator experiments, the Los Alamos experiment (LSND) has also shown positive evidence for oscillation of $\bar{\nu}_e$ to $\bar{\nu}_\mu$. This result has not been confirmed by KARMEN which has also looked for the same process. Currently MiniBoone experiment at Fermilab is searching for this process. If LSND is confirmed, it will require drastic change in our understanding of neutrinos, e.g. it will require the existence of sterile neutrinos that mix with the known neutrinos. We will ignore this evidence in the present paper.

Furthermore, in the domain of laboratory experiments, the negative results from the two experiments, CHOOZ and PALO VERDE provide upper limits on one of the mixing angles (to be called U_{e3} below) that has important implications for theories of neutrino masses. Further experiments are in progress or in planning stages to improve the limits on U_{e3}.

In this brief overview, I wish to draw attention to some of the theoretical ideas for understanding neutrino mass and mixing patterns in extensions of the standard model. This article will focus specifically on the see-saw mechanism that seems to provide the simplest way to understand small neutrino masses [1] and some attempts to understand large neutrino mixings within these models.

1.1 *Major theoretical issues in neutrino physics*

Major issues of interest in neutrino theory are driven by the following experimental results and conclusions derived from them. We will use the notation, where the flavor or weak eigenstates ν_α (with $\alpha = e, \mu, \tau$) are expressed in terms of the mass eigenstates ν_i ($i = 1, 2, 3$) as follows: $\nu_\alpha = \sum_i U_{\alpha i}\nu_i$. The $U_{\alpha i}$, the elements of the Pontecorvo–Maki–Nakagawa–Sakata matrix represent the observable mixing angles in the basis where the charged lepton masses are diagonal. In any other basis, one has $U = U_\ell^\dagger U_\nu$, where the matrices on the right-hand side are the ones that diagonalize the charged lepton and neutrino mass matrices respectively.

1.1.1 *Solar neutrinos.* Thanks to the SNO results on both charged and neutral currents, and the Kamland results, there now appears to be a winner among the various possible oscillation solutions to the solar neutrino puzzle. It seems that the so-called LMA MSW solution is preferred over the small angle as well as the low and pure vacuum solution [2]. The recently announced KAMLAND results [3] prefer the LMA solution over the others and it also rules out many of the non-oscillation as well as the magnetic moment solution to the solar neutrino problem [4]. The present range of preferred values of these parameters are: $2 \times 10^{-5} \leq \Delta m^2_\odot/\mathrm{eV}^2 \leq 4 \times 10^{-4}$ and $0.62 \leq \sin^2 2\theta_\odot \leq 0.99$ at 3σ confidence level. All non-oscillation mechanisms could however be present at a subdominant level and higher precision experiments are necessary to test for their presence.

1.1.2 *Atmospheric neutrinos.* Here evidence appears very convincing that the explanation of observed muon neutrino deficit in upward going muons as well as the azimuthal angle dependence of this spectrum involves oscillation of ν_μ to ν_τ, with $\Delta m^2_{\nu_\mu - \nu_\tau} \simeq 2.5 \times 10^{-3}$ eV^2 and maximal mixing $\sin^2 2\theta_A \geq 0.84$ at 99% c.l.

1.1.3 *Neutrinoless double beta decay.* Oscillation involves only mass differences and therefore do not give information on the overall scale of the neutrino masses. One may hope that neutrinoless double beta decay may provide this information. It however turns out that this hope is not completely justified even if the present limits on lifetime go up by two orders of magnitude as is contemplated in many experiments unless the neutrinos are quasi-degenerate with common mass in the range bigger than 0.05 eV.

Nevertheless, neutrinoless double beta decay is an experiment of fundamental significance since its observation will for the first time give evidence that neutrino is its own antiparticle and signal the breakdown of $B-L$ quantum number. Searches for $\beta\beta_{0\nu}$ decay has been going on for several years and a new round of higher precision experiments are on the verge of being launched. The most stringent limits on this decay are from the enriched ^{76}Ge experiment by the Heidelberg–Moscow as well as the IGEX Collaborations and can be converted to a constraint on masses and mixing angles as: $\sum_i U^2_{ei} m_i \leq 0.3$ eV, with an uncertainty of a factor of 2 to 3 due to nuclear matrix elements. Presently, planned experiments such as GENIUS, MAJORANA, CUORE, EXO, XMASS and MOON are expected to push this limit down by one order of magnitude.

1.1.4 U_{e3}. The reactor experiments CHOOZ and PALO VERDE imply that $U_{e3} \leq 0.16 - 0.2$.

All this information can be summarized in the following form for the **U** matrix (ignoring CP violation):

$$\mathbf{U} \simeq \begin{pmatrix} c & s & \epsilon \\ -\frac{s+c\epsilon}{\sqrt{2}} & \frac{c-s\epsilon}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s-c\epsilon}{\sqrt{2}} & \frac{-c-s\epsilon}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1}$$

where $\epsilon \equiv U_{e3}$.

As far as the mass pattern goes however, there are three possibilities all equally viable from experimental point of view:

- normal hierarchy: $m_1 \ll m_2 \ll m_3$;
- inverted hierarchy: $m_1 \simeq -m_2 \gg m_3$;
- approximately degenerate pattern $m_1 \simeq m_2 \simeq m_3$;

where m_i are the eigenvalues of the neutrino mass matrix. In the first case, the atmospheric and the solar neutrino data give direct information on m_3 and m_2 respectively. On the other hand, in the last case, the mass differences between the first and the second eigenvalues will be chosen to fit the solar neutrino data and the second and the third to fit the atmospheric neutrino data.

Three of the major theoretical challenges in neutrino physics now are:

- How does one understand the extreme smallness of the neutrino masses?
- How does one understand two large mixing angles among neutrinos given that there is so much similarity between quarks and leptons at the level of interactions and that the quark mixings are small?
- What is the mass pattern among the neutrinos and how does one understand them from a theoretical point of view simultaneously with the near bimaximal mixing pattern? In particular, why is $\Delta m^2_{\odot}/\Delta m^2_A \ll 1$?

2. See-saw mechanism for small neutrino masses

It is well-known that in the standard model the neutrino is massless due to a combination of two reasons: (i) its right-handed partner (ν_{R}) is absent and (ii) the model has exact global $B-L$ symmetry. Clearly, to understand a non-zero neutrino mass, one must give up one of the above assumptions. If one blindly included a $\nu_{\rm R}$ to the standard model as a singlet, the status of neutrino would be parallel to all other fermions in the model and one would be hard put to understand why its mass is so much smaller than that of other fermions. Clearly there must be some other new ingredient that must be added.

A first hint of this new ingredient came from the observation of Weinberg that if $B-L$ symmetry is broken by some high scale physics, in the effective low energy theory, one can have operators of the form $(LH)^2/M$, where M denotes the scale of new physics [5]. This, after electroweak symmetry breaking would lead to a neutrino mass $\sim v_{wk}^2/M$. The key question now is, what is the value of M?

In the absence of any $B-L$ violating physics all the way up to the Planck scale and assuming that non-perturbative Planck scale physics breaks all global symmetries such as the global $B-L$ symmetry present in the standard model, the above higher dimensional operators takes the form [6] $LHLH/M_{\rm Pl}$ (where L is a lepton doublet and H is the Higgs doublet). This operator leads to masses for neutrinos of order 10^{-5} eV or less and is therefore not adequate for understanding observations. Thus a non-trivial extension of the standard model is called for wherein, the requisite value for M to explain the atmospheric neutrino data (of order 10^{14} GeV or so) must be the scale of $B-L$ breaking. One then faces a 'naturalness' question similar to the Higgs mass problem of the standard model, i.e., why the radiative corrections do not send the mass M up to the Planck scale?

We will see below that there are at least two candidate symmetries which are compelling from other arguments and provide a reason for the stability of the new scale mass M. Both these symmetries are local symmetries and are connected with adding right-handed neutrinos to the standard model:

- local $B-L$ and/or;
- $SU(2)_H$ horizontal symmetry acting on the first two generations.

The most widely discussed example is the local $B-L$ symmetry but there are also very interesting arguments for the second and it may perhaps be a combination of both. The mass M in these examples is the Majorana mass of the right-handed neutrinos that break either or both these symmetries (i.e. in the exact symmetry limit the RH neutrinos have zero mass). In both cases we get what is known in the literature as the see-saw mechanism.

2.1 *Quark lepton symmetry and local $B-L$ symmetry*

As the first example of a model with right-handed neutrinos ($N_{\rm R}$), consider making the standard model completely quark lepton symmetric by adding one $N_{\rm R}$ per generation. This expands the gauge symmetry of the electroweak interactions to $SU(2)_{\rm L}\times U(1)_{I_{3\rm R}}\times U(1)_{B-L}$ or to its full left-right symmetric extension $SU(2)_{\rm L}\times SU(2)_{\rm R}\times U(1)_{B-L}$ symmetry. In the latter case, the fermion doublets $(u,d)_{\rm L,R}$ and $(\nu,e)_{\rm L,R}$ are assigned to the left–right gauge group in a parity symmetric manner. The electric charge formula for the model takes a very interesting form [7]: $Q = I_{3\rm L} + I_{3\rm R} + (B-L)/2$. It can be concluded from this that below the scale $v_{\rm R}$

where the $SU(2)_{\rm R} \times U(1)_{B-L}$ symmetry breaks down to the standard model and above the scale of M_W, one has the relation $\Delta I_{3\rm R} = -\Delta \frac{B-L}{2}$. This simple looking relation has the profound consequence that neutrinos must be Majorana fermions and that there must be lepton number violating interactions in nature. Furthermore it explains why the right-handed neutrino mass is so much smaller than the Planck mass- it is connected with the breaking of local $B-L$ symmetry.

2.2 *Type I versus type II see-saw*

To see how small neutrino masses are explained, note that the $\nu_{\rm L} - \nu_{\rm R}$ mass matrix for three generations takes the form:

$$M \;=\; \begin{pmatrix} M_{\rm LL} & M_{\rm LR} \\ M_{\rm LR}^T & M_{\rm RR} \end{pmatrix}, \tag{2}$$

where $M_{\rm RR} = \mathbf{f} v_{\rm R}$ is the Majorana mass matrix of the right-handed neutrinos, ($\mathbf{f}$ is the new Yukawa coupling matrix that determines the right-handed neutrino masses). The first term $M_{\rm LL} \simeq \mathbf{f}(v_{wk}^2/v_{\rm R})$ is the induced Majorana mass matrix for the left-handed neutrinos and is characteristic of the existence of asymptotic parity symmetry. (It would, for example, be absent if the local symmetry is $SU(2)_{\rm L} \times U(1)_{I_{3\rm R}} \times U(1)_{B-L}$.) The contribution $M_{\rm LR} \equiv M_{\rm D} = \mathbf{Y} v_{wk}$ is the Dirac mass matrix connecting the left- and the right-handed neutrinos. The diagonalization of this mass matrix leads to following form for the light neutrino masses:

$$M_\nu \simeq \;\mathbf{f}\frac{\lambda v_{wk}^2}{v_{\rm R}} - \frac{1}{v_{\rm R}} M_{\rm D}^T \mathbf{f}^{-1} M_{\rm D}, \tag{3}$$

$\mathbf{f}$, the Yukawa coupling matrix that is responsible for the masses of the heavy right-handed neutrinos characterizes the high scale physics, whereas all other parameters denote physics at the weak scale. We have called this generalized formula for neutrino masses, the type II see-saw [12] formula to distinguish it from the type I see-saw formula, the one that is commonly used in literature where the first term of eq. (2) is absent. An important feature of this formula is that both terms vanish as $v_{\rm R} \to \infty$ and since $v_{\rm R} \gg v_{wk}$, the neutrino masses are much smaller than the charged fermion masses. As was particularly emphasized in the third paper of ref. [1], the dominance of V-A interaction in the low energy weak processes is now connected to the smallness of neutrino masses.

If in the above see-saw formula, the second term dominates, this leads to the canonical type I see-saw formula and leads to the often discussed hierarchical neutrino masses, which in the approximation of small mixings lead to $m_{\nu_i} \simeq m_{f_i}^2/v_{\rm R}$, where f_i is either a charged lepton or a quark depending on the kind of model for neutrinos.

On the other hand, in models where the first term dominates, the neutrino masses can be almost generation independent unless f itself has the flavor structure of the charged fermions. For example, if there is indication for neutrinos being degenerate in mass from observations, one will have to resort to type II see-saw mechanism for its understanding.

A further advantage of the right-handed neutrino and see-saw mechanism is that it fits in very nicely into grand unified frameworks based on $SO(10)$ models. The coupling constant unification then provides a theoretical justification for the high see-saw scale and hence the small neutrino masses. Furthermore, the **16**-dim. spinor representation of $SO(10)$ has just the right quantum numbers to fit the $\nu_{\rm R}$ in addition to the standard model particles of each generation.

2.3 *Double see-saw with a low scale for $B-L$ symmetry*

As we saw from the previous discussion, the conventional see-saw mechanism requires rather high scale for the $B-L$ symmetry breaking and the corresponding right-handed neutrino mass (of order $\geq 10^{14}$ GeV). There is however no way at present to know what the scale of $B-L$ symmetry breaking is. There are for example models bases on string compactification [8] where the $B-L$ scale is quite possibly in the TeV range. In this case small neutrino mass can be implemented by a double see-saw mechanism suggested in ref. [9]. The idea is to take a right-handed neutrino N and a singlet neutrino S which has extra quantum numbers which prevent it from coupling to the left-handed neutrino. One can then write a three by three neutrino mass matrix in the basis (ν, N, S) of the form:

$$M = \begin{pmatrix} 0 & m_{\rm D} & 0 \\ m_{\rm D} & 0 & M \\ 0 & M & \mu \end{pmatrix}. \tag{4}$$

For the case $\mu \ll M \approx M_{B-L}$, (where M_{B-L} is the $B-L$ breaking scale) this matrix has one light and two heavy states. A generalization of this mechanism to the case of three generations is straightforward. One important point here is that to keep $\mu \sim m_{\rm D}$, one also needs some additional gauge symmetries, which often are a part of the string models. It can also be used in models with high scale $B-L$ breaking where the RH neutrino is forbidden by symmetries [10].

2.4 *$SU(2)_H$ local symmetry and 3×2 see-saw with two $N_{\rm R}$'s*

A symmetry among different generations has often been suspected as a possible way to understand the different properties of the quarks and leptons of different generations. This symmetry for the three generation case could be either a $U(1)$, $SU(2)$ or an $SU(3)$ local symmetry. Of these three possibilities, the third one requires that we include additional fermions to cancel anomalies. Of the remaining two, we choose $SU(2)_H$ since it has the following interesting property, i.e., if it operate on right-handed charged leptons, cancellation of global Witten anomaly requires that we must introduce at least two right-handed neutrinos $(N_{e\rm R}, N_{\mu\rm R})$ transforming as a doublet under the group. Thus two right-handed neutrinos is the minimal set required theoretically. Clearly, masses of the right-handed neutrinos are connected to the breaking of the $SU(2)_H$ symmetry [11]. An additional feature of these matrices is that they lead to a 3×2 see-saw as compared to the 3×3 see-saw in the case of the left–right symmetric (or $SO(10)$) models. This could of course be a part of the latter class of models if $v_H \ll M_{B-L}$. A distinct feature of the models with 3×2 seesaw is that one of the light neutrinos is massless. In this sense, in these models all parameters of a real neutrino mass matrix are determinable by only oscillation experiments.

3. Some attempts to understand large mixings

One of the major mysteries of neutrino physics is the need for large mixing angles to explain solar and the atmospheric neutrino oscillations. This is because of the simple fact that there is so much similarity in the interactions between the quarks and leptons and quarks mixings between different generations are of course well-known to be very small. In the see-saw framework one may attribute this to the fact that a central ingredient in understanding the neutrino mass matrix is the mass matrix of the right-handed neutrino which reflects high scale physics whereas quark physics is low scale physics and it can dictate only the pattern of the Dirac mass of the neutrinos. While this is qualitatively a reasonable argument, it is not much help in providing a quantitative understanding. The general strategy to make any headway towards a quantitative understanding is to search for mass matrices that fit observations and then search for symmetry or dynamical reasons for their origin.

To get useful mass patterns, one must first note that in the absence of CP violation, the symmetric Majorana mass matrix for the light neutrinos M_ν contains six parameters, whereas observations give only five pieces of observation, i.e., $\Delta m^2_{A,\cdot}$, $\theta_{12} \equiv \theta_\odot$, $\theta_{23} \equiv \theta_A$ and $U_{e3} \equiv \theta_{13}$. The absence of the sixth piece of information is essentially reflected in the fact that the precise mass pattern (normal, inverted or degenerate) of neutrinos is not known. So to make any progress, one may try to make ansatzes that reduce the number of parameters in a mass matrix either by making different elements equal or by putting them to zero in a basis where the charged leptons are diagonal.

An example of the first strategy is the zeroth order mass matrix discussed in [13]:

$$M_\nu = \begin{pmatrix} A+D & F & F \\ F & A & D \\ F & D & A \end{pmatrix}. \tag{5}$$

This leads to an exact bimaximal pattern but allows for all different mass patterns depending on the relative values of the parameters A, D and F. Since the present data implies that there are deviations from the exact bimaximal form, this mass matrix must have additional small corrections.

Three different mass patterns can emerge from this mass matrix in various limits: e.g. (i) for $F \ll A \simeq -D$, one gets the normal hierarchy; (ii) for $F \gg A, D$, one has the inverted pattern for masses and (iii) the parameter region $F, D \ll A$ leads to the degenerate case. An interesting symmetry of this mass matrix is the $\nu_\mu \leftrightarrow \nu_\tau$ interchange symmetry, which is obvious from the matrix; but in the limit where $A = D = 0$, there appears a much more interesting symmetry, i.e., the continuous symmetry $L_e - L_\mu - L_\tau$ [14]. If the inverted mass matrix is confirmed by future experiments, this symmetry will provide an important clue to new neutrino related physics beyond the standard model. Inverted mass pattern is the only case where such an interesting leptonic symmetry appears. Let us therefore discuss the implications of this symmetry further.

3.1 *Approximate $L_e - L_\mu - L_\tau$ symmetry and neutrino mixings*

In the exact $L_e - L_\mu - L_\tau$ symmetry limit, the model not only leads naturally to large solar and atmospheric mixing angles but it also leads to vanishing U_{e3} as well as $\Delta m^2_\odot/\Delta m^2_A = 0$. Therefore the model raises the hope that a small U_{e3} as well as the smallness of $\Delta m^2_\odot/\Delta m^2_A$ can be understood in a natural manner. One must therefore add small symmetry breaking terms to this model and examine the consequences.

This question was studied in two papers [15]. In the second paper of [15], the following mass matrix for neutrinos was considered that includes small $L_e - L_\mu - L_\tau$ violating terms.

$$M_\nu = m \begin{pmatrix} z & c & s \\ c & y & d \\ s & d & x \end{pmatrix}, \tag{6}$$

where $c = \cos\theta$ and $s = \sin\theta$. The charged lepton mass matrix is chosen to have a diagonal form in this basis and $L_e - L_\mu - L_\tau$ symmetric.

In the presence of the small symmetry breaking terms, we find the following sum rules involving the neutrino observables and the elements of the neutrino mass matrix. The non-trivial ones are:

$$\begin{aligned} \sin^2 2\theta_\odot &= 1 - \left(\frac{\Delta m^2_\odot}{4\Delta m^2_A} - z\right)^2 + O(\delta^3), \\ \frac{\Delta m^2_\odot}{\Delta m^2_A} &= 2(z + \vec{v}\cdot\vec{x}) + O(\delta^2), \\ U_{e3} &= \vec{A}\cdot(\vec{v}\times\vec{x}) + O(\delta^3), \end{aligned} \tag{7}$$

where $\vec{v} = (\cos^2\theta, \sin^2\theta, \sqrt{2}\sin\theta\cos\theta)$, $\vec{x} = (x, y, \sqrt{2}d)$ and $\vec{A} = \frac{1}{\sqrt{2}}(1,1,0)$. δ in the preceding equations represents the small parameters in the mass matrix.

Major consequences of these relations are that (i) there is a close connection between the measured value of the solar mixing angle and the neutrino mass measured in neutrinoless double beta decay, i.e., z; (ii) the present values for the solar mixing angle can be used to predict $m_{\beta\beta}$ for a value of $\Delta m^2_\odot$. For instance, for $\sin^2 2\theta_\odot = 0.9$, we would predict $((\Delta m^2_\odot/4\Delta m^2_A) - z) = 0.3$. For small $\Delta m^2_\odot$, this implies $m_{\beta\beta} \simeq 0.01$ eV. The second relation involving the $\Delta m^2_\odot/\Delta m^2_A$ in terms of x, y, z, d tells us that for this to be the case, we must have strong cancellation between the various small parameters. Given this, the above $m_{\beta\beta}$ value is expected to be within the reach of new double beta decay experiments contemplated. Note however that $\sin^2 2\theta_\odot$ cannot be smaller than 0.9 in the case of approximate $L_e - L_\mu - L_\tau$ symmetry.

If the value of $\sin^2 2\theta_\odot$ is ultimately determined to be less than 0.9, the question one may ask is whether the idea of $L_e - L_\mu - L_\tau$ symmetry is dead. The answer is in the negative since so far we have explored the breaking of $L_e - L_\mu - L_\tau$ symmetry only in the neutrino mass matrix. It was shown in the first paper of [15] that if the symmetry is broken in the charged lepton mass, one can lower $\sin^2\theta_\odot$ to 0.85 or so.

3.2 *Approximate $L_e - L_\mu - L_\tau$ symmetry from $SU(2)_H$ horizontal symmetry*

It can be shown that an $SU(2)_H$ model for leptons leads quite generally to an approximate $L_e - L_\mu - L_\tau$ symmetry for neutrinos. As already noted, a distinct feature of $SU(2)_H$ symmetry is that there are two right-handed neutrinos instead of three and therefore one has a 3×2 see-saw rather than the usual 3×3 one.

To see this in detail, first note that the gauge interactions have the symmetry $SU(2)_H \times U(1)_{e+\mu+\tau}$ global symmetry. The diagonal generator of $SU(2)_H$ is given by $L_e - L_\mu$. If we break horizontal symmetry by an $SU(2)_H$ triplet Higgs Δ_H, then $L_e - L_\mu$ survives as a gauge symmetry of leptons. We further break the symmetry by a doublet Higgs χ_H, then the allowed Yukawa couplings that contribute to neutrino masses are of the form $N^c\Delta_H N^c$, $L_\tau H_u\chi_H N^c$. Note that these two terms reduce the above global symmetry to $SU(2)_H \times U(1)_\tau$. The vevs of these Higgs fields, i.e., $\langle\Delta_H, 3\rangle \neq 0$ and $\chi_{H,2} \neq 0$ reduce this symmetry down to $L_e - L_\mu - L_\tau$. This is the major reason why this model leads to an inverted hierarchy and also two large mixings in zeroth order as desired. Thus if experiments confirm the inverted hierarchy and a possible $L_e - L_\mu - L_\tau$ symmetry for leptons, it may be a signal for the local $SU(2)_H$ symmetry at a high scale.

The charged lepton masses arise from the couplings of the form $LH_d\chi_H\tau^c$ and $L_\tau H_d\chi_H E^c$. The second term breaks $L_e - L_\mu - L_\tau$ symmetry and is responsible for departure from exact maximal mixing angle in the 12 sector as well as for solar mass splittings.

Using the discussion of the above paragraph, the Dirac mass of the neutrino as well as the right-handed neutrino mass matrix can be seen to lead [11] to 5×5 mass matrix for heavy and light neutrinos of the form

$$M_{\nu_{\rm L},\nu_{\rm R}} = \begin{pmatrix} 0 & 0 & 0 & h_0\kappa_0 & 0 \\ 0 & 0 & 0 & 0 & h_0\kappa_0 \\ 0 & 0 & 0 & h_1\kappa_1 & h_1\kappa_2 \\ h_0\kappa_0 & 0 & h_1\kappa_1 & 0 & fv'_H \\ 0 & h_0\kappa_0 & h_1\kappa_2 & fv'_H & 0 \end{pmatrix}. \tag{8}$$

After see-saw diagonalization, it leads to the light neutrino mass matrix of the form

$$M_\nu = -M_{\rm D}M_{\rm R}^{-1}M_{\rm D}^T, \tag{9}$$

where $M_{\rm D} = \begin{pmatrix} h_0\kappa_0 & 0 \\ 0 & h_0\kappa_0 \\ h_1\kappa_1 & h_1\kappa_2 \end{pmatrix}$; $M_{\rm R}^{-1} = \frac{1}{fv'_H}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The resulting light Majorana neutrino mass matrix M_ν is given by

$$M_\nu = -\frac{1}{fv'_H}\begin{pmatrix} 0 & (h_0\kappa_0)^2 & h_0h_1\kappa_0\kappa_2 \\ (h_0\kappa_0)^2 & 0 & h_0h_1\kappa_0\kappa_1 \\ h_0h_1\kappa_0\kappa_2 & h_0h_1\kappa_0\kappa_1 & 2h_1^2\kappa_1\kappa_2 \end{pmatrix}. \tag{10}$$

First of all, as discussed before, this leads to one neutrino which is massless. To get the physical neutrino mixings, we also need the charged lepton mass matrix defined by $\bar{\psi}_{\rm L} M_\ell \psi_{\rm R}$. This is given in our model by

$$M_\ell = \begin{pmatrix} h'_2\kappa_0 & 0 & -h'_1\kappa_2 \\ 0 & h'_2\kappa_0 & h'_1\kappa_1 \\ h'_4\kappa_1 & h'_4\kappa_2 & h'_3\kappa_0 \end{pmatrix}. \tag{11}$$

Note that in the limit of $\kappa_1 = 0$, the neutrino mass matrix has the $L_e - L_\mu - L_\tau$ symmetry and also there are mixing effects coming from the charged lepton sector so that one can get a lower value for $\sin^2 2\theta_\odot$.

4. Large mixings in models with quark–lepton unification

The $L_e - L_\mu - L_\tau$ model discussed above treats the quarks and leptons on a fundamentally different footing. On the other hand, it could be that at very short distances there is quark–lepton unification [16]. I give below two of a number of ideas, where models with quark–lepton symmetry can lead to large neutrino mixings. In the models discussed below large mixings arise dynamically and without need for any extra symmetries starting with small mixings at very short distances as would be dictated by quark–lepton symmetry.

4.1 *Radiative magnification of mixing angles*

In this class of models dynamics of radiative corrections plays an essential role in understanding the maximal mixing. The basic idea is that at the see-saw scale, all mixing angles are small, a situation quite natural if the pattern of **f** Yukawa coupling is similar to the quark sector. Since the observed neutrino mixings are weak scale observables, one must extrapolate [17] the see-saw scale mass matrices to the weak scale and recalculate the mixing angles.

The extrapolation formula is

$$M_\nu(M_Z) = \mathbf{I} M_\nu(v_{\rm R})\mathbf{I}, \tag{12}$$

where

$$\mathbf{I}_{\alpha\alpha} = \left(1 - \frac{h_\alpha^2}{16\pi^2}\right). \tag{13}$$

Note that since $h_\alpha = \sqrt{2} m_\alpha / v_{wk}$ (α being the charged lepton index), in the extrapolation only the τ lepton makes a difference. In the MSSM, this increases the $M_{\tau\tau}$ entry of the neutrino mass matrix and essentially leaves the others unchanged. It was shown in ref. [18] that if the muon and the τ neutrinos are nearly degenerate in mass at the see-saw scale, and in supersymmetric theories, $\tan\beta \geq 5$, the radiative corrections can become large enough so that at the weak scale the two diagonal elements of M_ν which were nearly equal but different at the see-saw scale become extremely degenerate. This leads to an enhancement of the mixing angle to become almost maximal and a solution to the atmospheric neutrino deficit emerges even though at the see-saw scale, the mixing angles were small. This happens only if the experimental observable $\Delta m^2_{23} \leq 0$ a possibility that can be tested in contemplated long base line experiments. Also for this mechanism to work, the overall scale of neutrino masses must be in the range of 0.1 eV or so making the idea testable in the forthcoming double beta decay experiments.

Several comments are in order: (i) to get a near degenerate mass spectrum without additional assumptions, one must use the type II see-saw mechanism as in eq. (3); (ii) an interesting question is whether this mechanism can be extended to the case of three generations and whether it can explain the bimaximal pattern also. There are examples where this can happen [19]. A second recent work [20] has used the techniques of ref. [18] to study radiative magnification of solar angle in texture zero neutrino mass matrices. In this example, the atmospheric neutrino mixing is an input but solar angle is dynamically magnified.

4.2 *A minimal SO(10) model*

Another suggestion for understanding large atmospheric mixing has been made within a class of $SO(10)$ models, which are strongly suggested by local $B-L$ symmetry, large see-saw scale and grand unification ideas. The basic ingredients of this suggestion are the following properties of the $SO(10)$ model: (i) that one can construct a minimal $SO(10)$ model with only two multiplets that couple to fermions, i.e., **10** and **126** and another that breaks $SO(10)$ down to the left–right model. The second breaks the $B-L$ symmetry and the first breaks the electroweak symmetry and (ii) a second property of $SO(10)$ models [21] is that **126** contains submultiplets that not only contribute to charged fermion but also to the left- and right-handed Majorana masses ($M_{\rm LL}, M_{\rm RR}$ respectively in eq. (2)) for the neutrinos. This leads to a tremendous reduction of the number of arbitrary parameters in the model, as we will see below.

There are only two Yukawa coupling matrices in this model: (i) h for the **10** Higgs and (ii) f for the **126** Higgs. $SO(10)$ has the property that the Yukawa couplings involving the **10** and **126** Higgs representations are symmetric. Therefore if we ignore CP violation and work on a basis where one of these two sets of Yukawa coupling matrices is diagonal, then it will have only nine parameters. Noting the fact that the (2,2,15) submultiplet of **126** has a standard model doublet that contributes to charged fermion masses, one can write the quark and lepton mass matrices as follows [21]:

$$M_u = h\kappa_u + fv_u,$$
$$M_d = h\kappa_d + fv_d,$$
$$M_\ell = h\kappa_d - 3fv_d,$$
$$M_{\nu_{\rm D}} = h\kappa_u - 3fv_u, \tag{14}$$
$$M_\nu = fv_{\rm L} - M_{\nu^{\rm D}} M_{\rm R}^{-1} M_{\nu^{\rm D}}, \tag{15}$$

where $\kappa_{u,d}$ are the vevs of the up and down Higgs vevs of the standard model doublets in **10** Higgs and $v_{u,d}$ are the corresponding vevs for the same doublets in **126**. Note that there are 13 parameters in the above equations and there are

13 inputs (six quark masses, three lepton masses and three quark mixing angles and weak scale). Thus all parameters of the model that go into fermion masses are determined.

To determine the light neutrino masses, we use the see-saw formula in eq. (3), where **f** is nothing but the **126** Yukawa coupling. Thus all parameters that give neutrino mixings except an overall scale are determined. These models were extensively discussed in the last decade [22]. Initially, CP phases were ignored and only recently CP phases have been included in the analysis.

A very interesting point regarding these models has been noted in ref. [23], where it is pointed out that if the direct triplet term in type II see-saw dominates, then it provides a very natural understanding of the large atmospheric mixing angle without invoking any symmetries. A simple way to see this is to note that when the triplet term dominates the see-saw formula, we have the neutrino mass matrix $M_\nu \propto f$, where f matrix is the **126** coupling to fermions discussed earlier. Using the above equations, one can derive the following sum rule (sum rule was already noted in the second reference of [22]):

$$M_\nu = c(M_d - M_\ell). \tag{16}$$

Now quark–lepton symmetry implies that for the second and third generation, the $M_{d,\ell}$ have the following general form:

$$M_d = \begin{pmatrix} \epsilon_1 & \epsilon_2 \\ \epsilon_2 & m_b \end{pmatrix}, \tag{17}$$

and

$$M_\ell = \begin{pmatrix} \epsilon_1' & \epsilon_2' \\ \epsilon_2' & m_\tau \end{pmatrix}, \tag{18}$$

where $\epsilon_i \ll m_{b,\tau}$ as required by low energy observations. It is well-known that in supersymmetric theories, when low energy quark and lepton masses are extrapolated to the GUT scale, $m_b \simeq m_\tau$. One then sees from the above sum rule for neutrino masses that all entries for the neutrino mass matrix are of the same order leading very naturally to the atmospheric mixing angle to be large. Thus one has a natural understanding of the large atmospheric neutrino mixing angle. No extra symmetries are assumed for this purpose. For this model to be a viable one for three generations, one must show that the minimal $SO(10)$ model with triplet vev dominated see-saw formula indeed can give a large θ_{12} and a small θ_{13}. This has been shown in a recent paper [24]. The predictions of this model for various oscillation parameters are given in figures 1, 2 and 3 in a self-expalanatory notation. Note specifically the prediction for U_{e3} which can be tested in MINOS as well as other planned long base line neutrino experiments such as Numi-Off-Axis, JHF etc.

5. Conclusion

The see-saw mechanism appears by far to be the simplest way to understand the small neutrino masses. The large right-handed neutrino mass implied by this also

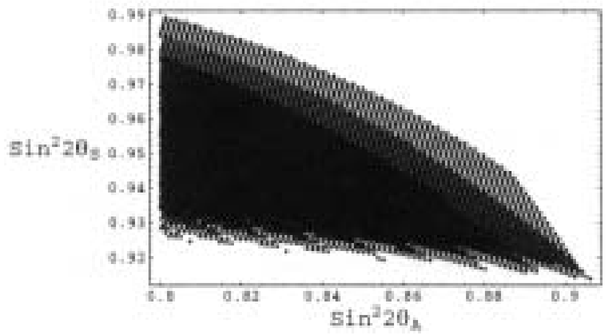

Figure 1. This figure shows the predictions for $\sin^2 2\theta_\odot$ and $\sin^2 2\theta_A$ for the range of quark masses at GUT scale. Note that $\sin^2 2\theta_\odot \geq 0.9$ and $\sin^2 2\theta_A \leq 0.9$.

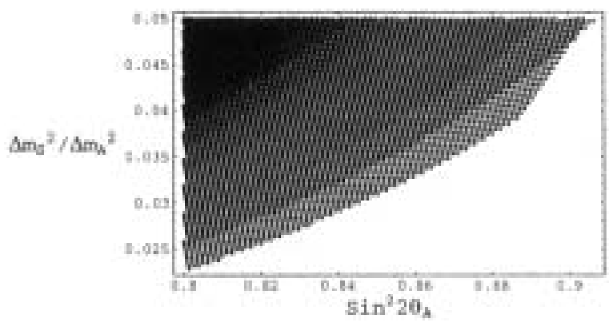

Figure 2. This figure shows the predictions for $\sin^2 2\theta_A$ and $\Delta m^2_\odot/\Delta m^2_A$ for the range of quark masses and mixings that fit charged lepton masses.

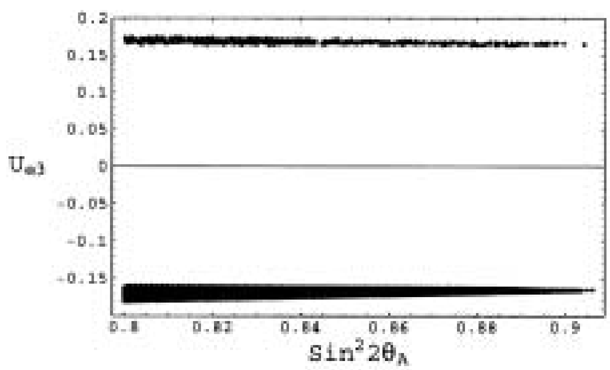

Figure 3. This figure shows the predictions of the model for $\sin^2 2\theta_A$ and U_{e3} for the allowed range of parameters in the model. Note that U_{e3} is very close to the upper limit allowed by the existing reactor experiments.

helps in understanding the origin of matter in the universe. Our understanding of mixings on the other hand, is at a very preliminary level. A particular challenge to theorists is to understand the so-called bimaximal mixing pattern, which is emerging as the favorite. Several symmetry and dynamical approaches to understand large mixings are noted. Also a minimal $SO(10)$ whose predictions are currently in accord but testable in the near future is also presented. On the experimental side, high precision search for U_{e3} and neutrinoless double beta decay will provide important ways to distinguish between the different models.

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