

Inflation, large scale structure and particle physics

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Abstract. We review experimental and theoretical developments in inflation and its application to structure formation, including the curvaton idea. We then discuss a particle physics model of supersymmetric hybrid inflation at the intermediate scale in which the Higgs scalar field is responsible for large scale structure, show how such a theory is completely natural in the framework extra dimensions with an intermediate string scale.

Keywords. Hybrid inflation; Higgs scalar field; structure formation; curvaton.

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1. Introduction

The standard hot big bang (SHBB) model of cosmology provides a convincing description of the early universe, from the time of nucleosynthesis, when the universe was a few seconds old, to the decoupling time 380,000 years later when the electrons were bound into atoms and the universe became transparent. The radiation that was emitted from the last scattering surface at this time is observed today 13.7 billion years later, redshifted by a factor of $z \approx 1090$ as the cosmic microwave background (CMB). The homogeneity and isotropy of the CMB implies very simple initial conditions for the early universe. The temperature fluctuations in the CMB, observed initially by COBE, have been measured most recently to high precision by WMAP as shown in figure 1 [1]. A compilation of data on the angular power spectrum is shown in the left panel of figure 2, and a best fit curve to these data is shown in the right panel of this figure.

The position of the first peak of the angular power spectrum shows that the universe is flat within experimental error, and the detailed fit to these data show that the necessary ingredients in the universe, must include about 23% cold dark matter (CDM) and 73% dark energy (DE), with baryons only providing 4% of the matter in the universe, of which only a tenth form luminous stars. The observed temperature fluctuations, together with DM and DE, seems sufficient to give the observed clumping of stars into galaxies, galaxies into clusters and clusters into superclusters, reproducing the large scale structure in the universe [1a].

From the point of view of the SHBB the current paradigm of our universe as described above presents several puzzles. The homogeneity and isotropy of the CMB is a major puzzle, since, according to the SHBB, the radiation from points of the surface of last photon scattering which are more than 1 degree apart could never have been in causal contact, as depicted in figure 3 (taken from [3]). This well-known puzzle is called the horizon problem. Another puzzle from the point of view of the SHBB is the fact that the universe is approximately flat, since deviations in the early universe from perfect flatness would have by now become magnified. The fact that the universe today is accurately flat only makes this so-called flatness problem even more puzzling. The temperature fluctuations of the CMB, although required as seeds of structure formation, also admit no explanation in the SHBB framework. There are additional puzzles associated with the matter and energy content of the universe, namely why is the baryon to photon ratio a number of order 10^{-10} and not zero? What is the origin of CDM? What is the source of DE and why should its density be of the same order as that of CDM at the present epoch?

Inflation is an attempt to solve the horizon and flatness problems, which also incorporates a quantum origin of temperature fluctuations. Moreover the basic physics of inflation is not too dissimilar from the behaviour of the present day universe, namely accelerated expansion due to vacuum energy. The original proposal of Guth in 1981 [4], posited that the energy density of the early universe was dominated by some scalar field which became hung up in a false vacuum for some time during which the energy density of the universe was approximately constant leading to approximately exponential expansion of space, thereby solving the horizon and flatness problems. Such inflation also effectively dilutes any unwanted cosmological relics produced at an earlier epoch such as magnetic monopoles predicted by certain gauge unified theories. The original version of inflation described above does not work since the false-vacuum tunnels to true vacuum due to a first order phase transition, and bubbles of true vacuum grow due to negative pressure, leading to a never ending sequence of inflating universes. However soon after this proposal versions of inflation were proposed [5,6] based on an inflaton field with a very flat potential higher than its minimum, along which the field slowly rolls during which

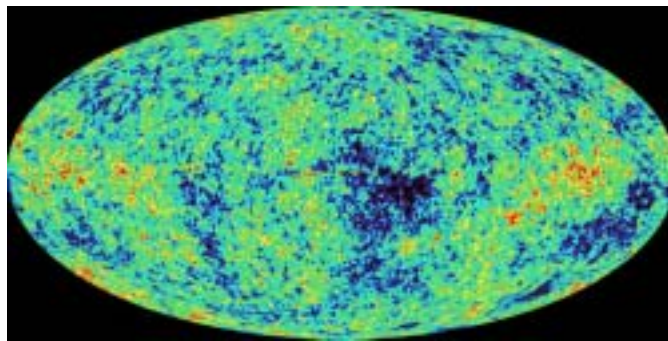


Figure 1. The CMB sky as seen by WMAP, showing the anisotropies at a few parts in 10^5 .

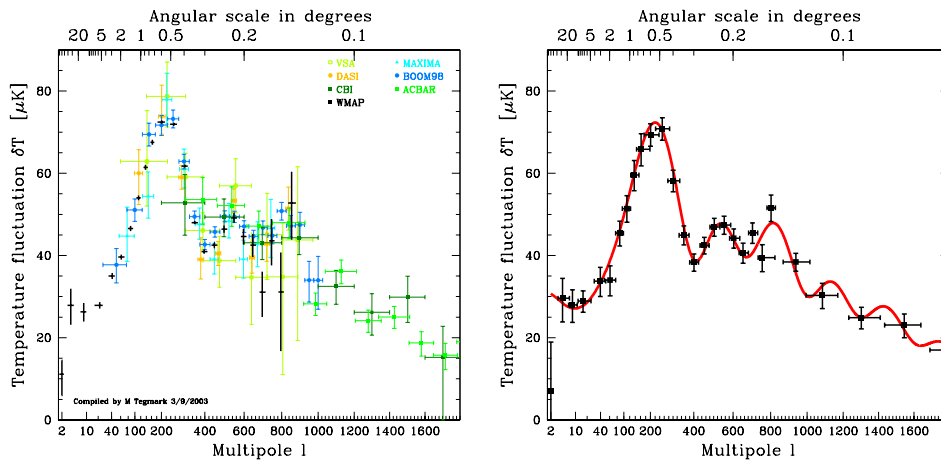


Figure 2. The angular power spectrum compiled from the most recent experiments (left), and the best fit theoretical curve (right) corresponding to a Λ CDM universe as discussed in the text.

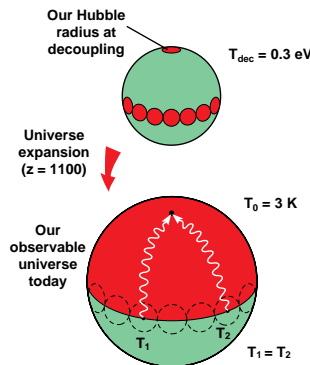


Figure 3. A depiction of the horizon problem (from [3]).

the universe inflates, before reaching the true ground state about which the inflaton field oscillates, ending inflation and reheating the universe as it decays to the minimum. This scenario called slow roll inflation is depicted in figure 4 (taken from [3]).

In such an inflationary approach, the very largest scales, which are now entering the horizon, would have been in causal contact at very early times, thereby solving the horizon problem, as shown in figure 5. It also accounts for the observed flatness of the universe $\Omega = 1$ (the flatness problem), consistent with the CMB data.

During the epoch of slow roll inflation in figure 4, the equation of motion for the field during the inflationary phase is given by

$$\ddot{\phi} + 3H\dot{\phi} = -V', \tag{1}$$

where the Hubble constant in this epoch is given by

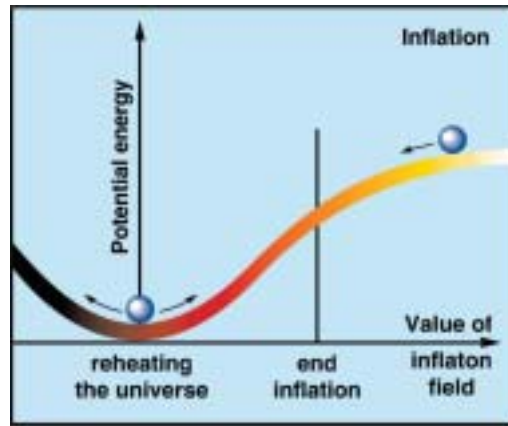


Figure 4. A depiction of slow roll inflation (from [3]).

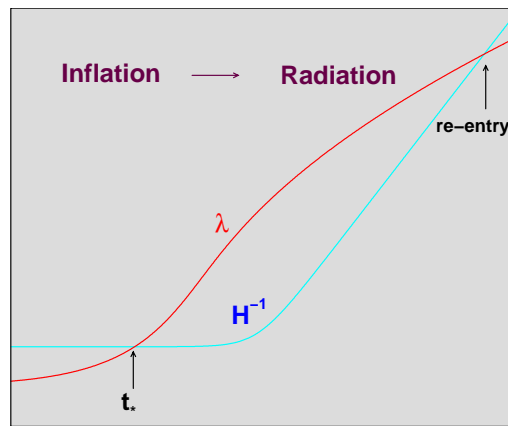


Figure 5. How inflation solves the horizon problem. Scales λ go out of causal contact at the time of horizon exit t_* , and re-enter the horizon much later.

$$H^2 \approx \frac{V}{2m_{\text{P}}^2}, \quad (2)$$

where m_{P} is the reduced Planck mass. The conventionally defined slow roll parameters are

$$\epsilon \equiv \frac{1}{2} m_{\text{P}}^2 \left(\frac{V'}{V} \right)^2 \ll 1, \quad (3)$$

$$|\eta| \equiv \left| \frac{m_{\text{P}}^2 V''}{V} \right| \ll 1, \quad (4)$$

where V' (V'') are the first (second) derivatives of the potential. Assuming the slow roll conditions are satisfied, the field equation approximates to

Table 1. Current status of inflation.

Prediction	Observation
$\Omega_0 = 1$	$\Omega_0 = 1.0 \pm 0.02$
Density perturbations	Observed perturbations
Acoustic peaks expected	Three acoustic peaks observed
Gaussian spectrum	No evidence for non-Gaussianity
Spectral index $n_s \approx 1$	One sigma range $n_s = 0.94-1.02$
Gravity waves $\epsilon \ll 1$	Gravity waves not observed $\epsilon < 0.022$

$$3H\dot{\phi} = -V'. \tag{5}$$

The current status of inflation is summarized in table 1.

2. Hybrid inflation

There are essentially three types of slow roll inflation which can be categorised as large field (chaotic) inflation, small field (new) inflation, and hybrid inflation. From the point of view of particle physics models perhaps the most interesting of these is hybrid inflation [8] which involves two (or more) scalar fields: a slowly rolling inflaton field ϕ plus other fields N, \dots which are held at zero (or small) field values during inflation, but which are destabilised when ϕ reaches a critical value called ϕ_c . In hybrid inflation, depicted in figure 6, during inflation only the inflaton field ϕ non-zero, and the potential takes an extremely simple form

$$V = V_0 + \frac{1}{2}m^2\phi^2, \tag{6}$$

where the idea is that the mass term $\frac{1}{2}m^2\phi^2$ is much smaller than the constant energy density term V_0 . The Hubble constant is given by $H \approx V_0^{1/2}/\sqrt{3}m_{\text{P}}$.

The slow roll parameters during inflation are given by

$$\epsilon \approx \frac{m_{\text{P}}^2}{2} \left(\frac{m^2\phi^2}{V_0} \right)^2, \quad \eta \approx \frac{m_{\text{P}}^2 m^2}{V_0}. \tag{7}$$

Assuming the slow roll conditions are satisfied $\epsilon, \eta \ll 1$ the field equation gives $\dot{\phi} \approx -(m^2\phi/3H)$.

To understand the origin of the critical value of ϕ which ends inflation, one must write down the full potential of the theory. This will be model dependent but will include a term like $(\phi^2 - \phi_c^2)N^2$ such that when $\phi > \phi_c$ the term is positive, corresponding to a positive mass squared for the N field, which holds this field such that $N = 0$, but when $\phi < \phi_c$ the term becomes negative, corresponding to a negative (tachyonic) mass squared for the N field, which causes it to become non-zero, effectively ending inflation, as shown in figure 6.

An interesting feature of hybrid inflation is that, since $\phi < m_{\text{P}}$, we have $\epsilon \ll \eta$ as is clear from eq. (7). Since the parameter ϵ is responsible for the tensor modes

associated with cosmological gravitational waves, in hybrid inflation gravitational waves are predicted to be of negligible effect.

The ratio of tensor to scalar amplitudes is conventionally defined to be R , and the scalar and tensor spectral indices n_s, n_t are then given in terms of the slow roll parameters as

$$R \approx -8n_t \approx 16\epsilon, \quad n_s \approx 1 + 2\eta - 6\epsilon. \tag{8}$$

The current experimental one and three σ limits on R, n_s are shown in figure 7 (taken from [9]), together with the theoretical expectations for different types of inflation. Since the point $R = 0, n_s = 1$ is permitted, it is clear that the present data does not discriminate between the different slow roll inflationary models.

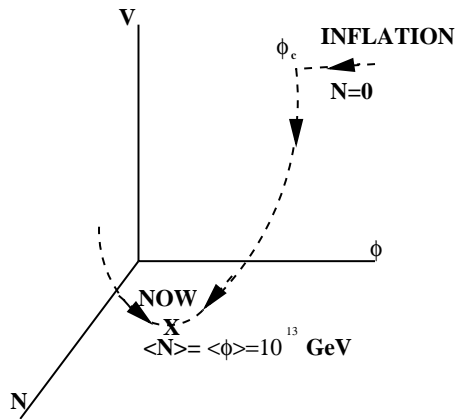


Figure 6. A depiction of hybrid inflation. The parameters at the global minimum correspond to the specific model discussed in the text.

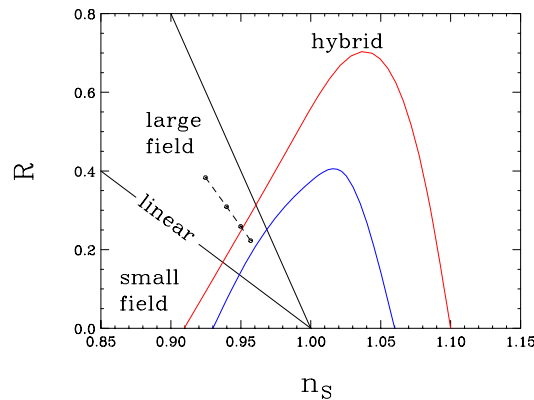


Figure 7. Experimental limits at one and three sigma in the (R, n_s) plane, as compared to the expectations from different types of inflation model (from [9]).

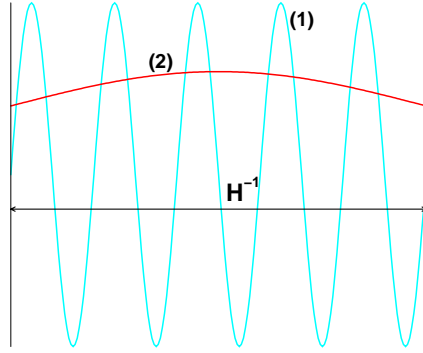


Figure 8. Quantum fluctuations of a scalar field during inflation different may be Fourier decomposed and behave differently according to their wavelength. The modes (1) shorter than the horizon size become redshifted, while the modes (2) longer than the horizon size get frozen in (figure from Mar Bastero-Gil).

3. Structure formation

Another commonly stated success of inflation is the fact that the observed primordial density perturbations, which were first observed by COBE on cosmological scales just entering the horizon, and which are supposed to be the seeds of large scale structure, could have originated from the quantum fluctuations of the inflaton field, the scalar field which is supposed to be responsible for driving inflation. In this scenario the quantum fluctuations of the inflaton field during the period of inflation become classical perturbations at horizon exit, giving a primordial curvature perturbation which remains constant until the approach of horizon entry.

The equation for the fluctuations of the inflaton field is

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2\delta\phi = 0. \tag{9}$$

The field fluctuations $\delta\phi$ may then be Fourier decomposed so that $\delta\phi_k$ refers to the fluctuations of the field associated with a particular mode. The spectrum of the fluctuations are then defined as

$$P_\phi \equiv \frac{k^3}{2\pi^2} \langle |\delta\phi_k|^2 \rangle = \left(\frac{H_*}{2\pi} \right)^2, \tag{10}$$

where $k = 2\pi/\lambda$ refers to the wavenumber of the particular mode of wavelength λ , and $*$ denotes quantities evaluated at the time of horizon exit. For wavelengths within the Hubble horizon distance $\lambda < H^{-1}$ the field fluctuations become redshifted as $\delta\phi_k \sim a^{-1}$, as depicted in figure 8. While for wavelengths greater than the Hubble horizon distance $\lambda > H^{-1}$ the fluctuation become frozen in and are given from eq. (10) as $\delta\phi_k(t_*) \sim H_*/\sqrt{2k^3}$.

The fluctuations $\delta\phi_k(t_*)$ are then frozen in as a classical perturbation on the density and hence the curvature perturbation \mathcal{R} is given by

$$\mathcal{R} = -\frac{H_*}{\dot{\phi}_*} \delta\phi_k = -\frac{H_*}{\dot{\phi}_*} \frac{H_*}{\sqrt{2k^3}}. \quad (11)$$

The primordial power spectrum, the seed of the observed linear power spectrum $P(k)$ is then given by $P_{\mathcal{R}} \equiv \frac{k^3}{2\pi^2} \langle |\mathcal{R}|^2 \rangle$. Experimentally, the primordial power spectrum is observed to be approximately scale invariant as

$$P_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1} \approx A_s, \quad (12)$$

where A_s is the scalar amplitude, and n_s is the spectral index which is observed to be approximately scale invariant and approximately equal to unity. From eq. (15) we find

$$P_{\mathcal{R}}^{1/2} \approx \frac{H_*^2}{2\pi\dot{\phi}_*} \approx \frac{1}{2\pi\sqrt{3}} \frac{V^{3/2}}{m_{\text{p}}^3 V'}. \quad (13)$$

The theoretical normalisation in eq. (13), evaluated from the inflaton potential, must then be compared to the experimentally measured normalisation from WMAP

$$P_{\mathcal{R}}^{1/2} \approx (4-5) \times 10^{-5}. \quad (14)$$

The comparison of eqs (13) and (14) provides a very strong restriction on the inflaton potential, much stronger than the requirements of slow roll.

4. The curvaton

The advantage of the scenario in which the inflaton fluctuations are responsible for large scale structure is that the prediction for the nearly scale-invariant spectrum depends only on the form of the inflaton potential, and is independent of what goes on between the end of inflation and horizon entry. The disadvantage is that it provides a strong restriction on models of inflation. The price of such simplicity, with one field being responsible for both inflation and the primordial curvature perturbation often translates into a severe restriction on the parameters of the inflaton potential. This often requires very small values for the couplings and/or the masses which apparently renders many such theories unnatural.

Recently it has been pointed out that in general it is unnecessary for the inflaton field to be responsible for generating the curvature perturbation [10–14]. It is possible that the inflaton only generates a very small curvature perturbation during the period of inflation, which instead may result from the isocurvature perturbations of a curvaton field which subsequently become converted into curvature perturbations in the period after inflation, but before horizon entry. Isocurvature perturbations simply mean perturbations which do not perturb the total curvature, usually because the curvaton field contributes a very small energy density ρ_σ during inflation. In the scenarios presented so far [10–14], the curvaton is assumed to be completely decoupled from inflationary dynamics, and is assumed to be some late-decaying scalar which decays before the time neutrinos become decoupled.

The reason why the curvaton is assumed to be late-decaying can be understood from the following argument. After reheating the total curvature perturbation can be written as

$$\mathcal{R} = (1 - f)\mathcal{R}_r + f\mathcal{R}_\sigma, \quad f = \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma} \quad (15)$$

and (on unperturbed hypersurfaces on super-horizon sized scales)

$$\mathcal{R}_i \approx -H \left(\frac{\delta\rho_i}{\dot{\rho}_i} \right) \sim \left(\frac{\delta\rho_i}{\rho_i} \right), \quad (16)$$

where the curvaton density ρ_σ and radiation density ρ_r , arising from the decay of the inflaton, each satisfy their own energy conservation equations and each \mathcal{R}_i remains constant on super-horizon scales. The time evolution of \mathcal{R} on these scales is then given by its time derivative

$$\dot{\mathcal{R}} \approx -Hf(1 - f)\frac{S_{\sigma r}}{3}, \quad S_{\sigma r} \simeq -3(\mathcal{R}_\sigma - \mathcal{R}_r), \quad (17)$$

where $S_{\sigma r}$ is the entropy perturbation. The curvaton generates an isocurvature perturbation because initially $\rho_r \gg \rho_\sigma$, and hence $f \ll 1$, so that from eq. (15) the curvature perturbation is dominated by \mathcal{R}_r . However as the universe expands and the scale factor a increases while the Hubble constant H decreases, the curvaton with mass m , whose oscillations have effectively been frozen in by the large Hubble constant, begins to oscillate and act as matter. After this happens ρ_r decreases as a^{-4} while the energy density in the curvaton field ρ_σ has a slower fall-off as a^{-3} . Eventually the curvaton energy density ρ_σ becomes comparable to the radiation density from the inflaton decay ρ_r , and when this happens we see that $f \sim 1$ and from eq. (17) this leads to the growth of the total curvature perturbation \mathcal{R} from the isocurvature perturbation $\mathcal{R}_\sigma > \mathcal{R}_r$. This mechanism, which allows the curvaton isocurvature perturbations to become converted into the total curvature perturbation, requires the curvaton scalar to be late-decaying.

5. A particle physics model of inflation and structure

We now turn to the problem of constructing a particle physics model of hybrid inflation. Such a model should be supersymmetric, because supersymmetry has flat directions which make good candidates for slow roll inflation. On the other hand supersymmetry is necessarily broken in the inflationary epoch by the F or D-term vevs which is necessary to generate the vacuum energy $V(0)$ and drive inflation. Nevertheless supersymmetry remains the best candidate for maintaining a flat potential and for safeguarding its flatness from radiative corrections in a controllable way. Assuming supersymmetry, the model must then provide an origin of the supersymmetric Higgs mass μ , and ideally should also provide a solution to the strong CP problem via the Peccei-Quinn mechanism. Hybrid inflation models which also include grand unification typically face problems with magnetic monopoles [15]. This is ironic since the avoidance of monopoles was one of Guth's

main motivations for introducing inflation. Following Guth's philosophy inflation can provide the solution to this problem providing inflation takes place at a scale below the GUT scale, when inflation will dilute the monopole abundance. To avoid the monopole problem, and shed light on the strong CP problem, and μ problem, we shall therefore assume that inflation happened at an intermediate scale, below the GUT scale.

We shall now discuss such an example of an intermediate scale supersymmetric hybrid inflation model which solves the μ problem and the strong CP problem [16–18]. The model is based on the superpotential:

$$W = \lambda N H_u H_d - \kappa \phi N^2, \quad (18)$$

where N and ϕ are singlet superfields, and $H_{u,d}$ are the Higgs superfields, and λ, κ are dimensionless couplings. Other cubic terms in the superpotential are forbidding by imposing a global $U(1)_{\text{PQ}}$ Peccei-Quinn symmetry. The superpotential in eq. (18) includes a linear superpotential for the inflaton field, ϕ , typical of hybrid inflation, as well as the singlet N coupling to Higgs doublets as in the NMSSM. In the original version of this model we assumed that during inflation N and H_u, H_d were set to zero, so that the inflationary trajectory was as in figure 6 due to the potential

$$V = V(0) + \frac{\kappa^2}{4} N^4 + \kappa^2 (\phi - \phi_c^+) (\phi - \phi_c^-) N^2 + \frac{1}{2} m_\phi^2 \phi^2. \quad (19)$$

The critical value of ϕ corresponds to the field dependent mass squared $\kappa^2 (\phi - \phi_c^+) (\phi - \phi_c^-) N^2$ changing sign and becoming negative. At the global minimum (NOW in figure 6) $\mu = \lambda \langle N \rangle \sim 1$ TeV, and the model incorporates an axionic solution to the strong CP problem with $f_a \sim \langle N \rangle \sim \langle \phi \rangle \sim 10^{13}$ GeV. Of course this leads to a naturalness problem since it requires $\lambda \sim \kappa \sim 10^{-10}$. The model also leads to $V_0^{1/4} \sim 10^8$ GeV and $H_* \sim 10$ MeV. Slow roll only requires $m_\phi \sim$ MeV but the COBE constraint requires $m_\phi \sim$ eV! This is an example of how the requirement of structure formation arising from the inflaton imposes severe constraints on the theory, and motivates an application of the curvaton approach to this model.

Recently [19] an alternative inflationary trajectory was discussed in which these fields may take small values away from the origin, consistent with slow roll inflation. The motivation for considering such a trajectory was to allow the Higgs fields to slowly roll during inflation, and so play the role of the curvaton. However, the Higgs fields are not late decaying scalars since they have gauge couplings which ensure rapid decay, so the mechanism which allows the Higgs fields to be responsible for large scale structure cannot be the curvaton mechanism as it was originally envisaged [10–14]. The new mechanism of structure formation that we proposed [19] relies on the observation that in hybrid inflation at the start of reheating, the vacuum energy present during inflation $V(0)$ gets redistributed among all the oscillating fields such that their energy densities become comparable. Thus any isocurvature perturbation in one of the hybrid inflation fields (in this case the Higgs field) may be converted into curvature perturbations during the onset of reheating. In such a scenario, the usual Higgs field responsible for the origin of mass in the supersymmetric standard model could also be responsible for generating the large scale structure in the universe!

In order to satisfy the D-flatness of the new inflationary trajectory we assumed [19] the values of the Higgs doublets during inflation to be equal, $H_u = H_d = h$. An important condition for inflation is that the inflaton mass m_ϕ (and also m_h) needs to be small enough in order to ensure the slow-roll of the inflaton. This implies $h \ll \phi$ and $\dot{h} \ll \dot{\phi}$. During inflation the curvature dominated by ϕ is very much less than the entropy which is dominated by h ,

$$\mathcal{R} \sim -\frac{H_*}{\dot{\phi}_*} \delta\phi_* \ll S \sim -\frac{H_*}{\dot{h}_*} \delta h_* \quad (20)$$

The isocurvature Higgs perturbations are then transferred to the curvature perturbations at the start of reheating when the energy densities become comparable, $\rho_\phi \sim \rho_h$, and the numerical solution to the field equations leads to a resulting curvature $\mathcal{R} \sim 0.1S$ [19]. The model gives non-Gaussianity below the Planck sensitivity, and a spectral index differing from unity by a number of order 0.1.

The above mechanism for structure formation allows $m_\phi \sim \text{MeV}$ rather than eV, which alleviates but does not solve all the naturalness problems of the model. However a completely natural model is possible using extra dimensions to give volume suppression factors for masses and couplings [20]. We suppose the Higgs and singlets N, ϕ are in an extra-dimensional bulk, and the matter fields live on our brane and feel the usual 3+1 dimensions. We set the string scale $M_* \sim 10^{13}$ GeV and the supersymmetry breaking scale $\sqrt{F_S} \sim 10^8$ GeV. Then we find natural values for all physical quantities. The vacuum energy is as required: $V(0)^{1/4} \sim \sqrt{F_S} \sim 10^8$ GeV. The small couplings are also as required $\lambda, \kappa \sim (M_*/m_P)^2 \sim 10^{-10}$. The soft masses on the supersymmetry breaking brane are $m_{\text{soft}} \sim F_S/M_* \sim \text{TeV}$, but the soft masses for bulk scalars are $m_\phi \sim m_h \sim m_N \sim (M_*/m_P)m_{\text{soft}} \sim \text{MeV}$, which satisfy the slow roll conditions.

The complete model [19,20] is therefore completely natural, and accounts for large scale structure via quantum fluctuations of the Higgs field during inflation, which are subsequently transferred to curvature perturbations at the onset of reheating.

6. Conclusion

These are exciting times for inflationary cosmology due to the new data from WMAP, and the prospect of the Planck launch in 2007, which will extend the angular power spectrum out to $l \sim 3000$, measure the polarization, measure the flatness to an accuracy $\Delta\Omega_0 \approx \pm 0.001$ and the spectral index to $\Delta n_s \approx \pm 0.008$. Inflation is already looking very good, with hybrid inflation at the intermediate scale looking most relevant for particle physics, and providing a link with the supersymmetric standard model [21]. Curvature perturbations need not have arisen from the inflaton, and could have originated from a curvaton or Higgs.

Acknowledgement

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