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Leptonic flavor and CP violation

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Abstract. Recent neutrino oscillation data teach us that the neutrinos have masses and that they mix. We discuss two ways that can be used to probe other non-standard leptonic physics. We show that non-standard neutrino interaction can be probed in neutrino oscillation experiments and discuss sneutrino–antisneutrino mixing.

Keywords. Lepton number violation; supersymmetry; neutrino oscillation.

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1. Introduction

The success of the standard model (SM) can be seen as a proof that it is an effective low energy description of nature. We are therefore interested in probing the more fundamental theory. One way to go is to search for new particles that can be produced in yet unreached energies. Another way to look for new physics is to search for indirect effects of heavy unknown particles. Here we explain how neutrino physics is used to probe such indirect signals of physics beyond the SM [1].

In the SM the neutrinos are exactly massless. This prediction, however, is rather specific to the SM. In almost all of the SM extensions the neutrinos are massive and they mix. The search for neutrino flavor oscillation, a phenomenon which is possible only for massive neutrinos, is a search for new physics beyond the SM. The recent experimental indications for neutrino oscillations are indirect evidences for new physics.

Neutrino masses and mixing may not be the only signal for new leptonic physics. Below we discuss two other probes for such new physics. First, we explain how neutrino oscillation experiments can be used to look for new CP violating neutrino interactions [2,3]. Second, we explain how sneutrino oscillations are sensitive to the neutrino Majorana masses [4].

2. New CP violation in neutrino oscillations

In the future, neutrino oscillation experiments will search for CP-violating effects. The SM, extended to include masses for the light, active neutrinos, predicts that CP is violated in neutrino oscillations through a single phase in the mixing matrix for leptons. This effect is suppressed by small mixing angles and small mass differences.

It is not unlikely, however, that the high-energy physics that is responsible for neutrino masses and mixing involves also new neutrino interactions. Such interactions provide new sources of CP violation. They could affect the production and/or detection processes in neutrino oscillation experiments. As we explain below they could manifest themselves in neutrino oscillations. In fact there are effects that are qualitatively different from the SM ones. In particular, we can use the time (or, equivalently, distance) dependence of the transition probability to distinguish between SM and new CP violation.

2.1 *Notations and formalism*

We denote by $|\nu_i\rangle$, $i = 1, 2, 3$, the three neutrino mass eigenstates. We denote by $|\nu_\alpha\rangle$ the weak interaction partners of the charged lepton mass eigenstates α^- ($\alpha = e, \mu, \tau$):

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle. \tag{1}$$

The neutrino mixing matrix, $U_{\alpha i}$ [5,6] is parametrized with three mixing angles, θ_{ij} with $i, j = 1, 2, 3$ and $i < j$ and one CP violating phase, δ. A convention-independent definition of the phase δ is given by

$$\delta \equiv \arg \left(\frac{U_{e3} U^*_{\mu 3}}{U_{e1} U^*_{\mu 1}} \right). \tag{2}$$

We consider new, possibly CP-violating, physics in the production and/or detection process [7,2,3]. We parametrize the new physics interaction in the source and in the detector by two sets of effective four-fermion couplings, $(G^s_{\rm NP})_{\alpha\beta}$ and $(G^d_{\rm NP})_{\alpha\beta}$, where $\alpha, \beta = e, \mu, \tau$. Here $(G^s_{\rm NP})_{\alpha\beta}$ refers to processes in the source where a ν_β is produced in conjunction with an incoming α^- or an outgoing α^+ charged lepton, while $(G^d_{\rm NP})_{\alpha\beta}$ refers to processes in the detector where an incoming ν_β produces an α^- charged lepton. While the $SU(2)_L$ gauge symmetry requires that the four-fermion couplings of the charged current weak interactions be proportional to $G_F \delta_{\alpha\beta}$, new interactions allow couplings with $\alpha \neq \beta$. Phenomenological constraints imply that the new interaction is suppressed with respect to the weak interaction,

$$|(G^s_{\rm NP})_{\alpha\beta}| \ll G_{\rm F}, \quad |(G^d_{\rm NP})_{\alpha\beta}| \ll G_{\rm F}. \tag{3}$$

For the sake of concreteness, we consider the production and detection processes that are relevant to neutrino factories. We therefore study an appearance experiment where neutrinos are produced in the process $\mu^+ \to e^+ \nu_\alpha \bar{\nu}_\mu$ and detected by the process $\nu_\beta d \to \mu^- u$, and antineutrinos are produced and detected by the corresponding charge-conjugate processes. Our results can be modified to any other neutrino oscillation experiment in a straightforward way. The relevant couplings are then $(G^s_{\rm NP})_{e\beta}$ and $(G^d_{\rm NP})_{\mu\beta}$. It is convenient to define small dimensionless quantities $\epsilon^{s,d}_{\alpha\beta}$ in the following way:

$$\epsilon^s_{e\beta} \equiv \frac{(G^s_{\rm NP})_{e\beta}}{\sqrt{|G_{\rm F} + (G^s_{\rm NP})_{ee}|^2 + |(G^s_{\rm NP})_{e\mu}|^2 + |(G^s_{\rm NP})_{e\tau}|^2}},$$
$$\epsilon^d_{\mu\beta} \equiv \frac{(G^d_{\rm NP})_{\mu\beta}}{\sqrt{|G_{\rm F} + (G^d_{\rm NP})_{\mu\mu}|^2 + |(G^d_{\rm NP})_{\mu e}|^2 + |(G^d_{\rm NP})_{\mu\tau}|^2}}. \qquad (4)$$

Since we assume that $|\epsilon^{s,d}_{\alpha\beta}| \ll 1$, we will only evaluate their effects to leading (linear) order. New flavor-conserving interactions affect neutrino oscillations only at $\mathcal{O}(|\epsilon|^2)$ and will be neglected from here on. (More precisely, the leading effects from flavor-diagonal couplings are proportional to ϵ(flavor-diagonal)$\times\epsilon$(flavor-changing) and can therefore be safely neglected.)

We use an explicit parameterization for only two of the ϵs, with the following convention:

$$\epsilon^s_{e\mu} \equiv |\epsilon^s_{e\mu}|{\rm e}^{i\delta_\epsilon}, \quad \epsilon^{d*}_{\mu e} \equiv |\epsilon^{d*}_{\mu e}|{\rm e}^{i\delta'_\epsilon}. \qquad (5)$$

Alternatively, we can define the phases δ_ϵ and δ'_ϵ in a convention-independent way:

$$\delta_\epsilon \equiv \arg\left(\frac{\epsilon^s_{e\mu}}{U_{e1}U^*_{\mu 1}}\right), \quad \delta'_\epsilon \equiv \arg\left(\frac{\epsilon^{d*}_{\mu e}}{U_{e1}U^*_{\mu 1}}\right). \qquad (6)$$

It is well-known that the three-generation mixing matrix for leptons depends, in the case of Majorana neutrinos, on three phases. Two of these, related to the fact that there is no freedom in redefining the phases of neutrino fields, do not affect neutrino oscillations and are therefore irrelevant to our discussion. The other one is analogous to the Kobayashi–Maskawa phase of the mixing matrix for quarks. The freedom of redefining the phases of charged lepton fields is fully used to reduce the number of relevant phases to one. Consequently, it is impossible to remove any phases from the $\epsilon^{s,d}_{\alpha\beta}$ parameters. Each of these parameters introduces a new, independent CP-violating phase. For example, when we discuss $\nu_e \to \nu_\mu$ oscillations, our results will depend on $\epsilon^s_{e\mu}$, $\epsilon^d_{\mu e}$ and the $U_{ei}U^*_{\mu i}$ $(i = 1, 2, 3)$ mixing parameters. This set of parameters depends on three independent phases, one of which is the δ of eq. (2), while the other two can be chosen to be δ_ϵ and δ'_ϵ of eq. (6).

2.2 *The transition probability in vacuum*

We denote by ν^s_e the neutrino state that is produced in the source in conjunction with an e^+, and by ν^d_μ the neutrino state that is signalled by μ^- production in the detector:

$$|\nu^s_e\rangle = \sum_i \left[U_{ei} + \epsilon^s_{e\mu}U_{\mu i} + \epsilon^s_{e\tau}U_{\tau i}\right]|\nu_i\rangle,$$
$$|\nu^d_\mu\rangle = \sum_i \left[U_{\mu i} + \epsilon^d_{\mu e}U_{ei} + \epsilon^d_{\mu\tau}U_{\tau i}\right]|\nu_i\rangle. \qquad (7)$$

(Note that the norm of the states so defined is one up to effects of $\mathcal{O}(|\epsilon|^2)$, which we consistently neglect.)

We obtain the following expression for the transition probability $P_{e\mu} = |\langle\nu_\mu^d|\nu_e^s(t)\rangle|^2$, where $\nu_e^s(t)$ is the time-evolved state that was purely ν_e^s at time $t=0$:

$$P_{e\mu} = \left|\sum_i \mathrm{e}^{-iE_i t}\left[\lambda_{e\mu} + \epsilon^s_{e\mu}\lambda_{\mu\mu} + \epsilon^{d*}_{\mu e}\lambda_{ee} + \epsilon^s_{e\tau}\lambda_{\tau\mu} + \epsilon^{d*}_{\mu\tau}\lambda_{\mu\tau}\right]\right|^2, \tag{8}$$

where $\lambda_{\alpha\beta} \equiv U_{\alpha i}U^*_{\beta i}$.

Our results will be given in terms of Δm^2_{ij}, Δ_{ij} and x_{ij}, which are defined as follows:

$$\Delta m^2_{ij} \equiv m_i^2 - m_j^2, \quad \Delta_{ij} \equiv \Delta m^2_{ij}/(2E), \quad x_{ij} \equiv \Delta_{ij}L/2, \tag{9}$$

where E is the neutrino energy and L is the distance between the source and the detector.

Equation (8) is the starting point of our calculations. To understand the essential features of our analysis it is useful to do the following. First, we separate $P_{e\mu}$ into a SM piece, $P^{\rm SM}_{e\mu}$, and a new physics piece, $P^{\rm NP}_{e\mu}$. What we mean by $P^{\rm SM}_{e\mu}$ is $P_{e\mu}(\epsilon^{s,d}_{\alpha\beta}=0)$. This is the contribution to $P_{e\mu}$ from the SM extended to include neutrino masses but no new interactions. In contrast, $P^{\rm NP}_{e\mu}$ contains all the $\epsilon^{s,d}_{\alpha\beta}$-dependent terms. Second, since the atmospheric and reactor neutrino data imply that $|U_{e3}|$ is small and the solar neutrino data imply that $\Delta m^2_{12}/\Delta m^2_{13}$ is small, we expand $P^{\rm SM}_{e\mu}$ to second order and $P^{\rm NP}_{e\mu}$ to first order in $|U_{e3}|$ and Δm^2_{12}.

For $P^{\rm SM}_{e\mu}$ we obtain:

$$\begin{aligned} P^{\rm SM}_{e\mu} = {} & 4x_{21}^2|U_{e2}|^2|U_{\mu2}|^2 + 4\sin^2 x_{31}|U_{e3}|^2|U_{\mu3}|^2 \\ & +4x_{21}\sin 2x_{31}\mathcal{R}e\big(U_{e2}U^*_{e3}U^*_{\mu2}U_{\mu3}\big) \\ & -8x_{21}\sin^2 x_{31}\mathcal{I}m\big(U_{e2}U^*_{e3}U^*_{\mu2}U_{\mu3}\big). \end{aligned} \tag{10}$$

The first term is the well-known transition probability in the two-generation case. The second term gives the well-known transition probability in the approximation that $\Delta m^2_{12}=0$. The last term is a manifestation of the SM CP violation.

For $P^{\rm NP}_{e\mu}$, we obtain

$$\begin{aligned} P^{\rm NP}_{e\mu} = {} & -4\sin^2 x_{31}\mathcal{R}e[U^*_{e3}U_{\mu3}\left(\epsilon^{d*}_{\mu e} + \epsilon^s_{e\mu}(1-2|U_{\mu3}|^2) - 2\epsilon^s_{e\tau}U^*_{\mu3}U_{\tau3}\right)] \\ & + 4x_{21}\sin 2x_{31}\mathcal{R}e[U^*_{e2}U_{\mu2}\left(\epsilon^s_{e\mu}|U_{\mu3}|^2 + \epsilon^s_{e\tau}U^*_{\mu3}U_{\tau3}\right)] \\ & - 4x_{21}\mathcal{I}m[U^*_{e2}U_{\mu2}\left(\epsilon^{d*}_{\mu e} + \epsilon^s_{e\mu}(1-|U_{\mu3}|^2) - \epsilon^s_{e\tau}U^*_{\mu3}U_{\tau3}\right)] \\ & - 2\sin 2x_{31}\mathcal{I}m[U^*_{e3}U_{\mu3}(\epsilon^{d*}_{\mu e} + \epsilon^s_{e\mu})] \\ & - 4x_{21}\cos 2x_{31}\mathcal{I}m[U^*_{e2}U_{\mu2}\left(\epsilon^s_{e\mu}|U_{\mu3}|^2 + \epsilon^s_{e\tau}U^*_{\mu3}U_{\tau3}\right)]. \end{aligned} \tag{11}$$

The last three terms in this expression are CP-violating and would be the basis for our results.

2.3 *CP violation in vacuum oscillations*

To measure CP violation, one need to compare the transition probability $P_{e\mu}$ evaluated in the previous section to that of the CP-conjugate process, $P_{\bar{e}\bar{\mu}}$. The latter will be measured in oscillation experiments where antineutrinos are produced in the source in conjunction with e^- and detected through μ^+ production. It is clear that a CP transformation relates the production processes, $\mu^- \to e^-\bar{\nu}_\alpha\nu_{\alpha'}$ and $\mu^+ \to e^+\nu_\alpha\bar{\nu}_{\alpha'}$. As concerns the detection processes, $\bar{\nu}_\beta u \to \mu^+ d$ and $\nu_\beta d \to \mu^- u$, the situation is less straightforward. We have $G^d_{\beta\mu} \propto \langle p\mu^-|\bar{\mu}^-\ \bar{u}\ \nu_\beta\ d|\nu_\beta n\rangle$ and $G^d_{\bar{\beta}\bar{\mu}} \propto \langle n\mu^+|\bar{\mu}^+\ \bar{d}\ \bar{\nu}_\beta\ u|\bar{\nu}_\beta p\rangle$. The relation is through CP and crossing symmetry, but for a four-fermion interaction this is equivalent to a CP transformation.

CP transformation of the Lagrangian takes the elements of the mixing matrix and the ϵ-terms into their complex conjugates. It is then straightforward to obtain the transition probability for antineutrino oscillations. Our interest lies in the CP asymmetry,

$$A_{\rm CP} = \frac{P_-}{P_+}, \qquad P_\pm = P_{e\mu} \pm P_{\bar{e}\bar{\mu}}. \tag{12}$$

We quote below the leading contributions for 'short' distances, $x_{31} \ll 1$. In some of the observables, we consider two limiting cases for $|U_{e3}|$:

$$\begin{aligned} &\text{The large } s_{13} \text{ limit}: \quad x_{21}/x_{31} \ll |(U_{e3}U_{\mu3})/(U_{e2}U_{\mu2})|, \\ &\text{The small } s_{13} \text{ limit}: \quad x_{21}/x_{31} \gg |(U_{e3}U_{\mu3})/(U_{e2}U_{\mu2})|. \end{aligned} \tag{13}$$

The CP conserving rate P_+ is always dominated by the SM. It is given by

$$P_+ = \begin{cases} 8x_{31}^2|U_{e3}U_{\mu3}|^2, & \text{large } s_{13}, \\ 8x_{21}^2|U_{e2}U_{\mu2}|^2, & \text{small } s_{13}. \end{cases} \tag{14}$$

The CP-violating difference between the transition probabilities within the SM can be obtained from Eq. (10):

$$P_-^{\rm SM} = -16x_{21}x_{31}^2\mathcal{I}m\big(U_{e2}U_{\mu2}^*U_{e3}^*U_{\mu3}\big). \tag{15}$$

As is well-known, CP violation within the SM is suppressed by both the small mixing angle $|U_{e3}|$ and the small mass-squared difference Δm^2_{12}. More generally, it is proportional to the Jarlskog measure of CP violation, $J = \mathcal{I}m\big(U_{e2}U_{\mu2}^*U_{e3}^*U_{\mu3}\big)$. For short distances ($x_{21}, x_{31} \ll 1$), the dependence of $P_-^{\rm SM}$ on the distance is L^3. Since it is CP-violating, it should be odd in L. The absence of a term linear in L comes from the fact that the SM requires for CP to be violated, that all three mass-squared differences do not vanish, that is, $P_- \propto \Delta_{21}\Delta_{31}\Delta_{32}$. In the limit $x_{21}/x_{31} \ll |(U_{e3}U_{\mu3})/(U_{e2}U_{\mu2})|$, we obtain the following SM asymmetry:

$$A_{\rm CP}^{\rm SM} = -2x_{21}\mathcal{I}m\left(\frac{U_{e2}U_{\mu2}^*}{U_{e3}U_{\mu3}^*}\right). \tag{16}$$

In the small s_{13} limit, the standard CP violation is unobservably small. The CP-violating difference between the transition probabilities that arises from the new physics interactions can be obtained from eq. (11):

$$P_-^{\rm NP} = \begin{cases} -8x_{31}\mathcal{I}m[U_{e3}^* U_{\mu 3}(\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s)], & \text{large } s_{13}, \\ -8x_{21}\mathcal{I}m[U_{e2}^* U_{\mu 2}(\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s)], & \text{small } s_{13}. \end{cases} \tag{17}$$

We learn that CP violation beyond the weak interactions requires only that either $|U_{e3}|$ or Δm_{21}^2 be different from zero, but not necessarily both. Also the dependence on the distance is different: for short distances, $P_-^{\rm NP} \propto L$. From eqs (14) and (17) we obtain the following new physics contribution to the CP asymmetry:

$$A_{\rm CP}^{\rm NP} = \begin{cases} -\frac{1}{x_{31}}\mathcal{I}m\left(\frac{\epsilon_{\mu e}^{d*}+\epsilon_{e\mu}^s}{U_{e3}U_{\mu 3}^*}\right), & \text{large } s_{13}, \\ -\frac{1}{x_{21}}\mathcal{I}m\left(\frac{\epsilon_{\mu e}^{d*}+\epsilon_{e\mu}^s}{U_{e2}U_{\mu 2}^*}\right), & \text{small } s_{13}. \end{cases} \tag{18}$$

The apparent divergence of $A_{\rm CP}^{\rm NP}$ for small L is only due to the approximations that we used. Specifically, there is an $\mathcal{O}(|\epsilon|^2)$ contribution to P_+ that is constant in L [7], namely $P_+ = \mathcal{O}(|\epsilon|^2)$ for $L \to 0$. In contrast, $P_- = 0$ in the $L \to 0$ limit to all orders in $|\epsilon|$.

Equations (17) and (18) lead to several interesting conclusions:

(i) It is possible that, in CP-violating observables, the new physics contributions compete with or even dominate over the SM ones in spite of the superweakness of the interactions ($|\epsilon| \ll 1$). Given that for the proposed experiments $x_{31} \lesssim 1$, it is sufficient that

$$\max\left(|\epsilon_{e\mu}^s|, |\epsilon_{\mu e}^d|\right) \geq \min\left(|U_{e3}|, x_{21}\right), \tag{19}$$

for the new contribution to the CP-violating difference P_- to be larger than the standard one.

(ii) The different distance dependence of $P_-^{\rm NP}$ and $P_-^{\rm SM}$ will allow, in principle, an unambiguous distinction to be made between new physics contributions of the type described here and the contribution from lepton mixing.

(iii) The $1/L$ dependence of $A_{\rm CP}^{\rm NP}$ suggests that the optimal baseline to observe CP violation from new physics is shorter than the one optimized for the SM.

2.4 *Conclusions and discussion*

CP-violating observables are particularly sensitive to new physics. The reason is that the standard CP violation that comes from the lepton mixing matrix gives effects that are particularly suppressed by small mass differences and mixing angles. Some of these suppression factors do not apply to new contributions.

The effects of new physics in the production and detection processes depend on the source–detector distance in a way that is different from the standard one. One consequence of this situation is that, at least in principle, it is possible to disentangle standard and new effects. Another consequence is that in short distance experiments the new effects are enhanced.

3. Sneutrino oscillation

Consider a supersymmetric SM with Majorana neutrino masses [4,8]. In such models, lepton number violation can generate interesting phenomena in the slepton sector. In addition to generating small neutrino masses, the $\Delta L = 2$ operators introduce a mass splitting and mixing in the sneutrino–antisneutrino system. This is analogous to the neutral meson systems [9]. For example, in the B^0 system the effect of a small $\Delta B = 2$ perturbation to the leading $\Delta B = 0$ mass term results in a mass splitting between the heavy and light B^0, which are no longer pure B^0 and $\bar{B}^0$ states. The very small mass splitting, $\Delta m_B/m_B = 6 \times 10^{-14}$ [10], can be measured by observing flavor oscillations. The flavor is tagged in B-decays by the final state lepton charge. Since $x_d \equiv \Delta m_B/\Gamma_B \approx 0.7$ [10], there is time for the flavor to oscillate before the meson decays. When B mesons are produced in pairs (for example in e^+e^- collider operating at the $\Upsilon(4S)$ resonance) the same sign dilepton signal indicates that one of the B oscillated. This time-integrated same sign dilepton sample is used to determine the tiny mass splitting.

The sneutrino system can exhibit similar behavior. The lepton number is tagged in sneutrino decay using the charge of the outgoing lepton. When the sneutrino has time to mix before it decays, namely when

$$x_{\tilde{\nu}} \equiv \frac{\Delta m_{\tilde{\nu}}}{\Gamma_{\tilde{\nu}}} \gtrsim 1, \tag{20}$$

and when the branching ratio of the decay of the sneutrino into a charged lepton is significant, then we can directly measure a non-zero sneutrino mass splitting via the same sign dilepton signal. When the sneutrinos are pair produced, e.g. in e^+e^- collisions, the two leptons from the sneutrino decays are used. When the sneutrino is produced together with a charged lepton, e.g. in hadron collider via cascade decays, the lepton from the sneutrino decay and the associated produced lepton are used. In both cases a measurable same sign dilepton signal is expected.

The neutrino mass and the sneutrino mass splitting are both consequences of the small breaking of lepton number. Therefore, they are expected to be related. Thus, we can use the measured neutrino masses to set bounds on the sneutrino mass splitting.

3.1 *The supersymmetric see-saw model*

Consider an extension of the MSSM where one adds a right-handed neutrino superfield, $\hat{N}$, with a bare mass $M \gg m_Z$. We consider a one generation model (i.e., we ignore lepton flavor mixing) and assume CP conservation. We employ the most general R-parity conserving renormalizable superpotential and attendant soft-supersymmetry breaking terms. Here, the relevant terms in the superpotential are (following the notation of ref. [11])

$$W = \epsilon_{ij}\left[\lambda \hat{H}_2^i \hat{L}^j \hat{N} - \mu \hat{H}_1^i \hat{H}_2^j\right] + \tfrac{1}{2} M \hat{N}\hat{N}\,. \tag{21}$$

The D-terms are the same as in the MSSM.

The relevant terms in the soft-supersymmetry-breaking scalar potential are

$$V_{\text{soft}} = m_{\tilde{L}}^2 \tilde{\nu}^* \tilde{\nu} + m_{\tilde{N}}^2 \tilde{N}^* \tilde{N} + (\lambda A_\nu H_2^2 \tilde{\nu} \tilde{N}^* + M B_N \tilde{N} \tilde{N} + \text{h.c.}) . \tag{22}$$

When the neutral Higgs field vacuum expectation values are generated [$\langle H_i^i \rangle = v_i/\sqrt{2}$, with $\tan\beta \equiv v_2/v_1$ and $v_1^2 + v_2^2 = v^2 = (246\text{ GeV})^2$], one finds that the light neutrino mass is given by the usual one generation see-saw result

$$m_\nu = \frac{m_D^2}{M} , \tag{23}$$

where $m_D \equiv \lambda v_2$ and we drop higher order terms in m_D/M. The sneutrino masses are obtained by diagonalizing a 4×4 squared-mass matrix. Here, it is convenient to define: $\tilde{\nu} = (\tilde{\nu}_1 + i\tilde{\nu}_2)/\sqrt{2}$ and $\tilde{N} = (\tilde{N}_1 + i\tilde{N}_2)/\sqrt{2}$. Then, the sneutrino squared-mass matrix separates into CP-even and CP-odd blocks:

$$-\mathcal{L}_{\text{mass}} = \tfrac{1}{2} \begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix} \begin{pmatrix} \mathcal{M}_+^2 & 0 \\ 0 & \mathcal{M}_-^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} , \tag{24}$$

where $\phi_i \equiv \begin{pmatrix} \tilde{\nu}_i & \tilde{N}_i \end{pmatrix}$ and

$$\mathcal{M}_\pm^2 = \begin{pmatrix} m_{\tilde{L}}^2 + \frac{1}{2} m_Z^2 \cos 2\beta + m_D^2 & m_D[A_\nu - \mu \cot\beta \pm M] \\ m_D[A_\nu - \mu \cot\beta \pm M] & M^2 + m_D^2 + m_{\tilde{N}}^2 \pm 2B_N M \end{pmatrix} . \tag{25}$$

In the following derivation we assume that M is the largest mass parameter. Then, to first order in $1/M$, the two light sneutrino eigenstates are $\tilde{\nu}_1$ and $\tilde{\nu}_2$, with corresponding squared masses:

$$m_{\tilde{\nu}_{1,2}}^2 = m_{\tilde{L}}^2 + \tfrac{1}{2} m_Z^2 \cos 2\beta \mp \tfrac{1}{2} \Delta m_{\tilde{\nu}}^2 , \tag{26}$$

where the squared mass difference $\Delta m_{\tilde{\nu}}^2 \equiv m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2$ is of order $1/M$. Thus, in the large M limit, we recover the two degenerate sneutrino states of the MSSM, usually chosen to be $\tilde{\nu}$ and $\bar{\tilde{\nu}}$. For finite M, these two states mix with a 45° mixing angle, since the two light sneutrino mass eigenstates must also be eigenstates of CP. The sneutrino mass splitting is easily computed using $\Delta m_{\tilde{\nu}}^2 = 2 m_{\tilde{\nu}} \Delta m_{\tilde{\nu}}$, where $m_{\tilde{\nu}} \equiv \frac{1}{2}(m_{\tilde{\nu}_1} + m_{\tilde{\nu}_2})$ is the average of the light sneutrino masses. We find that the ratio of the light sneutrino mass difference relative to the light *neutrino* mass (eq. (23)) is given by (to leading order in $1/M$)

$$r_\nu \equiv \frac{\Delta m_{\tilde{\nu}}}{m_\nu} \simeq \frac{2(A_\nu - \mu \cot\beta - B_N)}{m_{\tilde{\nu}}} . \tag{27}$$

The magnitude of r_ν depends on various supersymmetric parameters.

Naturalness constrains supersymmetric mass parameters associated with particles with non-trivial electroweak quantum numbers to be roughly of order m_Z [12]. Thus, we assume that μ, A_ν, and $m_{\tilde{L}}$ are of the order of the electroweak scale. The parameters M, $m_{\tilde{N}}$, and B_N are fundamentally different since they are associated with the $SU(2) \times U(1)$ singlet superfield $\hat{N}$. In particular, $M \gg m_Z$, since this

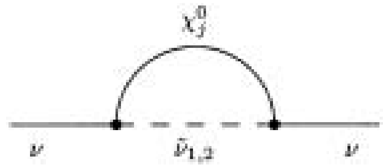

Figure 1. One-loop contribution to the neutrino mass due to sneutrino mass splitting.

drives the see-saw mechanism. Since M is a supersymmetry-conserving parameter, the see-saw hierarchy is technically natural. The parameters $m_{\tilde{N}}$ and B_N are soft-supersymmetry-breaking parameters; their order of magnitude is less clear.

Since $\hat{N}$ is an electroweak gauge group singlet superfield, supersymmetry-breaking terms associated with it need not be directly tied to the scale of electroweak symmetry breaking. Thus, it is possible that $m_{\tilde{N}}$ and B_N are much larger than m_Z. Since B_N enters directly into the formula for the light sneutrino mass splitting (eq. (27)), its value is critical for sneutrino phenomenology. If $B_N \sim \mathcal{O}(m_Z)$, then $r_\nu \sim \mathcal{O}(1)$, which implies that the sneutrino mass splitting is of order the *neutrino* mass. However, if $B_N \gg m_Z$, then the sneutrino mass splitting is significantly enhanced.

3.2 *Loop effects*

In the previous sections, we took into account only tree-level contributions to the neutrino and sneutrino mass matrices. However, in some cases, one-loop effects can substantially modify r_ν.

In general, the existence of a sneutrino mass splitting generates a one-loop contribution to the neutrino mass. Note that this effect is generic, and is independent of the mechanism that generates the sneutrino mass splitting. Similarly, the existence of a Majorana neutrino mass generates a one-loop contribution to the sneutrino mass splitting. Since in general $r_\nu \gtrsim 1$ at tree level, the latter effect can be safely neglected. In contrast, the one-loop correction to the neutrino mass is potentially significant, and may dominate the tree-level mass. We have computed exactly the one-loop contribution to the neutrino mass $[m_\nu^{(1)}]$ from neutralino/sneutrino loops shown in figure 1. In the limit of $m_\nu, \Delta m_{\tilde{\nu}} \ll m_{\tilde{\nu}}$, the formulae simplify, and we find

$$m_\nu^{(1)} = \frac{g^2 \Delta m_{\tilde{\nu}}}{32\pi^2 \cos^2\theta_W} \sum_j f(y_j) |Z_{jZ}|^2 , \tag{28}$$

where $f(y_j) = \sqrt{y_j}\,[y_j - 1 - \ln(y_j)]/(1-y_j)^2$, with $y_j \equiv m_{\tilde{\nu}}^2/m_{\tilde{\chi}_j^0}^2$, and $Z_{jZ} \equiv Z_{j2}\cos\theta_W - Z_{j1}\sin\theta_W$ is the neutralino mixing matrix element that projects out the $\tilde{Z}$ eigenstate from the jth neutralino. One can check that $f(y_j) < 0.566$, and for typical values of y_j between 0.1 and 10, $f(y_j) > 0.25$. Since Z is a unitary matrix, we find $m_\nu^{(1)} \approx 10^{-3} m_\nu^{(0)} r_\nu^{(0)}$, where $r_\nu^{(0)}$ is the tree-level ratio and $m_\nu^{(0)}$

is the tree-level neutrino mass. If $r_\nu^{(0)} \gtrsim 10^3$, then the one-loop contribution to the neutrino mass cannot be neglected. Moreover, r_ν cannot be arbitrarily large without unnatural fine-tuning. Writing the neutrino mass as $m_\nu = m_\nu^{(0)} + m_\nu^{(1)}$, and assuming no unnatural cancellation between the two terms, we conclude that

$$r_\nu \equiv \frac{\Delta m_{\tilde{\nu}}}{m_\nu} \lesssim 2 \times 10^3. \tag{29}$$

3.3 *Phenomenological consequences*

Based on the analysis presented above, we take $1 \lesssim r_\nu \lesssim 10^3$. The sneutrino mass splitting can be probed using the same sign dilepton signal if $x_{\tilde{\nu}} \gtrsim 1$. Here we must rely on sneutrino oscillations. Assume that the sneutrino decays with significant branching ratio via chargino exchange: $\tilde{\nu} \to \ell^\pm + X$. Since this decay conserves lepton number, the lepton number of the decaying sneutrino is tagged by the lepton charge. Then in $e^+e^- \to \tilde{\nu}_1\tilde{\nu}_2$, the probability of a same sign dilepton signal is

$$P(\ell^+\ell^+) + P(\ell^-\ell^-) = \chi_{\tilde{\nu}} \left[\mathrm{BR}(\tilde{\nu} \to \ell^\pm + X)\right]^2, \tag{30}$$

where

$$\chi_{\tilde{\nu}} \equiv \frac{x_{\tilde{\nu}}^2}{2(1+x_{\tilde{\nu}}^2)}, \tag{31}$$

is the integrated oscillation probability, which arises in the same way as the corresponding quantity that appears in the analysis of B meson oscillations [9].

At hadron collider, where the sneutrino are produced mainly via $\chi_2^+ \to \tilde{\nu}\ell^+$ the probability of a same sign dilepton signal is

$$P(\ell^+\ell^+) + P(\ell^-\ell^-) = \chi_{\tilde{\nu}} \left[\mathrm{BR}(\tilde{\nu} \to \ell^\pm + X)\right]. \tag{32}$$

We have considered the constraints on the supersymmetric model imposed by the requirements that $x_{\tilde{\nu}} \sim \mathcal{O}(1)$ and $\mathrm{BR}(\tilde{\nu} \to \ell^\pm + X) \sim 0.5$. We examined two cases depending on whether the dominant $\tilde{\nu}$ decays involve two-body or three-body final states. If the dominant sneutrino decay involves two-body final states, then we must assume that $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}^+} < m_{\tilde{\nu}}$. Then, the widths of the two leading sneutrino decay channels, with the latter summed over both final state charges, are given by [13,14]

$$\begin{aligned}
\Gamma(\tilde{\nu} \to \tilde{\chi}_j^0 \nu) &= \frac{g^2|Z_{jZ}|^2 m_{\tilde{\nu}}}{32\pi\cos^2\theta_W} B(m_{\tilde{\chi}_j^0}^2/m_{\tilde{\nu}}^2)\,, \\
\Gamma(\tilde{\nu} \to \tilde{\chi}^\pm \ell^\mp) &= \frac{g^2|V_{11}|^2 m_{\tilde{\nu}}}{8\pi} B(m_{\tilde{\chi}^+}^2/m_{\tilde{\nu}}^2)\,,
\end{aligned} \tag{33}$$

where $B(x) \equiv (1-x)^2$, V_{11} is one of the mixing matrix elements in the chargino sector, and Z_{jZ} is the neutralino mixing matrix element defined below eq. (28), and we take $m_\ell = 0$. For example, for $m_{\tilde{\nu}} \sim \mathcal{O}(m_Z)$ we find

$$\Gamma(\tilde{\nu} \to \chi_j^0 \nu) \approx \mathcal{O}\left(|Z_{jZ}|^2 B(m^2_{\tilde{\chi}_j^0}/m^2_{\tilde{\nu}}) \times 1\,\mathrm{GeV}\right),$$
$$\Gamma(\tilde{\nu} \to \chi^+ \ell) \approx \mathcal{O}\left(|V_{11}|^2 B(m^2_{\tilde{\chi}^+}/m^2_{\tilde{\nu}}) \times 1\,\mathrm{GeV}\right). \tag{34}$$

Typically, $B \gtrsim 10^{-2}$ in eq. (33). Thus, the same-sign dilepton signal is very small for neutrino masses below 1 eV.

If no open two-body decay channel exists, then we must consider the possible sneutrino decays into three-body final states. In this case we require that $m_{\tilde{\nu}} < m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}^+}$. Again, we assume that there exists a significant chargino-mediated decay rate with charged leptons in the final state. The latter occurs in models in which the $\tilde{\tau}_R$ is lighter than the sneutrino. In this case, the rate for chargino-mediated three-body decay $\tilde{\nu}_\ell \to \tilde{\tau}_R \nu_\tau \ell$ can be significant. The $\tilde{\tau}_R$ with $m_{\tilde{\tau}_R} < m_{\tilde{\nu}}$ can occur in radiative electroweak breaking models of low-energy supersymmetry if $\tan\beta$ is large. However, in the context of the MSSM, such a scenario would require that $\tilde{\tau}_R$ is the lightest supersymmetric particle (LSP), a possibility strongly disfavored by astrophysical bounds on the abundance of stable heavy charged particles. Thus, we go beyond the usual MSSM assumptions and assume that $\tilde{\tau}_R$ decays. This can occur in gauge-mediated supersymmetry breaking models [15] where $\tilde{\tau}_R \to \tau \tilde{g}_{3/2}$, or in R-parity violating models where $\tilde{\tau}_R \to \tau\nu$. Here, we have assumed that intergenerational lepton mixing is small; otherwise the $\Delta L = 2$ sneutrino mixing effect is diluted.

We have computed the chargino- and neutralino-mediated three-body decays of $\tilde{\nu}_\ell$. In the analysis presented here, we have not considered the case of $\ell = \tau$, which involves a more complex final state decay chain containing two τ-leptons. For simplicity, we present analytic formulae in the limit where the mediating chargino and neutralinos are much heavier than the $\tilde{\tau}_R$. In addition, we assume that the lightest neutralino is dominated by its bino component. We have checked that our conclusions do not depend strongly on these approximations. Then, the rates for the chargino- and neutralino-mediated sneutrino decays (the latter summed over both final state charges) are

$$\Gamma(\tilde{\nu}_\ell \to \ell^- \tilde{\tau}^+ \nu_\tau) = \frac{g^4 m_{\tilde{\nu}}^3 m_\tau^2 \tan^2\beta\, f_{\tilde{\chi}^+}(m^2_{\tilde{\tau}}/m^2_{\tilde{\nu}})}{1536\pi^3 (m_W^2 \sin 2\beta - M_2\mu)^2},$$
$$\Gamma(\tilde{\nu}_\ell \to \tau^\pm \tilde{\tau}^\mp \nu_\ell) = \frac{g'^4 m_{\tilde{\nu}}^5\, f_{\tilde{\chi}^0}(m^2_{\tilde{\tau}}/m^2_{\tilde{\nu}})}{3072\pi^3 M_1^4}, \tag{35}$$

for $\ell = \mu$, e, where the M_i are gaugino mass parameters and

$$f_{\tilde{\chi}^+}(x) = (1-x)(1+10x+x^2) + 6x(1+x)\ln x,$$
$$f_{\tilde{\chi}^0}(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \tag{36}$$

As an example, for $\tan\beta = 20$ (consistent with a light $\tilde{\tau}_R$ as noted above) and $m^2_{\tilde{\tau}}/m^2_{\tilde{\nu}} = 0.64$, reasonable values for the other supersymmetric parameters can be found such that $\Gamma(\tilde{\nu}_\ell \to \ell^\pm \tilde{\tau}^\mp \nu_\tau) \sim \Gamma(\tilde{\nu}_\ell \to \tau^\pm \tilde{\tau}^\mp \nu_\ell) \sim \mathcal{O}(1\,\mathrm{eV})$. In this case, for $r_\nu \sim 1$ [10^3], a significant like-sign dilepton signal could be observed for light neutrino masses as low as 1 eV (10^{-3} eV).

3.4 *Discussion and conclusions*

Non-zero Majorana neutrino masses imply the existence of $\Delta L = 2$ phenomena. In particular, in supersymmetric models, we expect sneutrino–antisneutrino mixing. The resulting sneutrino mass splitting is generally of the same order as the light neutrino mass, although an enhancement of up to three orders of magnitude is conceivable. Remarkably, model parameters exists where sneutrino mixing phenomena are detectable for *neutrino* masses as low as $m_\nu \sim 10^{-3}$ eV.

Thus, sneutrino mixing and oscillations could provide a novel opportunity to probe lepton-number violating phenomena in the laboratory.

4. Conclusions

The recent neutrino oscillation data imply that neutrinos are massive. The new physics that generate their masses may generate other non-standard effects. In this talk we discuss two such effects. We explain how neutrino oscillation can be used to look for non-standard neutrino interaction. We also explain how sneutrino–antisneutrino mixing can be used to probe non-zero Majorana neutrino masses.

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