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GUT precursors and fixed points in higher-dimensional theories

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Abstract. Within the context of traditional logarithmic grand unification at $M_{\rm GUT} \approx 10^{16}$ GeV, we show that it is nevertheless possible to observe certain GUT states such as X and Y gauge bosons at lower scales, perhaps even in the TeV range. We refer to such states as 'GUT precursors'. Such states offer an interesting alternative possibility for new physics at the TeV scale, even when the scale of gauge coupling unification remains high, and suggest that it may be possible to probe GUT physics directly even within the context of high-scale gauge coupling unification. More generally, our results also suggest that it is possible to construct self-consistent 'hybrid' models containing widely separated energy scales, and give rise to a Kaluza–Klein realization of non-trivial fixed points in higher-dimensional gauge theories.

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1. Introduction

One of the most important theoretical challenges in physics is to determine the nature of fundamental theories. Such fundamental theories include theories of grand unification, quantum gravity, and even strings, with each theory carrying its own intrinsic energy scale.

The traditional view of such theories stipulates that their intrinsic energy scales are exceedingly high. In such cases, experimental evidence in favor of such theories is at best indirect. More recently, however, it has been suggested [1–4] that the presence of large extra dimensions might significantly lower the energy scales associated with such theories, perhaps all the way to the TeV range.

In these proceedings, we report on a 'hybrid' possibility [5]. Specifically, we wish to consider a higher-dimensional scenario in which the fundamental theories of physics retain their traditional high characteristic energy scales, but in which it is nevertheless possible to obtain *direct*, low-energy evidence of their existence. As we shall see, this will be possible because of the emergence of a non-trivial fixed point which enables a large separation of scales to exist within a single model.

For concreteness, we shall concentrate on the case of grand unification, and consider a scenario in which the unification of gauge couplings retains its traditional logarithmic behavior, with unification occurring near $M_{\rm GUT} \approx 10^{16}$ GeV. However, we shall demonstrate that even within such a scenario, it is possible that certain states associated with the emergence of a grand unified theory (GUT) at this energy scale can actually be extremely light, perhaps even in the TeV range. We shall refer to such states as 'GUT precursors'.

2. GUT symmetry breaking, orbifolds, and 'GUT precursors'

In theories which exhibit a unification of the standard model (SM) gauge couplings, it is natural to imagine the emergence of a grand unified theory at the scale of unification. The gauge symmetry group associated with this GUT (e.g., $SU(5)$ or $SO(10)$) must then be large enough to contain the SM gauge symmetry group as a subgroup. In each case, we then find that the corresponding GUT contains not only the usual standard model particles, but also additional particles which are directly associated with the GUT. These particles necessarily include the so-called X and Y gauge bosons associated with the enlarged GUT gauge symmetry, and may also include additional matter particles (such as colored Higgs triplets).

There are two basic methods by which GUT symmetries can be broken below the scale of unification. The first method is intrinsically field-theoretic: one imagines that a certain GUT field obtains a non-vanishing expectation value $v \approx M_{\rm GUT}$ in such a way that the standard model fields remain light while the extra GUT fields become heavy. This is the standard Higgs mechanism for breaking a GUT symmetry, and it is characterized by the fact that the masses of the extra GUT fields – the true signatures of the existence of the GUT – are parametrically tied to the GUT scale.

The second method, by contrast, is essentially string-theoretic, and involves truncating the full string Fock space in such a way that the large initial gauge symmetry is broken down to a smaller residual gauge symmetry. This method has often been referred to as 'GUT breaking by orbifolds', and has been discussed within the context of large extra dimensions in [3] as well as more recently in [6–8].

This method works as follows: For simplicity, let us first imagine compactification of a single extra dimension on the circle S^1 defined by identifying $y \approx y + 2\pi R$ (where y is the coordinate along the compact extra dimension). Any field $\Phi(x^\mu, y)$ on the circle can then be Fourier-decomposed as $\Phi = \Phi_+ + \Phi_-$ where

$$\Phi_+(x^\mu, y) = \sum_{n=0}^{\infty} \Phi_+^{(n)}(x^\mu) \cos\left(\frac{ny}{R}\right),$$
$$\Phi_-(x^\mu, y) = \sum_{n=1}^{\infty} \Phi_-^{(n)}(x^\mu) \sin\left(\frac{ny}{R}\right). \quad (1)$$

Note that Φ_+ is even under $y \to -y$, while Φ_- is odd; moreover Φ_- lacks a zero mode. The mass of the Kaluza–Klein mode $\Phi_\pm^{(n)}$ is n/R. Given this Kaluza–Klein circle decomposition, it is then straightforward to compactify on the orbifold defined by S^1/Z_2 where the Z_2 action is $y \to -y$. We simply retain only the even or odd modes in the above decomposition. It is this truncation which reduces the Fock space. For example, if Φ refers to a standard model field, we arrange our orbifold so as to retain the even components Φ_+, whereas if Φ refers to a GUT field which is not present in the standard model, we retain the odd components Φ_-. In this way, the spectrum of zero-modes accessible to the low-energy observer at energies $E \ll R^{-1}$ consists of only the standard model fields.

For our purposes, however, the important feature of this method of GUT symmetry breaking is the energy scale at which the first signatures of the full GUT symmetry appear. Unlike the Higgs breaking mechanism, where masses of the GUT fields beyond the standard model are parametrically tied to $M_{\rm GUT}$, in this case the masses of the first Kaluza–Klein modes for these GUT particles are set by the inverse radius of the orbifold! Thus, in cases for which $R^{-1} < M_{\rm GUT}$, we actually begin to observe GUT particles (such as X and Y gauge bosons) *before* we detect actual gauge coupling unification. In other words, these low-lying Kaluza–Klein modes of the GUT particles appear as 'GUT precursors' [3], signalling the future emergence of a full gauge coupling unification at an even higher energy scale.

The obvious question, then, is to determine how light these GUT precursors can be. How far below $M_{\rm GUT}$, the scale of gauge coupling unification, can these states sit?

Clearly, the answer to this question depends on the particular model under discussion. In the models of accelerated power-law unification in [3], the ratio between the scale of extra dimensions and the scale of accelerated gauge coupling unification is never significantly more than one order of magnitude: $M_{\rm GUT}R \lesssim 20$. Likewise, in the more recent models of [7] for which $M_{\rm GUT}$ takes a high value, i.e., $M_{\rm GUT} > 10^{16}$ GeV, this ratio is somewhat larger: $M_{\rm GUT}R \approx 100$. Thus, both classes of models predict the appearance of GUT precursors well in advance of actual gauge coupling unification.

Despite these facts, both classes of models predict the appearance of GUT precursors which are not drastically separated from their corresponding fundamental scales of gauge coupling unification.

When the effects of the GUT precursors are included, it follows that the states at each excited Kaluza–Klein mass level fall into complete GUT multiplets. As originally noticed in [3], this still leads to gauge coupling unification. However, as pointed out in [7], the presence of complete GUT multiplets at each excited level implies that the power-law running is *universal* for each gauge coupling. Thus, the *unification* of the gauge couplings continues to be logarithmic, occurring just as it does in four dimensions in the absence of Kaluza–Klein states. Indeed, this feature is the hallmark of the models of [7], and is one of the reasons why these models can apparently tolerate the larger value of $M_{\rm GUT}R \approx 100$.

The chief danger inherent in such a unification, however, is that the gauge couplings are each individually still experiencing power-law evolution. Thus, even though the *unification* of these gauge couplings is completely logarithmic, the gauge couplings themselves might flow towards strong coupling, thereby invalidating the perturbative unification calculation. Indeed, this is the primary feature which ultimately limits the separation between the scale at which the GUT precursors appear and the scale of gauge coupling unification.

3. Power-law running and perturbativity

We shall now demonstrate that this restriction does not arise for models in which all matter is restricted to orbifold fixed points and in which only the standard model and GUT gauge bosons propagate in the bulk. In such cases, we shall show that it is possible to separate R^{-1}, the scale of the GUT precursors, *by an arbitrary amount* from $M_{\rm GUT}$, at least as far as gauge coupling unification is concerned.

In theories with extra dimensions, the evolution of the gauge couplings takes the approximate form [3]

$$\alpha_i^{-1}(\Lambda) \approx \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi}\ln\frac{\Lambda}{M_Z} + \frac{\tilde{b}_i}{2\pi}\ln \Lambda R - \frac{\tilde{b}_i X_\delta}{2\pi\delta}\left[(\Lambda R)^\delta - 1\right]. \quad (2)$$

The emergence of power-law behavior is expected in a higher-dimensional gauge theory [3,9], and can equivalently be viewed as logarithmic running with a beta-function coefficient that continually changes as successive Kaluza–Klein thresholds are crossed. We refer the reader to [5] for the various definitions entering into (2).

In cases where $\tilde{b}_i$ are unequal, we see from eq. (2) that the power-law evolution of the gauge couplings is different for each gauge coupling. This implies that the relative *differences* between the gauge couplings also evolve with power-law behavior. However, when $\tilde{b}_i$ are all equal, we see from eq. (2) that this power-law behavior is universal for all gauge couplings. The relative differences of gauge couplings then evolve purely logarithmically, exactly as in four dimensions.

Despite this fact, it is still important to verify that the individual gauge couplings themselves remain perturbative over the entire energy range from R^{-1} to $\Lambda \equiv M_{\rm GUT}$. Otherwise, the use of the one-loop result in eq. (2) is no longer valid. Towards this end, let us assume that $\tilde{b}_i \equiv \tilde{b} < 0$ for all i. Since $\tilde{b} < 0$, the power-law contributions to the gauge couplings push the couplings towards extremely weak values. Indeed, in the limit where $\Lambda R \gg 1$, we find from eq. (2) that each of the gauge couplings scales in the ultraviolet as

$$\alpha(\Lambda) \;\approx\; -\frac{2\pi\delta}{\tilde{b}X_\delta}(\Lambda R)^{-\delta}. \quad (3)$$

However, even though these couplings are extremely weak, the true loop expansion parameter in such a situation is $\alpha_{\rm eff} \equiv N\alpha$ where $N \equiv X_\delta(\Lambda R)^\delta$ is the number of Kaluza–Klein levels that have been crossed. Thus, for true perturbativity, we must demand $\alpha_{\rm eff} \ll 4\pi$. This constraint is satisfied no matter how large ΛR becomes. Indeed, we find that $\alpha_{\rm eff} \approx -2\pi\delta/\tilde{b}$ as $\Lambda R \to \infty$, so that the condition for perturbativity becomes $-\delta/(2\tilde{b}) \ll 1$. Thus, as long as $\tilde{b}$ is sufficiently large and negative, this condition can be satisfied even if $\Lambda R \gg 1$. Actually, for observables which have large contributions beyond one-loop, this conclusion is not quite true. Indeed, for an $SU(N)$ gauge group, the number of 'colors' multiplies also the perturbativity parameter and the effective coupling is actually always non-perturbative. For quantities protected by supersymmetry, like gauge couplings, however, the relevant correction is at the one-loop level and our considerations do apply.

As an example, let us consider a scenario in which, as discussed above, the zero-mode fields are those of the MSSM and only the GUT gauge bosons sit in the bulk.

For simplicity, we shall take our unified gauge group to be $SU(5)$, and we shall also assume that $\delta = 1$. Since our low-energy theory is $\mathcal{N} = 1$ supersymmetric, the bulk fields necessarily fall into $\mathcal{N} = 2$ supermultiplets. Our bulk fields therefore consist of $\mathcal{N} = 2$ vector multiplets transforming in the adjoint of $SU(5)$, leading to $\tilde{b}_i = \tilde{b} = -10$ for all i. We then find that the effective gauge interaction strength at unification is $\alpha_{\text{eff}} \approx 0.63$. Note that this remains true even if $\Lambda R \approx 10^{13}$. Thus it is possible for the GUT precursors to appear at the TeV scale even though the (logarithmic) gauge coupling unification does not occur until the usual scale $M_{\text{GUT}} \approx 10^{16}$ GeV. Note that this scenario requires beta-function coefficients $\tilde{b}_i$ which are universal and *negative*.

One might worry that two-loop effects might be significant in such a scenario. However, two-loop effects essentially *vanish* in the $\Lambda R \to \infty$ limit, since the presence of $\mathcal{N} = 2$ supersymmetry in the bulk ensures that the higher-loop power-law effects are suppressed by a factor of $1/\Lambda R$ relative to the one-loop effects [3,10]. Likewise, we remark in this context that the potentially damaging brane surface kinetic terms discussed in [11] become vanishingly small in this context, since the volume of the bulk becomes 10^{13} as large as the volume on the brane.

It is also important to understand what happens if $\tilde{b} > 0$. In this case, the gauge couplings become stronger rather than weaker as we evolve upwards in energy, ultimately hitting a Landau pole where $\alpha^{-1}(\Lambda) = 0$. In such cases, depending on the specific value of $\tilde{b}$, there is therefore a maximum allowed value of $M_{\text{GUT}}R$ which can be tolerated if we imagine varying R while holding M_{GUT} fixed. For example, in some of the 'minimal' models of [7], the bulk contains not only the $SU(5)$ gauge bosons, but also two **5** representations (for the Higgses) and four **10** representations (for the first two generations). Since these are all $\mathcal{N} = 2$ supermultiplets, this leads to a value $\tilde{b} = +4$. We then find that perturbativity in such models requires a maximum value $M_{\text{GUT}}R \lesssim 44$ in order to avoid a Landau pole (or equivalently, to avoid negative inverse gauge couplings).

4. Non-trivial ultraviolet fixed points

In the set-up described in the previous section, the asymptotic ultraviolet power-law scaling of gauge couplings towards weak values is exactly compensated by the asymptotic power-law growth of the number of degrees of freedom in the theory in such a way that the product of these two quantities remains a constant. Thus, the effective strength of the gauge interactions appears to approach a non-trivial fixed point in the ultraviolet. Such behavior for gauge couplings with $\tilde{b} < 0$ was also observed previously in [12].

It is already apparent that as long as $\Lambda R \gtrsim 100$, our theory essentially becomes 'scale invariant' in the sense that the ultraviolet physics becomes independent of the low-energy scale R^{-1} at which the GUT precursors appear. We may also rephrase this observation directly in terms of the effective couplings $\alpha_{\text{eff},i} \equiv N\alpha_i$ where $N \equiv X_\delta(\Lambda R)^\delta$. Given the evolution equations for the couplings α_i in eq. (2), it is straightforward to show that the effective couplings $\alpha_{\text{eff},i}$ evolve according to

$$\Lambda \frac{\mathrm{d}\alpha_{\mathrm{eff},i}^{-1}}{\mathrm{d}\Lambda} = -\left(\delta\alpha_{\mathrm{eff},i}^{-1} + \frac{\tilde{b}_i}{2\pi}\right) + \left(\frac{\tilde{b}_i - b_i}{2\pi X_\delta}\right)(\Lambda R)^{-\delta} + \frac{c_i}{2\pi}\,\frac{\alpha_{\mathrm{eff},i}}{4\pi}(\Lambda R)^{-\delta} + \cdots, \tag{4}$$

where, in the second line we have written the dominant two-loop contributions arising from the bulk and boundary fields running in the loops (with c_i representing a two-loop beta-function coefficient). Thus, even though the individual gauge couplings α_i themselves evolve with power-law behavior, we see from eq. (4) that for $\Lambda R \gg 1$, the *effective* gauge couplings $\alpha_{\mathrm{eff},i}$ each approach an ultraviolet fixed point at $\alpha_{\mathrm{eff},i} = -2\pi\delta/\tilde{b}_i$. Moreover, if $\tilde{b}_i \equiv \tilde{b}$ for all i, we see that even though the differences of the gauge couplings continue to evolve logarithmically, the fixed-point values of the *effective* gauge couplings all become equal.

It is natural to interpret these results as indicating the emergence of a non-trivial (interacting) ultraviolet fixed point corresponding to a supersymmetric, higher-dimensional, unified gauge theory. Indeed, such higher-dimensional fixed-point gauge theories are known to exist in uncompactified five [13] and six [14] dimensions. Since we expect the ultraviolet (short-distance) limit of our compactified theory to reproduce the physics of an uncompactified higher-dimensional theory, it is tempting to identify the ultraviolet limit of our theory as one of the interacting fixed-point theories discussed in [13,14].

As an example, let us consider the case of $SU(N)$ gauge theory in five dimensions. If the only matter consists of n_f 'quarks' transforming in the fundamental representation, then the necessary and sufficient condition [13] for the existence of an interacting ultraviolet fixed point is $n_f \leq 2N$. This is equivalent to our requirement that $\tilde{b} \leq 0$. It is important to stress that for unitary groups, the conditions for the existence of such non-trivial fixed points are generally *stronger* than merely demanding $\tilde{b} < 0$. For example, if we also introduce bulk matter which transforms in the *antisymmetric tensor* representation (e.g., the **10** representation of $SU(5)$), then further restrictions on the number of fundamental representations must be imposed in order to guarantee the fixed-point behavior of the theory in the ultraviolet [13]. These conditions are consistent with $\tilde{b} \leq 0$, but provide further constraints on the precise matter content.

Thus far, we have presented a general GUT scenario in which GUT precursor states can be extremely light compared with the scale of gauge coupling unification. Our purpose has been to illustrate the emergence and utilization of non-trivial fixed points as a means of incorporating widely separated energy scales within a single model. However, in order to build a fully consistent model, we must choose an explicit orbifold and take into account certain additional contributions that arise [7,15–17].

Let us first consider the case of the five-dimensional $SU(5)$ theory compactified on the $S_1/\mathbb{Z}_2$ orbifold presented in §2. Under the $\mathbb{Z}_2$ action, standard-model gauge supermultiplets are even while GUT supermultiplets such as the X and Y gauge supermultiplets are odd. However, since this is a five-dimensional theory, the X and Y supermultiplets also have fifth components X_5 and Y_5, and consistency of the $\mathbb{Z}_2$ orbifold requires that these components be even. The zero modes of such fields then produce additional non-universal logarithmic contributions to the runnings of the gauge couplings, with coefficients $(b_1, b_2, b_3) = (5, 3, 2)$. However, it is easy to verify that taking $R^{-1} \sim$ TeV still leads to an approximate unification at $M_{\rm GUT} \approx 2 \times 10^{13}$ GeV which, although not as precise as the MSSM unification, is nevertheless more precise than the unification in the standard model. Of course, at a phenomenological level, it still remains necessary to find a mechanism to give masses to these X_5 and Y_5 zero modes, since they represent colored and fractionally charged scalars.

Another option is to compactify our theory on an $S_1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold with two distinct $\mathbb{Z}_2$ discrete actions [6,7] associated with $y \to -y$ and $y \to \pi R - y$. Under the $(\mathbb{Z}_2, \mathbb{Z}_2')$ orbifold actions, the standard-model gauge fields A_μ have $(+,+)$ eigenvalues (with corresponding cosine modings as in eq. (1) with $n \in 2\mathbb{Z}$ only), while A_5 has $(-,-)$ eigenvalues (resulting in sine modings with $n \in 2\mathbb{Z}$). Likewise, the GUT precursors X_μ and Y_μ have $(-,+)$ eigenvalues (resulting in sine modings with $n \in 2\mathbb{Z}+1$), while X_5 and Y_5 have $(+,-)$ eigenvalues (resulting in cosine modings with $n \in 2\mathbb{Z}+1$). With this orbifold choice, only the standard model fields have zero modes, but this occurs at the expense of splitting the complete GUT multiplets at each Kaluza–Klein level into a subset at even levels, with beta-function coefficients $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (0, -4, -6)$, and a subset at odd levels, with $(\tilde{b}_1', \tilde{b}_2', \tilde{b}_3') = (-10, -6, -4)$. This results in a staggered, 'zig-zag' running for the gauge couplings which averages to a universal power-law running with an effective radius $R/2$, along with a non-universal logarithmic correction. Specifically, the gauge couplings now run according to

$$\alpha_i^{-1}(\Lambda) \approx \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi}\ln\frac{\Lambda}{M_Z} + \frac{\tilde{b}}{4\pi}\ln\frac{\Lambda R}{2} - \frac{\tilde{b}}{2\pi}\left[\left(\frac{\Lambda R}{2}\right) - 1\right] - \frac{\tilde{b}_i'}{2\pi}Y, \tag{5}$$

where $\tilde{b} \equiv \tilde{b}_i + \tilde{b}_i' = -10$ for all i, where we have neglected various universal additive constants, and where the non-universal logarithm is given by

$$Y \;\equiv\; \sum_{n=0}^{(\Lambda R-2)/2} \ln\frac{2n+2}{2n+1} \;\approx\; \tfrac{1}{2}\ln\frac{\pi\Lambda R}{2} \tag{6}$$

with the last approximation holding in the $\Lambda R \gg 1$ limit. Given this running, we then find that the three gauge couplings continue to experience an approximate unification. With $R^{-1} \sim$ TeV, the unification scale is unfortunately quite high ($M_{\rm GUT} \approx 10^{21}$ GeV), but increasing R^{-1} not only improves the accuracy of the resulting unification but also lowers the unification scale. Asymptotically, with $R^{-1} \approx 10^{15}$ GeV, we obtain an essentially *exact* unification at $M_{\rm GUT} \approx 10^{17}$ GeV. Although this resembles the energy scales in the scenario in [7], we stress that the bulk theory here contains only gauge fields, and our gauge couplings become *weak* rather than strong in the ultraviolet limit.

A final option is to place a single Higgs five-plet in the bulk. After compactifying on the $S_1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold, the bulk Higgs doublets (triplets) are at even (odd) Kaluza–Klein levels. We thus have $\tilde{b} = -9$, $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (3/5, -3, -6)$, and $(\tilde{b}_1', \tilde{b}_2', \tilde{b}_3') = (-48/5, -6, -3)$. Note that these values of $\tilde{b}_i$ are the same as those of [3]. This yields a unification of gauge couplings which is similar to (and approximately as accurate as) the case with only gauge bulk fields discussed above; in each case, small (few per cent) threshold corrections at the unification scale are sufficient to render the unification exact for all values of R^{-1}. Of course, in each case one must place the remaining Higgs field(s) on a standard-model brane lacking the GUT symmetry in order to avoid the doublet/triplet problem.

5. Discussion

We have shown that it is possible for certain 'GUT precursor' states to appear with masses that are significantly below the scale at which grand unification occurs. As we have seen, the fixed-point structure which makes this possible can be viewed as a general tool which permits the construction of generalized 'hybrid' models in which widely separated energy scales can coexist in a natural way.

Needless to say, these observations prompt a number of important questions, both phenomenological and theoretical. Among the most important phenomenological questions is the issue of proton decay. Ordinarily, light X and Y gauge boson precursors will mediate rapid proton decay. However, as in all low-scale extensions to the standard model, this problem may be cured through the use of split fermions on the branes [18] or through the introduction of extra discrete symmetries [3]. Likewise, other phenomenological issues include doublet/triplet splitting and general issues of flavor physics. Although we have not attempted to make a complete GUT model that accommodates these phenomena, one could imagine doing so following the lines of [6–8] except that we now have the interesting option of extending the energy scales of such models into the TeV range. Moreover, we are also free to introduce further matter into the bulk beyond what we have discussed here, provided we ensure that $\tilde{b}$ remains sufficiently large and negative and provided the conditions [13] for an ultraviolet fixed point are maintained. Of course, we have not speculated on the unknown dynamics which might ultimately be responsible for generating and stabilizing such a large radius; ideas along these lines can be found, e.g., in [19].

Our results in this paper also raise a number of theoretical issues. The most important issue concerns the ultraviolet limit of our theory. Although we have shown that the evolution of the gauge couplings is consistent with perturbativity even when the effective higher-dimensional energy interval is large, one must actually verify that *all* correlation functions in the theory remain finite and under control over this large energy range. This is clearly connected with the over-riding question discussed in §4 concerning the manner in which we approach a scale-invariant fixed point in the ultraviolet. By counting Kaluza–Klein states and vertex factors in diagrams with arbitrary numbers of loops and external legs, it is straightforward to demonstrate that all diagrams in this theory necessarily scale as $(N\alpha)^k\sqrt{\alpha}^\ell$ where k and ℓ are non-negative integers. Thus, in the ultraviolet limit, such diagrams either vanish (if $\ell \neq 0$) or approach a fixed finite value (if $\ell = 0$).

For example, the four-fermion amplitude for tree-level Kaluza–Klein exchange in the $\delta = 1$ case becomes

$$A(s) \sim \alpha(s) \sum_n \frac{1}{s - n^2/R^2} \sim \frac{\alpha(s)}{\sqrt{s}} \, R \cot(\pi R\sqrt{s})$$
$$\longrightarrow \frac{\alpha_{\text{eff}}}{s} \cot(\pi R\sqrt{s}), \tag{7}$$

where we have taken the limit $sR^2 \gg 1$ and identified $\alpha_{\text{eff}} \sim (R\sqrt{s})\alpha(s)$ as $s \to \infty$. Thus, since $A(s)$ continues to have the asymptotic energy dependence $\sim 1/s$, no unitarity bounds are violated in the ultraviolet. (The divergence when $R\sqrt{s} \in \mathbb{Z}$ merely reflects a fine-tuned Kaluza–Klein resonant pole.) Moreover, as we have already noted, the presence of $\mathcal{N} = 2$ supersymmetry in the bulk ensures the vanishing of many diagrams which would otherwise lead to large and potentially uncontrollable corrections.

Another important theoretical issue for our models concerns gravity. For detailed discussions about this issue, see [5].

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