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Physics with large extra dimensions

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Abstract. The recent understanding of string theory opens the possibility that the string scale can be as low as a few TeV. The apparent weakness of gravitational interactions can then be accounted by the existence of large internal dimensions, in the sub-millimeter region. Furthermore, our world must be confined to live on a brane transverse to these large dimensions, with which it interacts only gravitationally. In my lecture, I describe briefly this scenario which gives a new theoretical framework for solving the gauge hierarchy problem and the unification of all interactions. I also discuss a minimal embedding of the standard model, gauge coupling unification and proton stability.

Keywords. Low scale string; extra dimensions; standard model.

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1. Early motivation for large extra dimensions

Attempts to construct a consistent theory for quantum gravity have led only to one candidate: the string theory. The only vacuum of string theory free of any pathologies are supersymmetric. Not being observed in nature, supersymmetry should be broken. In contrast to ordinary supergravity, where supersymmetry breaking can be introduced at an arbitrary scale, through for instance the gravitino, gaugini and other soft masses, in string theory this is not possible (perturbatively). The only way to break supersymmetry at a scale hierarchically smaller than the (heterotic) string scale is by introducing a large compactification radius whose size is set by the breaking scale. This has to be therefore of the order of a few TeV in order to protect the gauge hierarchy [1]. This is one of the very few general predictions of perturbative (heterotic) string theory that leads to the spectacular prediction of the possible existence of extra dimensions accessible to future accelerators [2]. The main theoretical problem though is that the heterotic string coupling becomes necessarily strong.

The strong coupling problem can be understood from the effective field theory point of view from the fact that at energies higher than the compactification scale, the KK excitations of gauge bosons and other standard model (SM) particles will start being produced and contribute to various physical amplitudes. Their multiplicity very rapidly turns the logarithmic evolution of gauge couplings into a power dependence [3], invalidating the perturbative description, as expected in a higher-dimensional non-renormalizable gauge theory. A possible way to avoid this problem is to impose conditions which prevent the power corrections to low-energy couplings [2]. For gauge couplings, this implies the vanishing of the corresponding β-functions, which is the case for instance when the KK modes are organized in multiplets of $N = 4$ supersymmetry, containing for every massive spin-1 excitation, two Dirac fermions and six scalars. Examples of such models are provided by orbifolds with no $N = 2$ sectors with respect to the large compact coordinate(s).

The simplest example of a one-dimensional orbifold is an interval of length πR, or equivalently S^1/Z_2 with Z_2 the coordinate inversion. The Hilbert space is composed of the untwisted sector, obtained by the Z_2-projection of the toroidal states, and of the twisted sector which is localized at the two end-points of the interval, fixed under the Z_2 transformations. This sector is chiral and can thus naturally contain quarks and leptons, while gauge fields propagate in the 5d bulk.

Similar conditions should be imposed to Yukawa's and in principle to higher (non-renormalizable) effective couplings in order to ensure a soft ultraviolet (UV) behavior above the compactification scale. We now know that the problem of strong coupling can be addressed using string S-dualities which invert the string coupling and relate a strongly coupled theory with a weakly coupled one. For instance, the strongly coupled heterotic theory with one large dimension is described by a weakly coupled type-II theory with a tension at intermediate energies of $\sim 10^{11}$ GeV [4]. Furthermore, non-Abelian gauge interactions emerge from tensionless strings whose effective theory describes a higher-dimensional non-trivial infrared fixed point of the renormalization group. This theory incorporates all conditions to low-energy couplings that guarantee a smooth UV behavior above the compactification scale. In particular, one discovers that KK modes of gauge bosons form $N = 4$ supermultiplets, while matter fields are localized in four dimensions. It is remarkable that the main features of these models were captured already in the context of the heterotic string despite its strong coupling [2].

In the case of two or more large dimensions, the strongly coupled heterotic string is described by a weakly coupled type-II or type-I theory [4]. Moreover, the tension of the dual string will be of the order or even lower than the compactification scale. In fact, the string tension becomes an arbitrary parameter [5]. It can be anywhere below the Planck scale and as low as a few TeV [6]. The main advantage of having the string tension at the TeV, besides its obvious experimental interest, is that it offers an automatic protection to the gauge hierarchy, alternative to low-energy supersymmetry or technicolor [7–9].

2. Type-I string theory and D-branes

Type-I (in general type-I$'$) is a ten-dimensional theory of closed and open unoriented strings. Closed strings describe gravity, while gauge interactions are described by open strings whose ends are confined to propagate on p-dimensional sub-spaces defined as Dp-branes. Assuming that the standard model is localized on a p-brane with $p \geq 3$, the internal space has six compactified dimensions, $p - 3$ longitudinal and $9 - p$ transverse to the Dp-brane.

The gauge and gravitational interactions appear at different order in string loops perturbation theory, leading to different powers of the string coupling $g_{\rm s}$ in the corresponding effective action:

$$S_I = \int {\rm d}^{10}x \frac{1}{g_{\rm s}^2 l_{\rm s}^8}\mathcal{R} + \int {\rm d}^{p+1}x \frac{1}{g_{\rm s} l_{\rm s}^{p-3}} F^2 \,, \tag{1}$$

where $l_{\rm s}$ is the string length ($l_{\rm s} \equiv M_{\rm s}^{-1}$ with $M_{\rm s}$ the string scale). The $1/g_{\rm s}$ factor in front of the gauge kinetic terms corresponds to the lowest order open string diagram represented by a disk. Upon compactification in four dimensions, the Planck length $l_{\rm p} = M_{\rm p}^{-1}$ and gauge couplings g_{YM} are given to leading order by

$$\frac{1}{l_{\rm p}^2} = \frac{V_\parallel V_\perp}{g_{\rm s}^2 l_{\rm s}^8}, \quad \frac{1}{g_{YM}^2} = \frac{V_\parallel}{g_{\rm s} l_{\rm s}^{p-3}} \,, \tag{2}$$

where $V_\parallel$ ($V_\perp$) denotes the compactification volume longitudinal (transverse) to the Dp-brane. From the second relation above, it follows that the requirements of weak coupling, $g_{YM} \sim \mathcal{O}(1)$, $g_s < 1$, imply that the size of the longitudinal space must be of order of the string length ($V_\parallel \sim l_{\rm s}^{p-3}$), while the transverse volume $V_\perp$ remains unrestricted. Using the longitudinal volume in string units $v_\parallel \gtrsim 1$, and assuming an isotropic transverse space of $n = 9 - p$ compact dimensions of radius $R_\perp$, we can rewrite these relations as

$$M_{\rm p}^2 = \frac{1}{g_{YM}^4 v_\parallel} M_{\rm s}^{2+n} R_\perp^n, \quad g_{\rm s} = g_{YM}^2 v_\parallel . \tag{3}$$

From relations (3), it follows that the type-I string scale can be chosen hierarchically smaller than the Planck mass at the expense of introducing extra large transverse dimensions that are felt only by the gravitationally interacting light states, while keeping the string coupling weak [8]. The weakness of 4d gravity compared to gauge interactions ($M_W/M_{\rm p}$) is then attributed to the largeness of the transverse space $R_\perp/l_{\rm s}$. An important property of these models is that gravity becomes $(4+n)$-dimensional with a strength comparable to those of gauge interactions at the string scale. The first relation of eq. (3) can be understood as a consequence of the $(4+n)$-dimensional Gauss law for gravity, with

$$G_{\rm N}^{(4+n)} = g_{YM}^4 l_{\rm s}^{2+n} v_\parallel \tag{4}$$

the Newton's constant in $4+n$ dimensions. Taking the type-I string scale $M_{\rm s}$ to be at 1 TeV, one finds the size for the transverse dimensions $R_\perp$ varying from 10^8 km, 0.1 mm (10^{-3} eV), down to 0.1 Fermi (10 MeV) for $n = 1, 2$, or 6 large dimensions, respectively. This shows that while $d_\perp = 1$ is excluded, $d_\perp \geq 2$ are allowed by the present experimental bounds on gravitational forces [10].

3. Ultraviolet–infrared correspondence

In addition to the open strings describing the gauge degrees of freedom, consistency of string theory requires the presence of closed strings associated with gravitons and different kinds of moduli fields m_a. There are two types of extended objects: D-branes and orientifolds. The former are hypersurfaces on which open strings end while the latter are hypersurfaces located at fixed points when acting simultaneously with a Z_2 parity on the transverse space and world-sheet coordinates.

Closed strings can be emitted by D-branes and orientifolds, the lowest order diagrams being described by a cylindric topology. In this way D-branes and orientifolds appear as the lowest order classical point-like sources in the transverse space. For weak type-I string coupling this can be described by a Lagrangian of the form

$$\int \mathrm{d}^n x_\perp \left[\frac{1}{g_\mathrm{s}^2}(\partial_{x_\perp} m_a)^2 + \frac{1}{g_\mathrm{s}} \sum_\mathrm{s} f_\mathrm{s}(m_a)\delta(x_\perp - x_{\perp\mathrm{s}})\right], \tag{5}$$

where $x_{\perp\mathrm{s}}$ is the location of the source s (D-branes and orientifolds) while $f_\mathrm{s}(m_a)$ encodes the coupling of this source to the moduli m_a. As a result, while m_a have constant values in the four-dimensional space, their expectation values will generically vary as a function of the transverse coordinates $x_\perp$ of the n directions with size $\sim R_\perp$ large compared to the string length l_s.

In a compact space where flux lines cannot escape to infinity, the Gauss law implies that the total charge, thus global tadpoles, should vanish, while local tadpoles may not. In that case, obtained for generic positions of the D-branes, the tadpole contribution leads to the following behavior in the large radius limit for the moduli m_a [9]:

$$m_a(x_{\perp\mathrm{s}}) \sim \begin{cases} O(R_\perp M_\mathrm{s}) & \text{for} \quad d_\perp = 1 \\ O(\ln R_\perp M_\mathrm{s}) & \text{for} \quad d_\perp = 2 \\ O(1) & \text{for} \quad d_\perp > 2 \end{cases}, \tag{6}$$

which is dictated by the large-distance behavior of the two-point Green function in the $d_\perp$-dimensional transverse space. Some important implications of these results are:

- The tree-level exchange diagram of a closed string can also be seen as one-loop exchange of open strings. While from the former point of view, a long cylinder represents an infrared limit where one computes the effect of exchanging light closed strings at long distances, in the second point of view the same diagram is conformally mapped to an annulus describing the one-loop running in the ultraviolet limit of very heavy open strings streching between the two boundaries of the cylinder. Thus, from the brane gauge theory point of view, there are ultraviolet effects that are not cut-off by the string scale M_s but instead by the winding mode scale $R_\perp M_\mathrm{s}^2$.
- In the case of one large dimension $d_\perp = 1$, the corrections are linear in $R_\perp$. Such correction appears for instance for the dilaton field which sits in front of gauge kinetic terms, that drive the theory rapidly to a strong coupling singularity and, thus, forbid the size of the transverse space to become much larger than the string length. It is possible to avoid such large corrections if the tadpoles cancel locally. This happens when D-branes are equally distributed at the two fixed points of the orientifold.
- The case $d_\perp = 2$ is particularly attractive because it allows the effective couplings of the brane theory to depend logarithmically on the size of the

transverse space, or equivalently on $M_{\rm p}$, exactly as in the case of softly broken supersymmetry at $M_{\rm s}$. Both higher derivative and higher string loop corrections to the bulk supergravity Lagrangian are expected to be small for slowly (logarithmically) varying moduli. The *classical* equations of motion of the effective 2d supergravity in the transverse space are analogous to the renormalization group equations used to resum large corrections to the effective field theory parameters with appropriate boundary conditions.

4. Supersymmetry breaking and scales hierarchy

When decreasing the string scale, the question of hierarchy of scales, i.e., of why the Planck mass is much bigger than the weak scale, is translated into the question of why there are transverse dimensions much larger than the string scale. For a string scale in the TeV range, from eq. (3), the required hierarchy $R_\perp/l_{\rm s}$ varies from 10^{15} to 10^5, when the number of extra dimensions in the bulk varies from $n=2$ to $n=6$, respectively.

We have seen in the last section that although the string scale is very low, large quantum corrections might arise, depending on the size of the large dimensions transverse to the brane. This is as if the UV cut-off of the effective field theory on the brane is not the string scale but the winding scale $R_\perp M_{\rm s}^2$, dual to the size of the large transverse dimensions and which can be much larger than the string scale. In particular such correction could spoil the nullification of gauge hierarchy that remains the main theoretical motivation of TeV scale strings. Another important issue is to understand the dynamical question on the origin of the hierarchy.

TeV scale strings offer a solution to the technical (at least) aspect of gauge hierarchy without the need of supersymmetry, provided there is no effective propagation of bulk fields in a single transverse dimension, or else closed string tadpoles should cancel locally. The case of $d_\perp=2$ leads to a logarithmic dependence of the effective potential on $R_\perp/l_{\rm s}$ which allows the possible radiative generation of the hierarechy between $R_\perp$ and $l_{\rm s}$ as for no-scale models. Moreover, it is interesting to notice that the ultraviolet behavior of the theory is very similar to the one with soft supersymmetry breaking at $M_{\rm s}\sim$ TeV. It is then natural to ask the question whether there is any motivation left over for supersymmetry or not. This brings us to the problems of the stability of the new hierarchy and of the cosmological constant [8].

In fact, in a non-supersymmetric string theory, the bulk energy density behaves generically as $\Lambda_{\rm bulk}\sim M_{\rm s}^{4+n}$, where n is the number of transverse dimensions which is much larger than the string length. In the type-I context, this induces a cosmological constant on our world-brane which is enhanced by the volume of the transverse space $V_\perp\sim R_\perp^n$. When expressed in terms of the 4d parameters using the mass-relation (3), it is translated to a quadratically dependent contribution on the Planck mass:

$$\Lambda_{\rm brane}\sim M_{\rm s}^{4+n}R_\perp^n\sim M_{\rm s}^2M_{\rm p}^2. \tag{7}$$

This contribution is in fact the analogue of the quadratic divergent term ${\rm Str}\mathcal{M}^2$ in softly broken supersymmetric theories, with $M_{\rm s}$ playing the role of the supersymmetry breaking scale.

The brane energy density (7) is far above the (low) string scale M_s and in general destabilizes the hierarchy that one tries to enforce. One way out is to resort to special models with broken supersymmetry and vanishing or exponentially small cosmological constant [11]. Alternatively, one could conceive a different scenario, with supersymmetry broken primordially on our world-brane maximally, i.e., at the string scale which is of the order of a few TeV. In this case the brane cosmological constant would be, by construction, $\mathcal{O}(M_\mathrm{s}^4)$, while the bulk would only be affected by gravitationally suppressed radiative corrections and thus would be almost supersymmetric [8,12]. In particular, one would expect the gravitino and other soft masses in the bulk to be extremely small, i.e., $O(M_\mathrm{s}^2/M_\mathrm{p})$. In this case, the cosmological constant induced in the bulk would be

$$\Lambda_{\mathrm{bulk}} \sim M_\mathrm{s}^4/R_\perp^n \sim M_\mathrm{s}^{6+n}/M_\mathrm{p}^2, \tag{8}$$

i.e., of order $(10\ \mathrm{MeV})^6$ for $n=2$ and $M_\mathrm{s} \simeq 1$ TeV.

In the absence of gravity, brane supersymmetry breaking can occur in a non-BPS system of D-branes. The simplest examples are based on orientifold projections of type-IIB, in which some of the orientifold 5-planes have opposite charge, requiring an open string sector living on anti-D5 branes in order to cancel the RR (Ramond–Ramond) charge. As a result, supersymmetry is broken on the intersection of D9 and anti-D5 branes that coincides with the world volume of the latter. The simplest construction of this type is a T^4/Z_2 orientifold with a flip of the Ω-projection (world-sheet parity) in the twisted orbifold sector. It turns out that several orientifold models, where tadpole conditions do not admit naive supersymmetric solutions, can be defined by introducing non-supersymmetric open sector containing anti-D-branes. A typical example of this type is the ordinary $Z_2 \times Z_2$ orientifold with discrete torsion.

The resulting models are chiral, anomaly-free, with vanishing RR tadpoles and no tachyons in their spectrum [12]. Supersymmetry is broken at the string scale on a collection of anti-D5 branes while, to lowest order, the closed string bulk and the other branes are supersymmetric. In higher orders, supersymmetry breaking is of course mediated to the remaining sectors, but is suppressed by the size of the transverse space or by the distance from the brane where supersymmetry breaking primarily occurred. The models contain in general uncancelled NS (Neveu–Schwarz) tadpoles reflecting the existence of a tree-level potential for the NS moduli, which is localized on the (non-supersymmetric) world volume of the anti-D5 branes.

As a result, this scenario implies the absence of supersymmetry on our world-brane but its presence in the bulk, a millimeter away! The bulk supergravity is needed to guarantee the stability of gauge hierarchy against large gravitational quantum radiative corrections.

5. D-brane standard model

One of the main questions with such a low string scale is to understand the observed values of the low energy gauge couplings. One possibility is to have the three gauge group factors of the standard model (SM) arising from different collections of coinciding branes. This is unattractive since the three gauge couplings correspond in this case to different arbitrary parameters of the model. A second possibility is to maintain unification by imposing all the SM gauge bosons to arise from the same collection of D-branes. The large difference in the actual values of gauge couplings could then be explained either by introducing power-law running from a few TeV to the weak scale [13], or by an effective logarithmic evolution in the transverse space in the special case of two large dimensions [14]. However, no satisfactory model built along these lines has so far been presented.

Here, we will discuss a third possibility [15], which is alternative to unification but nevertheless maintains the prediction of the weak angle at low energies. Specifically, we consider the strong and electroweak interactions to arise from two different collections of coinciding branes, leading to two different gauge couplings [16]. Assuming that the low energy spectrum of the (non-supersymmetric) SM can be derived by a type-I string vacuum, the normalization of the hypercharge is determined in terms of the two gauge couplings and leads naturally to the right value of $\sin^2\theta_W$ for a string scale of the order of a few TeV. The electroweak gauge symmetry is broken by the vacuum expectation values of two Higgs doublets, which are both necessary in the present context to give masses to all quarks and leptons.

Another issue of this class of models with TeV string scale is to understand proton stability. In the model presented here, this is achieved by the conservation of the baryon number which turns out to be a perturbatively exact global symmetry, remnant of an anomalous $U(1)$ gauge symmetry broken by the Green–Schwarz mechanism. Specifically, the anomaly is canceled by shifting a corresponding axion field that gives mass to the $U(1)$ gauge boson. Moreover, the two extra $U(1)$ gauge groups are anomalous and the associated gauge bosons become massive with masses of the order of the string scale. Their couplings to the standard model fields up to dimension five are fixed by charges and anomalies.

5.1 *Hypercharge embedding and the weak angle*

The gauge group closest to the standard model one can hope to derive from type-I string theory in the above context is $U(3)\times U(2)\times U(1)$. The first factor arises from three coincident 'color' D-branes. An open string with one end on them is a triplet under $SU(3)$ and carries the same $U(1)$ charge for all three components. Thus, the $U(1)$ factor of $U(3)$ has to be identified with *gauged* baryon number. Similarly, $U(2)$ arises from two coincident 'weak' D-branes and the corresponding Abelian factor is identified with *gauged* weak-doublet number. *A priori*, one might expect that $U(3)\times U(2)$ would be the minimal choice. However, it turns out that one cannot give masses to both up- and down-quarks in that case. Therefore, at least one additional $U(1)$ factor corresponding to an extra '$U(1)$' D-brane is necessary in order to accommodate the standard model. In principle this $U(1)$ brane can be chosen to be independent of the other two collections with its own gauge coupling. To improve the predictability of the model, here we choose to put it on top of either the color or the weak D-branes. In either case, the model has two independent gauge couplings g_3 and g_2 corresponding, respectively, to the gauge groups $U(3)$ and $U(2)$. The $U(1)$ gauge coupling g_1 is equal to either g_3 or g_2.

Let us denote by Q_3, Q_2 and Q_1 the three $U(1)$ charges of $U(3)\times U(2)\times U(1)$, in a self explanatory notation. Under $SU(3)\times SU(2)\times U(1)_3\times U(1)_2\times U(1)_1$, the members of a family of quarks and leptons have the following quantum numbers:

$$\begin{aligned}
&Q \quad (\mathbf{3},\mathbf{2};1,w,0)_{1/6},\\
&u^c \quad (\bar{\mathbf{3}},\mathbf{1};-1,0,x)_{-2/3},\\
&d^c \quad (\bar{\mathbf{3}},\mathbf{1};-1,0,y)_{1/3},\\
&L \quad (\mathbf{1},\mathbf{2};0,1,z)_{-1/2},\\
&l^c \quad (\mathbf{1},\mathbf{1};0,0,1)_{1}.
\end{aligned}\tag{9}$$

Here, we normalize all $U(N)$ generators according to $\mathrm{Tr}T^aT^b=\delta^{ab}/2$, and measure the corresponding $U(1)_N$ charges with respect to the coupling $g_N/\sqrt{2N}$, with g_N the $SU(N)$ coupling constant. Thus, the fundamental representation of $SU(N)$ has $U(1)_N$ charge unity. The values of the $U(1)$ charges x,y,z,w will be fixed below so that they lead to the right hypercharges, shown for completeness as subscripts.

The quark doublet Q corresponds necessarily to a massless excitation of an open string with its two ends on the two different collections of branes. The Q_2 charge w can be either $+1$ or -1 depending on whether Q transforms as a $\mathbf{2}$ or a $\bar{\mathbf{2}}$ under $U(2)$. The antiquark u^c corresponds to fluctuations of an open string with one end on the color branes and the other on the $U(1)$ brane for $x=\pm1$, or on other branes in the bulk for $x=0$. Ditto for d^c. Similarly, the lepton doublet L arises from an open string with one end on the weak branes and the other on the $U(1)$ brane for $z=\pm1$, or in the bulk for $z=0$. Finally, l^c corresponds necessarily to an open string with one end on the $U(1)$ brane and the other in the bulk.

The weak hypercharge Y is a linear combination of the three $U(1)$s [17]:

$$Y=c_1Q_1+c_2Q_2+c_3Q_3\,. \tag{10}$$

$c_1=1$ is fixed by the charges of l^c in eq. (9), while for the remaining two coefficients and the unknown charges x,y,z,w, we obtain four possibilities:

$$\begin{aligned}
&c_2=\mp\frac{1}{2},\quad c_3=-\frac{1}{3};\quad x=-1,\quad y=0,\quad z=0/-1,\quad w=\mp1,\\
&c_2=\mp\frac{1}{2},\quad c_3=\frac{2}{3};\quad x=0,\quad y=1,\quad z=0/-1,\quad w=\mp1.
\end{aligned}\tag{11}$$

To compute the weak angle, $\sin^2\theta_W$, we use eq. (10) to find

$$\sin^2\theta_W\equiv\frac{g_Y^2}{g_2^2+g_Y^2}=\frac{1}{1+4c_2^2+2g_2^2/g_1^2+6c_3^2g_2^2/g_3^2}\,, \tag{12}$$

with $g_1=g_2$ or $g_1=g_3$ at the string scale.

We now show that the above prediction agrees with the experimental value for $\sin^2\theta_W$ for a string scale in the region of a few TeV. For this comparison, we use the evolution of gauge couplings from the weak scale M_Z as determined by the one-loop beta functions of the SM with three families of quarks and leptons and one Higgs doublet. In order to compare the theoretical relations for $g_1=g_2$ and $g_1=g_3$ with the experimental value of $\sin^2\theta_W$ at M_s, we plot in figure 1 the corresponding

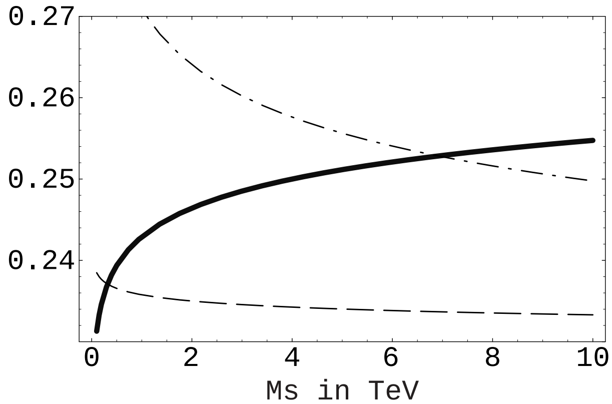

Figure 1. The experimental value of $\sin^2\theta_W$ (thick curve), and the theoretical predictions (12).

curves as functions of $M_{\rm s}$. The solid line is the experimental curve. The dashed line is the plot of the function (12) for $g_1 = g_2$ with $c_3 = -1/3$ while the dotted–dashed line corresponds to $g_1 = g_3$ with $c_3 = 2/3$. The other two possibilities are not shown because they lead to a value of $M_{\rm s}$ which is too high to protect the hierarchy. Thus, the second case, where the $U(1)$ brane is on top of the color branes, is compatible with low energy data for $M_{\rm s} \sim$ 6–8 TeV and $g_{\rm s} \simeq 0.9$. This selects the last two possibilities of charge assignments in eq. (11).

From the general solution (11) and the requirement that the Higgs doublet has hypercharge 1/2, one finds the following possible assignments:

$$c_2 = \mp\frac{1}{2}: \quad H \quad (\mathbf{1},\mathbf{2};0,\pm1,1)_{1/2} \quad H' \quad (\mathbf{1},\mathbf{2};0,\mp1,0)_{1/2}. \tag{13}$$

It is straightforward to check that the allowed (trilinear) Yukawa terms are:

$$\begin{aligned} c_2 = -\frac{1}{2}: &\quad H'Qu^c,\ H^\dagger Ll^c, H^\dagger Qd^c; \\ c_2 = \frac{1}{2}: &\quad H'Qu^c, H'^\dagger Ll^c, H^\dagger Qd^c. \end{aligned} \tag{14}$$

Thus, in each case, two Higgs doublets are necessary and sufficient to give masses to all quarks and leptons. The presence of the second Higgs doublet changes the curves of figure 1 and very little and consequently our previous conclusions about $M_{\rm s}$. Two important comments are in order:

(i) The spectrum we assumed in eq. (9) does not contain right-handed neutrinos on the branes. They could in principle arise from open strings in the bulk [18]. Their interactions with the particles on the branes would then be suppressed by the large volume of the transverse space. More specifically, conservation of the three $U(1)$ charges allows for the following Yukawa couplings involving the right-handed neutrino $\nu_{\rm R}$:

$$c_2 = -\frac{1}{2}: \quad H'L\nu_{\rm L}; \quad c_2 = \frac{1}{2}: \quad HL\nu_{\rm R}. \tag{15}$$

These couplings lead to Dirac-type neutrino masses between $\nu_{\rm L}$ from L and the zero mode of $\nu_{\rm R}$, which is naturally suppressed by the volume of the bulk.

(ii) From eq. (12) and figure 1, we find the ratio of the $SU(2)$ and $SU(3)$ gauge couplings at the string scale to be $\alpha_2/\alpha_3 \sim 0.4$. This ratio can be arranged by an appropriate choice of the relevant moduli. For instance, one may choose the color and $U(1)$ branes to be D3 branes while the weak branes to be D7 branes. Then the ratio of couplings above can be explained by choosing the volume of the four compact dimensions of the seven branes to be $V_4 = 2.5$ in string units. This predicts an interesting spectrum of KK states, different from the naive choices that have appeared hitherto: the only SM particles that have KK descendants are the W bosons as well as the hypercharge gauge boson. However, since the hypercharge is a linear combination of the three $U(1)$s, the massive $U(1)$ gauge bosons do not couple to hypercharge but to doublet number.

5.2 *The fate of* $U(1)$*s and proton stability*

The model under discussion has three $U(1)$ gauge interactions corresponding to the generators Q_1, Q_2, Q_3. From the previous analysis, the hypercharge was shown to be either one of the two linear combinations: $Y = Q_1 \mp \frac{1}{2}Q_2 + \frac{2}{3}Q_3$. It is easy to see that the remaining two $U(1)$ combinations orthogonal to Y are anomalous. In particular there are mixed anomalies with the $SU(2)$ and $SU(3)$ gauge groups of the standard model. These anomalies are canceled by two axions coming from the closed string sector, via the standard Green–Schwarz mechanism [19]. The mixed anomalies with the non-anomalous hypercharge are also canceled by dimension five Chern–Simmons-type of interactions [15]. The presence of such interactions has so far escaped attention in the context of string theory.

An important property of the above Green–Schwarz anomaly cancellation mechanism is that the two $U(1)$ gauge bosons A and A' acquire masses leaving behind the corresponding global symmetries. This is in contrast with what would had happened in the case of an ordinary Higgs mechanism. These global symmetries remain exact to all orders in type-I string perturbation theory around the orientifold vacuum. This follows from the topological nature of Chan–Paton charges in all string amplitudes. On the other hand, one expects non-perturbative violation of global symmetries and consequently exponentially small in the string coupling, as long as the vacuum stays at the orientifold point. Once we move sufficiently far away from it, we expect the violation to become the order of unity. So, as long as we stay at the orientifold point, all the three charges Q_1, Q_2, Q_3 are conserved and since Q_3 is the baryon number, proton stability is guaranteed.

To break the electroweak symmetry, the Higgs doublets in eq. (13) should acquire non-zero VEVs. Since the model is non-supersymmetric, this may be achieved radiatively [20]. From eq. (14), to generate masses for all quarks and leptons, it is necessary for both Higgses to get non-zero VEVs. The baryon number conservation remains intact because both Higgses have vanishing Q_3. However, the linear combination which does not contain Q_3, will be broken spontaneously, as follows from their quantum numbers in eq. (13). This leads to an unwanted massless Goldstone boson of the Peccei–Quinn type. The way out is to break this global symmetry explicitly, by moving away from the orientifold point along the direction of the associated modulus so that baryon number remains conserved. Instanton effects in that case will generate the appropriate symmetry breaking couplings in the potential.

6. Gravity modification and sub-millimeter forces

Besides the spectacular experimental predictions in particle accelerators, string theories with large volume compactifications and/or low string scale predict also possible modifications of gravitation in the sub-millimeter range, which can be tested in 'table-top' experiments that measure gravity at short distances. There are three categories of such predictions:

(i) Deviations from the Newton's law $1/r^2$ behavior to $1/r^{2+n}$, for n extra large transverse dimensions, which can be observable for $n = 2$ dimensions of (sub)-millimeter size. This case is particularly attractive on theoretical grounds because of the logarithmic sensitivity of standard model couplings on the size of transverse space [9], which allows to determine the desired hierarchy [21], but also for phenomenological reasons since the effects in particle colliders are maximally enhanced [22]. Notice also the coincidence of this scale with the possible value of the cosmological constant in the Universe that recent observations seem to support.

(ii) New scalar forces in the sub-millimeter range, motivated by the problem of supersymmetry breaking, and mediated by light scalar fields φ with masses [8,12,23,24]:

$$m_\varphi \simeq \frac{m^2_{\rm SUSY}}{M_{\rm p}} \simeq 10^{-4} - 10^{-6}\ {\rm eV}, \tag{16}$$

for a supersymmetry breaking scale $m_{\rm SUSY} \simeq$ 1–10 TeV. These correspond to Compton wavelengths in the range of 1 mm to 10 μm. $m_{\rm SUSY}$ can be either the KK scale $1/R$ if supersymmetry is broken by compactification [23,24], or the string scale if it is broken 'maximally' on our world brane [8,12]. A model independent and universal attractive scalar force is mediated by the radius modulus (in Planck units)

$$\varphi \equiv \ln R, \tag{17}$$

with R the radius of the longitudinal ($\|$) or transverse ($\perp$) dimension(s). In the former case, the result (16) follows from the behavior of the vacuum energy density $\Lambda \sim 1/R^4_\|$ for large $R_\|$ (up to logarithmic corrections). In the latter case, supersymmetry is broken primarily on the brane only, and thus its transmission to the bulk is gravitationally suppressed, leading to masses (16). Note that in the case of two-dimensional bulk, there may be an enhancement factor of the radion mass by $\ln R_\perp M_{\rm s} \simeq 30$ which decreases its wavelength by roughly an order of magnitude [21].

The coupling of the radius modulus (17) to matter relative to gravity can be easily computed and is given by

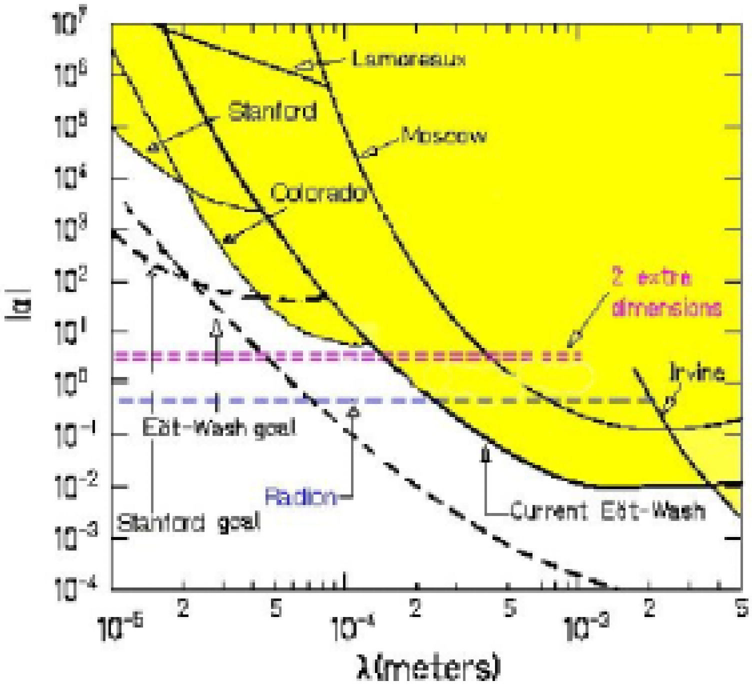

Figure 2. Limits on non-Newtonian forces at short distances, compared to new forces mediated by the graviton in the case of two large extra dimensions, and by the radion.

$$\sqrt{\alpha_\varphi} = \frac{1}{m}\frac{\partial m}{\partial \varphi}; \quad \alpha_\varphi = \begin{cases} \frac{\partial \ln \Lambda_{\rm QCD}}{\partial \ln R} \simeq \frac{1}{3} & \text{for } R_\parallel \\ \frac{n}{n+2} = 1/2 - 3/4 & \text{for } R_\perp \end{cases}, \tag{18}$$

where m denotes a generic physical mass. In the case of a longitudinal radius, $R_{||}$, the coupling arises dominantly through the radius dependence of the QCD gauge coupling [24], while in the case of transverse radius, $R_\perp$, it can be deduced from the rescaling of the metric which changes the string to the Einstein frame and depends on the dimensionality of the bulk n (varying from $\alpha = 1/2$ for $n = 2$ to $\alpha = 3/4$ for $n = 6$) [21]. Moreover, in the case of $n = 2$, there may be again model dependent logarithmic corrections of the order of $(g_{\rm s}/4\pi)\ln RM_{\rm s} \simeq \mathcal{O}(1)$. Such a force can be tested in microgravity experiments and should be contrasted with the change of Newton's law due to the presence of extra dimensions that is observable only for $n = 2$ [10]. In principle there can be other light moduli which couple with even larger strengths. For example the dilaton, whose VEV determines the (logarithm of the) string coupling constant, if it does not acquire large mass from some dynamical supersymmetric mechanism, can lead to a force of strength 2000 times bigger than gravity [25].

(iii) Non-universal repulsive forces much stronger than gravity, mediated by possible Abelian gauge fields in the bulk [26,27]. Such gauge fields may acquire tiny masses of the order of M_s^2/M_p, as in (16), due to brane localized anomalies [27]. Although the corresponding gauge coupling is infinitesimally small, $g_A \sim M_s/M_p \simeq 10^{-16}$, it is still bigger than the gravitational coupling $\sim E/M_p$ for typical energies, E, of the order of the proton mass, and the strength of the new force would be 10^6–10^8 stronger than gravity. This is an interesting region which will be soon explored in microgravity experiments (see figure 2). Note that in this case the supernova constraints impose that there should be at least four large extra dimensions in the bulk [26].

In figure 2 we depict the actual information from previous, present and upcoming experiments [21]. The vertical axis is the strength, α, of the force relative to gravity; the horizontal axis is the Compton wavelength, λ, of the exchanged particle. The solid lines indicate the present limits from the experiments indicated. The excluded regions lie above these solid lines. Measuring gravitational strength forces at such short distances is quite challenging. The most important background is the Van der Walls force which becomes equal to the gravitational force between two atoms when they are about 100 microns apart. Since the Van der Walls force falls off as the seventh power of the distance, it rapidly becomes negligible compared to gravity at distances exceeding 100 μm. The dashed thick lines give the expected sensitivity of the present and upcoming experiments, which will improve the actual limits by roughly two orders of magnitude, while the horizontal dashed lines correspond to the theoretical predictions for the graviton in the case of two large extra dimensions and for the radion in the case of transverse radius.

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