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Anti-screening in magnetically quantized plasmas*

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Abstract. It is shown that in magnetically quantized plasmas, static Debye screening is changed. Furthermore, it is shown that under special circumstances, two electrons in such media may attract each other.

Keywords. Quantized magnetic field; magnetized plasmas.

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1. Introduction

It is known that in an isotropic electron plasma, quantum effects manifest in the longitudinal wave dispersion [1]. They are stipulated by individual quantum oscillation of free electrons with frequency, $\omega_e = \hbar k^2/2m$ where m is the electron mass, $\vec{k}$ is the wave vector and $\hbar$ is the Plank constant divided by 2π. This effect becomes important when the energy of the aforementioned oscillation exceeds electron thermal energy (or Fermi energy in the degenerate case). It is clear that this condition may be valid in a magneto-active plasma. In the presence of an external magnetic field B_0, there is another characteristic frequency related to the individual electron motion, i.e., Larmor frequency $\Omega_e = eB_0/mc$. Therefore, it is expected that quantum effects manifest when Larmor frequency energy exceeds thermal energy (or Fermi energy in the degenerate case). Under this condition, the transverse motion of electrons with respect to the external magnetic field gets also quantized.

Thus, for observing quantum effects in a plasma, the condition $\hbar\Omega_\alpha \gg T_\alpha$ should be satisfied. This can occur only in the presence of a very strong magnetic field.

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Here $\Omega_\alpha = (|e_\alpha| B_0 / m_\alpha c)$ is the Larmor frequency for $\alpha = e, i$ species and T_α is the thermal energy or Fermi energy for the non-degenerate or degenerate plasma, respectively. For electrons, this condition in metals in $B_0 > 10^6$ Gauss and in semiconductors in $B_0 > 10^4$ Gauss and $T_e \sim 10^2$ K is satisfied. For ions we need $B_0 > 10^{10}$ Gauss.

In earlier works [2,3], the general electrodynamic properties of an electron gas in a quantizing magnetic field was developed, when $\hbar\Omega_e \gg T_e$, i.e., when Larmor quanta energy is much larger than particle thermal energy. Also in [4,5] the spectra of electromagnetic and spin waves of a homogeneous and infinite electron gas in a quantizing magnetic field were studied. It was shown that the quantum motions of electrons produce a few new qualitative effects in oscillation spectrum of an electron gas and a few classical branches disappear and new quantum branches, called quantum waves, appear. In this sense there are two regions of energy:

(1) Both electrons and ions or holes have quantum motions, i.e.,

$$\hbar\Omega_e \gg T_e, \quad \hbar\Omega_i \gg T_i. \tag{1}$$

(2) Only electrons have quantum motions, i.e.,

$$\hbar\Omega_i \ll T_i, \quad \hbar\Omega_e \gg T_e. \tag{2}$$

In the present paper we assume that electrons only are magnetized. In treating the quantum waves, we also assume that $(B_0^2/8\pi) \gg \Sigma_\alpha N_\alpha T_\alpha$ where N_α is the α-type particle density and $\alpha = e, i$. This condition allows us to omit the inhomogeneity of the external magnetic field and assume the plasma oscillation to be approximately potential. This condition is easily satisfied in the strong magnetic field.

On the other hand, it is well-known that the electric potential of static charged particles with density $\rho_0(\vec{r})$ in a medium can be written as [6,7]

$$\varphi(r) = \frac{1}{2\pi^2} \int \frac{\mathrm{e}^{i\vec{k}\cdot\vec{r}}}{k^2 \epsilon(0, \vec{k})} \rho_0(\vec{k}) \mathrm{d}\vec{k}, \tag{3}$$

where $\epsilon(0, \vec{k}) = k_i k_j \epsilon_{ij}(0, \vec{k})/k^2$ is the static longitudinal dielectric permittivity of the medium and $\rho_0(\vec{k})$ is the Fourier transformation of the charged particle density.

In the classical consideration, for an isotropic equilibrium plasma with either Maxwell or Fermi distribution of charged particles, we obtain [6,7]

$$\epsilon(0, \vec{k}) = 1 + \frac{1}{k^2 r_{\mathrm{D}}^2}, \tag{4}$$

where r_{D} is the Debye radius. The potential field of a point-like charged particle q in such a plasma has the form [6,7]

$$\varphi(r) = \frac{q}{r} \exp\left(-\frac{r}{r_{\mathrm{D}}}\right). \tag{5}$$

Here, it is assumed that the static field of the charged particles is purely potential and there is no other static magnetic field due to the charged particles. Therefore, the material equation

$$D_i(0,\vec{k}) = \lim_{\omega\to 0} \epsilon_{ij}(\omega,\vec{k})E_j(\omega,\vec{k}) = -\iota\epsilon_{ij}(0,\vec{k})k_j\varphi(\vec{k}), \tag{6}$$

can be reduced to eq. (1) by Maxwell equations.

In anisotropic plasmas, the foregoing assumption and consequently eq. (1) will not be valid. It is well-known that static charged particles, besides the static electric field, can produce the static magnetic field as well in a plasma with anisotropic temperature [6,7]. This fact shows that the form of eq. (1) should be changed.

In the present paper, we show how the form of eq. (1) is changed in an anisotropic plasma in the presence of an external magnetic field. Furthermore, we obtain the static electric and magnetic field, produced by static charged particles in such a plasma. In addition, it is shown that the variation of the electron density brings about the variation of current density and, as a result, produces a magnetic field in a medium. The magnetic field, in turn, affects the distribution of the current and potential and it is not screened directly by plasma electrons. This leads to the asymptotic potential term in the scalar potential showing anti-screening effect. Consequently, we found that the anti-screening effect is caused by an anisotropy due to the presence of a strong magnetic field.

This paper is organized in four sections. In §2 we formulate our problem. In §3, we study a plasma in the presence of an external magnetic field. Finally, a summary and conclusion is presented in §4.

2. Field equations

We begin the study of anisotropic media with non-stationary scalar and vector potentials, $\varphi(\vec{r},t)$ and $\vec{A}(\vec{r},t)$, where [8]

$$\vec{B} = \nabla\times\vec{A}, \quad \vec{E} = -\frac{1}{c}\,\frac{\partial\vec{A}}{\partial t} - \nabla\varphi(\vec{r},t), \tag{7}$$

and after that pass to the static limit. Making use of Fourier transformation with respect to t and $\vec{r}$ and taking into account $\nabla\cdot\vec{A}=0$, instead of eq. (6), we find

$$D_i(0,\vec{k}) = -\iota \lim_{\omega\to 0} \epsilon_{ij}(\omega,\vec{k})\left[k_j\varphi - \frac{\iota\omega}{k^2 c}(\vec{k}\times\vec{B})\right]. \tag{8}$$

Hence, it is clear that if the conductivity of a medium is non-zero in the static limit, then eq. (6) is different from eq. (4). In this case, the induction $\vec{D}(0,\vec{k})$ is determined not only by the potential electric field but also by the magnetic field probably produced by static charged particles in the media.

By Fourier transformation of eq. (7) and writting the electrodynamic equation without any external sources and after that going to the static limit ($\omega=0$), we will have

$$\begin{aligned} k^2\phi &= 4\pi[\rho(0,\vec{k})], \\ k^2\vec{A} &= \frac{4\pi}{c}[\vec{j}(0,\vec{k})], \end{aligned} \tag{9}$$

where $\rho(0,\vec{k})$ and $\vec{j}(0,\vec{k})$ are the static limit of dynamic quantities $\rho(\omega,\vec{k})$ and $\vec{j}(\omega,\vec{k})$ which satisfy the continuity equation.

To close the above equation system, it is necessary to write the material equation connecting $\rho(0,\vec{k})$ and $\vec{j}(0,\vec{k})$, induced in the medium, to the electric and magnetic fields in a spatially infinite medium. Then in the linear approximation from eqs (7) and (9), we can write material equations for $\rho(0,\vec{k}) = \rho(\vec{k})$ and $\vec{j}(0,\vec{k}) = \vec{j}(\vec{k})$:

$$\begin{aligned} \rho &= -k^2\alpha\phi - \frac{1}{c}\tilde{\tau}_i A_i, \\ j_i &= \tau_i\phi + \Pi_{ij}A_j, \end{aligned} \tag{10}$$

where $\alpha, \Pi_{ij}, \tilde{\tau}_i$ and τ_i are static response functions related to the conductivity tensor:

$$\begin{aligned} \alpha(\vec{k}) &= \lim_{\omega\to 0}\frac{\iota}{\omega}\,\frac{k_ik_j}{k^2}\sigma_{ij}(\omega,\vec{k}), \quad \Pi_{ij}(\vec{k}) = \lim_{\omega\to 0}\frac{\iota\omega}{c}\sigma_{ij}(\omega,\vec{k}), \\ \tilde{\tau}_i(\vec{k}) &= \lim_{\omega\to 0}\frac{1}{\iota}k_j\sigma_{ji}(\omega,\vec{k}), \quad \tau_i(\vec{k}) = \lim_{\omega\to 0}\frac{1}{\iota}\sigma_{ij}(\omega,\vec{k})k_j. \end{aligned} \tag{11}$$

Taking into account the material eq. (10), from the system of eqs (9), potentials of static charged particles take the forms

$$\begin{aligned} \phi(\vec{k}) &= \frac{4\pi\rho_0(\vec{k})}{k^2\tilde{\epsilon}(0,\vec{k})}, \\ A_i(\vec{k}) &= \frac{4\pi}{c}\left(k^2\delta_{ij} - \frac{4\pi}{c}\Pi_{ij}(\vec{k})\right)^{-1}\tau_j(\vec{k})\phi(\vec{k}), \end{aligned} \tag{12}$$

where $\tilde{\epsilon}(0,\vec{k})$ is the effective static permittivity

$$\tilde{\epsilon}(0,\vec{k}) = 1 + 4\pi\alpha + \left(\frac{4\pi}{ck}\right)^2\tilde{\tau}_i\left(k^2\delta_{ij} - \frac{4\pi}{c}\Pi_{ij}\right)^{-1}\tau_j. \tag{13}$$

Here, $1 + 4\pi\alpha(\vec{k}) = \epsilon(0,k) = (k_ik_j/k^2)\epsilon_{ij}(0,\vec{k})$ is the static longitudinal dielectric permittivity.

From eqs (11), we can see that if the condition $\tau_i(\vec{k}) = \tilde{\tau}_i(\vec{k}) \equiv 0$ is satisfied in a medium, electrostatics is separated from magnetostatics and they form two independent sub-systems. In the opposite case, static charged particles can generate static magnetic fields and static currents can generate static electric fields. On the other hand, in this case, electric field potential of static charged particles no longer can be described by the longitudinal dielectric permittivity $\epsilon(0,\vec{k})$ only. Moreover, in this case a magnetic field is induced besides the electric field in the medium, i.e., the medium can find magnetic properties. This means that static charged particles, apart from medium polarization produced by induced charged particles, induce current.

It can be shown that for every classical gas with equilibrium distribution function (Maxwell or Fermi), τ_i and $\tilde{\tau}_i$ are equal to zero and, as a result, electrostatics is not related to magnetostatics. Moreover, conductivity, in the static limit, has finite value for a classical equilibrium medium and consequently $\Pi_{ij} = 0$. This means that such a medium does not have magnetic properties and the magnetostatics of such a medium is not different from vacuum's.

It should be noted that the static conductivity $\sigma_{ij}(\omega, \vec{k})$ is different from zero for a classical magnetoactive plasma of charged particles. However, one can simply show that the response functions $\tau_i(\vec{k})$ and $\tilde{\tau}_i(\vec{k})$ for such a classical magnetoactive plasma with an arbitrary isotropic distribution of particles are zero. Consequently, in such a plasma, eqs (1) and (4) are valid.

3. A quantizing magnetic field

In this section, we consider an electron gas or electron plasma in the presence of a very strong magnetic field $\vec{B}_0$. Electromagnetic properties of such a gas were studied in [1–4,9–12]. Making use of the dielectric permittivity tensor of such a gas [2,4,10], we can calculate τ_i and $\tilde{\tau}_i$. It was shown that in such a gas the local change of electron density leads to the change of the magnetic field and vice versa [13]. This fact should appear in the static fields produced by point-like charges, even in equilibrium plasmas. In fact, due to such effects one can estimate the static field around a point-like charge in the presence of a quantizing magnetic field (see for example [13–15]).

We study a non-degenerate electron gas or plasma with Maxwellian distribution in the presence of a quantizing magnetic field. We will see how eq. (1) is changed in the presence of an external quantizing magnetic field. It was shown that in such a gas the local change of electron density leads to the change of the magnetic field and vice versa [13]. This fact should appear in the static fields produced by point-like charged particles, even in equilibrium plasmas.

To study the aforementioned system and find its electric potential, we should obtain its static response functions (11) by making use of the dielectric permittivity tensor of such a gas. It is well-known that the distribution function of a non-degenerate electron gas in the presence of a quantizing magnetic field B_0 along the z-axis has the form [7,16]

$$f_0(\vec{p}) = \frac{n}{(2\pi m)^{3/2} T_e^{1/2} T_\perp} \exp\left(-\frac{p_z^2}{2mT_e} - \frac{p_\perp^2}{2mT_\perp}\right), \tag{14}$$

where

$$T_\perp = \frac{\hbar\Omega_e}{2} \coth\frac{\hbar\Omega_e}{2T_e} \simeq \begin{cases} T_e, & T_e \gg \hbar\Omega_e, \\ \hbar\Omega_e, & T_e \ll \hbar\Omega_e. \end{cases} \tag{15}$$

Here $T_\perp$ plays the role of transverse temperature. It should be noted that the non-degeneracy condition in this case may be written as $E_{\rm F} \ll T_e^{1/3} T_\perp^{2/3}$ where $E_{\rm F}$ is the electron Fermi energy. Generalizing the expression for the longitudinal dielectric permittivity of an electron plasma in the presence of an external magnetic field [2], from eq. (14) in the same way as done in [2,4,10,11] we find the dielectric permittivity

$$\epsilon(\omega, \bar{k}) = 1 + \frac{\Omega_e^2}{k^2 v_{Te}^2}\left\{1 - \sum_{n=1} \frac{\omega A_n(k_\perp^2/\lambda_q^2)}{\omega - n\Omega_e} J_+\left(\frac{\omega - n\Omega_e}{k_z v_{Te}}\right)\right\}, \tag{16}$$

for an electron gas in the presence of a strong quantizing magnetic field when $T_e \ll \hbar\Omega_e$. Here, $A_n(Z) = \exp(-Z)I_n(Z)$ and $I_n(Z)$ is the modified Bessel function; $k_\perp$ is the wave number component in the xy plane; the role of Larmor radius is played by quantum length $\lambda_q = \sqrt{\hbar/m\Omega_e}$; and $J_+(x)$ is the plasma dispersion function where $J_+(x) = x\exp(-x^2/2)\int_{i\infty}^{x} \exp(t^2/2)\mathrm{d}t$ [7].

Also we assume that $kr_\mathrm{D} \ll 1$ and $k_\| \ll mv_{Te}/\hbar$ when $\omega \to 0$ (a non-degenerate Maxwellian electron gas) where $k_\|$ is the wave number component along the z-axis. This means that we restrict our treatment on distances which are very longer than the Debye and quantum radii ($r \gg r_{\mathrm{D}e}$ and $r \gg \hbar/mv_{Te}$; the second limit holds when $r \gg 10^{-8}$ cm). As a result, from eqs (10) and (11) we find static response functions as follows:

$$\begin{aligned}
\alpha &= \frac{1}{4\pi r_{\mathrm{D}e}^2},\\
\tau_i &= \hat{\tau}_i = \frac{\iota\hbar}{2m}\,\frac{k_{\perp i}}{4\pi r_{\mathrm{D}e}^2}\left(\frac{1}{\eta} - \frac{1}{\sinh\eta\cosh\eta}\right),\\
\Pi_{ij} &= \frac{\hbar^2}{8\pi m^2 c r_{\mathrm{D}e}^2}\,\frac{k_{\perp i}k_{\perp j}}{\sinh^2\eta}\left(1 - \frac{\sinh\eta}{\eta}\right),
\end{aligned} \tag{17}$$

where $\eta = \hbar\Omega/2T_e$.

Substituting eq. (17) into eqs (11) and (12), we obtain electric potential $\phi(\vec{r})$ of a point-like charged particle q:

$$\phi(\vec{r}) = \beta\frac{q r_{\mathrm{D}e}^2}{r^3}(1 - 3\cos^2\theta) + \frac{q}{r}\exp\left(-\frac{r}{r_{\mathrm{D}e}}\right),$$

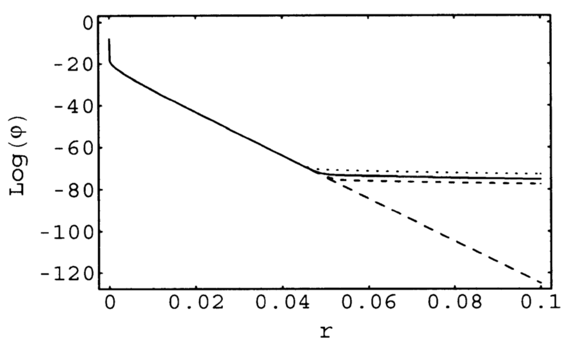

Figure 1. Plot of log φ as a function of r in cm for gas discharge plasmas with $\omega_{pe} \approx 5.5 \times 10^{10}$ s^{-1} and $r_{\mathrm{D}e} \approx 10^{-3}$ cm in the presence of the quantizing magnetic field when $\eta = 1000$ (solid line), $\eta = 10,000$ (dash line) or $\eta = 100$ (dotted line) and is compared with Debye potential given by eq. (5) (long dash line).

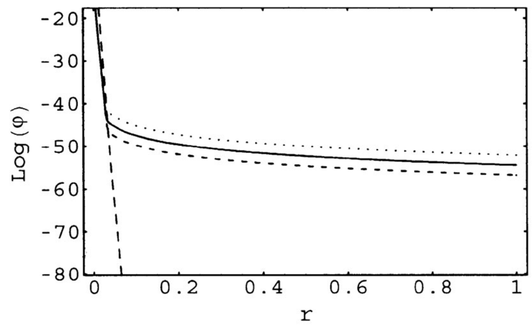

Figure 2. Plot of log φ as a function of r for semiconductor plasmas with $\omega_{pe} \approx 0.5$–1.5×10^{16} s^{-1} and $r_{\mathrm{D}e} \approx 10^{-7}$ cm in the presence of the quantizing magnetic field when $\eta = 1000$ (solid line), $\eta = 10,000$ (dash line) or $\eta = 100$ (dotted line) and is compared with Debye potential given by eq. (5) (long dash line). Here $r = 1$ is 10^{-5} cm.

$$\beta = \left(\frac{\hbar}{2mc}\,\frac{1}{r_{\mathrm{D}e}}\right)^2 \left(\frac{1}{\eta} - \frac{1}{\sinh\eta\cosh\eta}\right), \tag{18}$$

where θ is the angle between the radius $\vec{r}$ and the magnetic field vector $\vec{B}_0$.

In eq. (18) when $r \gg r_{\mathrm{D}e}$, we can neglect the exponential term. In this case, the first term affects the screening characteristics in the given plasma and shows the non-screening effect. Moreover, the quantity β may become large ($\sim$1) when the magnetic instability occurs in a non-degenerate electron gas in the presence of a quantizing magnetic field [10,11]. It should be noted that in eq. (18) if $3\cos^2\theta > 1$, the potential between electrons becomes negative which means electrons can attract each other.

On the other hand, in a degenerate gas, it is expected that the static interaction effects will be stronger for distances larger than Debye radius.

This anti-screening effect is illustrated in figures 1 and 2. Making use of eq. (18), $\log\varphi$ is plotted vs. r in figure 1 for a gas discharge plasma with $\omega_{pe} \approx 5.5 \times 10^{10}$ s^{-1} and $r_{\mathrm{D}e} \approx 10^{-3}$ cm when $\eta = 1000$ (solid line), η =10,000 (dash line) or $\eta = 100$ (dotted line) and is compared with Debye potential given by eq. (5) (long dash line). The same diagrams are plotted in figure 2 for semiconductor plasmas with $\omega_{pe} \approx 0.5$–1.5×10^{16} s^{-1} and $r_{\mathrm{D}e} \approx 10^{-7}$ cm. From these figures, the non-screening role of the first term in eq. (18) caused by the quantizing magnetic field present in the given plasma is clear.

4. Summary

From foregoing results we can conclude that static charged particles, besides the electric fields, may produce the stationary magnetic field in the medium. This, in turn, changes the characteristics of interaction between charged particles in plasmas. This means that, apart from the electric interaction, the magnetic interaction plays an important role in large distances. Specially, the potential field of charged particles in such a plasma is not determined by eq. (1) and it has a non-exponential damping term as well. It should be noted that such a situation occurs in a classical plasma only when it is in a thermodynamic non-equilibrium state. The aforementioned effect disappears in a thermodynamic equilibrium classical plasma.

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