PRAMANA
— journal of
physics

Vol. 62, No. 1
January 2004
pp. 31–36

Atom in a femtosecond bichromatic laser field

K BATRA[1,†], R KUNDLIYA and MAN MOHAN*
Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India
[1]Corresponding address: 1/10355, Lane-1, West Gorakh Park, Shahdara, Delhi 110 032, India
Email: *sneh@del2.vsnl.net.in; manmohan@physics.du.ac.in; †kriti4oct@yahoo.co.in

MS received 16 April 2003; revised 25 August 2003; accepted 30 August 2003

Abstract. The ionization of real hydrogen atom is studied under the action of a two-color pulsed laser field of duration 2.12 fs consisting of a fundamental frequency ω and one of its harmonics. We study the effect of phase difference ϕ between ω and its harmonic on the ionization probability. It is found that ionization can be switched on and off by varying the relative phase ϕ between ω and its harmonic.

Keywords. Bichromatic field; L^2 technique.

PACS Nos 32.80.Fb; 32.80.Qk

Recently with the advancement of laser technology, intense ($I \geq 10^{14}$ W/cm^2) short pulses are becoming standard experimental tools [1]. Atoms exposed to such high intense field undergo ionization [2]. Control of electronic motion in ionization process due to change of phase in a bichromatic field is an important and outstanding problem of current interest [3,4].

In the present work, we have investigated the coherence phase control effect for the two-color ionization of atomic hydrogen. We study the dependence of the probability of atomic ionization on the phase difference ϕ between the laser field components.

One of the major techniques being used to study the dynamics of excitation and ionization is the direct solution of the time-dependent Schrödinger equation for an atom or molecule in a pulsed laser field. This approach has the advantage that no restrictions need to be imposed on the type of laser pulse and that solutions can, in principle, be obtained for all regimes of frequency and intensity. However, it has the disadvantage that it is computationally very intensive. Various approaches have been used to reduce the computer time in these studies. Simple methods of reducing the amount of computation needed to perform these calculations are either to consider only one dimension [5,6] or to use linearly polarized light [7,8] so that the system has axial symmetry in the dipole approximation and the dimensions considered can be reduced to one.

Here in this paper, we have numerically integrated the time-dependent Schrödinger equation for hydrogen atom in a femtosecond bichromatic laser pulse. We have expanded the electron wave function in terms of the eigenfunctions of the unperturbed Hamiltonian which are in turn expanded in terms of a discretized L^2 basis [9–11]. As the size of the basis required for the proper convergence of the results is small, this limited number of L^2 wave functions replaces the infinite target states thereby reducing the computational effort to a great extent. The energy spectrum of the target atom in this basis of size N contains both positive and negative values thereby representing all the bound states and the continuum levels adequately.

The Schrödinger equation in the presence of the dipole field is given by

$$\imath \frac{\partial \psi(r,t)}{\partial t} = H(t)\psi(r,t), \tag{1}$$

where

$$\begin{aligned} &H(t) = H_0 + H_1(t), \\ &H_0 = \text{ Unperturbed Hamiltonian}, \\ &H_1(t) = \vec{E} \cdot \vec{r}. \end{aligned} \tag{2}$$

The electric field vector consists of a fundamental and its pth harmonic, i.e.,

$$E = E_0 f(t) \left[\sin(\omega t) + \frac{1}{4} \sin(p\omega t + \phi) \right], \tag{3}$$

where ϕ is the phase difference between the fundamental and the harmonic, ω is the fundamental laser frequency, E_0 is the maximum field amplitude with corresponding intensity $I_0 = E_0^2$ and $f(t)$ describes shape of the laser pulse. The laser pulse is turned on and off (sine squared ramps) in three optical cycles. In between, the constant laser field lasted for eight optical cycles. The pulse is therefore called 3-8-3 pulse. The frequency of laser oscillation is 1 a.u. and the total duration of the pulse is 14 optical cycles or 2.12 fs.

Let $\psi_{nl}(r)$ be a set of wave functions satisfying

$$H_0 \psi_{nl}(r) = E_{nl} \psi_{nl}(r). \tag{4}$$

We expand $\psi(r,t)$ in a basis of $\psi_{nl}(r)$ as

$$\psi(r,t) = \sum_{nl} b_{nl} \psi_{nl}(r) \exp(-\imath E_{nl} t). \tag{5}$$

Substitution of eq. (5) into eq. (1) gives

$$\imath \frac{\mathrm{d}b_{nl}(t)}{\mathrm{d}t} = \sum_{n'l'} H_{nln'l'}(t) \exp[\imath(E_{nl} - E_{n'l'})t] b_{n'l'}(t) \tag{6}$$

with

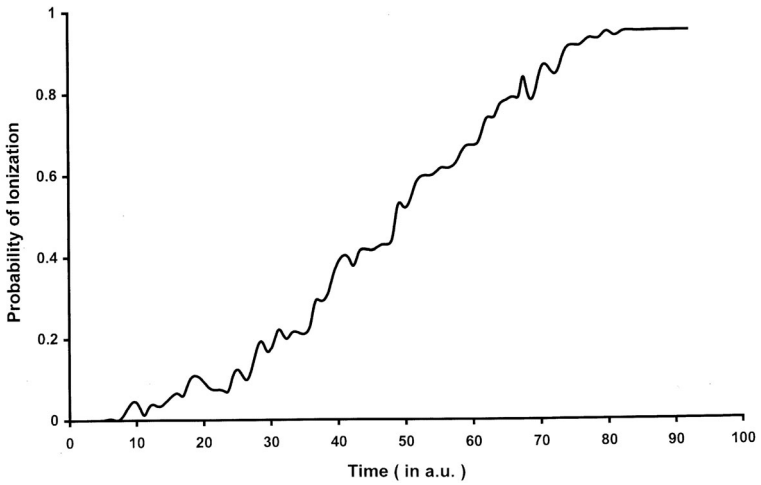

Figure 1. Probability of ionization is plotted as a function of time for a laser with $\omega = 1$ a.u., $I_0 = 1$ a.u. and $\phi = 0$. The laser pulse is turned on and off (sine squared ramps) in three optical cycles. In between, the constant field lasted for eight optical cycles.

$$H_{nln'l'}(t) = \int \psi_{nl}(r)\vec{E}\cdot\vec{r}\psi_{n'l'}(r)\mathrm{d}^3r. \tag{7}$$

This is a set of coupled integro-differential equations which are solved by using Runge–Kutta methods. The probability of ionization after time t is given by

$$P_{\mathrm{ion}}(t) = 1 - P_{\mathrm{bound}}(t) = 1 - \sum_{E<0} |b_{nl}(t)|^2. \tag{8}$$

We diagonalize the target Hamiltonian using a basis, which is discrete and complete, and expand the target wave functions $\psi_{nl}(r)$ in terms of the basis functions. The Laguerre basis taken is defined by

$$\phi_n^l(r) = (2\lambda r)^{l+1}\exp(-\lambda r)L_{n-1}^{2l+2}(2\lambda r), \tag{9}$$

where l is the orbital angular momentum and λ is the basis parameter.

On diagonalizing the target Hamiltonian in the basis (9) of size N, one obtains N energy eigenvalues spanning both positive and negative energies representing the bound and the continuum states of the target. The target wave functions are expanded as

$$\psi_{nl}(r) = \sum_{v=1}^{N} \frac{C_{nl}^v \phi_v^l(r)}{r}. \tag{10}$$

In the present paper, we study the response of real hydrogen atom (rather than the atomic models) in a bichromatic laser field. The proper convergence of the

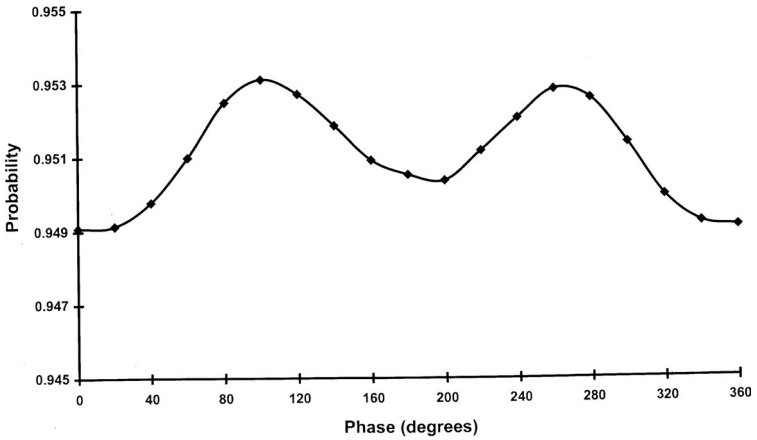

Figure 2. End-of-pulse ionization is plotted as a function of the phase difference ϕ for $I_0 = 1$ a.u. and $\omega = 1$ a.u. for the 1ω–2ω field and for the 3-8-3 pulse.

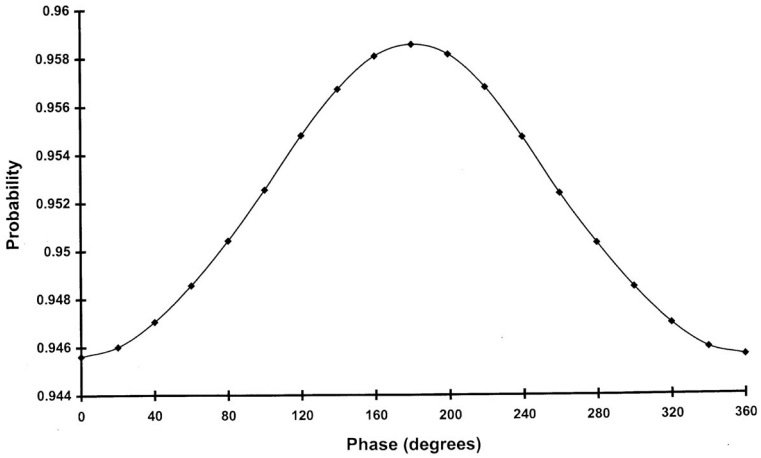

Figure 3. End-of-pulse ionization is plotted as a function of the phase difference ϕ for $I_0 = 1$ a.u. and $\omega = 1$ a.u. for the 1ω–3ω field and for the 3-8-3 pulse.

results depends on the number of angular momentum states taken as well as on the number of basis functions taken per angular momentum. For the present calculations, we have taken $l = 0$ to $l = 4$ and for each l value we have taken 25 basis functions, i.e., we have taken $N = 125$ basis which fairly represents both bound and continuum states giving rise to 125 complex coupled equations. For solving these equations we have used the efficient Runge–Kutta method by assuming that the system is initially in the ground state.

Figure 1 shows the probability of ionization as a function of time for laser intensity =1 a.u. and the fundamental laser frequency =1 a.u. for the 1ω–2ω field. As can

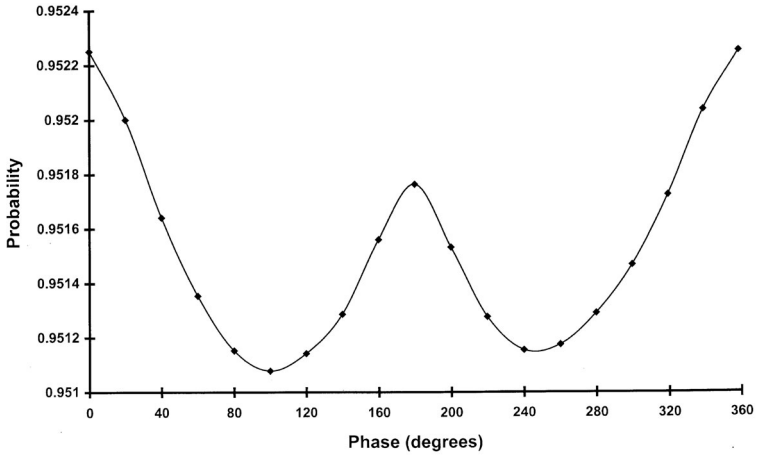

Figure 4. End-of-pulse ionization is plotted as a function of the phase difference ϕ for $I_0 = 1$ a.u. and $\omega = 1$ a.u. for the 1ω–4ω field and for the 3-8-3 pulse.

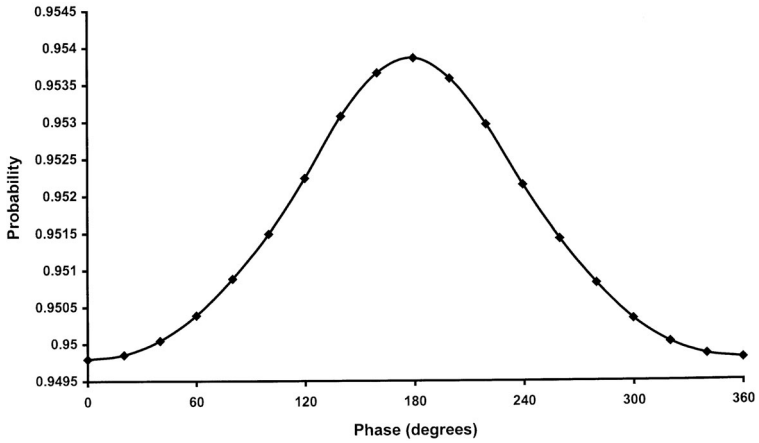

Figure 5. End-of-pulse ionization is plotted as a function of the phase difference ϕ for $I_0 = 1$ a.u. and $\omega = 1$ a.u. for the 1ω–5ω field and for the 3-8-3 pulse.

be seen from the figure, the ionization probability grows almost linearly in time after the pulse is turned on. The slope of the curve corresponds to the ionization rate.

In figures 2–5, we give the variation of the end-of-pulse ionization as a function of the phase difference ϕ between the two components of the laser field for the 1ω–2ω, 1ω–3ω, 1ω–4ω, 1ω–5ω fields respectively. In the above calculations we have taken the laser intensity as 1 a.u. and the fundamental laser frequency as 1 a.u. It is observed that the addition of a small component of the harmonic to the fundamental can enhance or suppress the ionization depending upon its relative

phase with respect to the fundamental. For example, in figure 2, we show the phase dependence of the total ionization probability at constant intensity for the 1ω–2ω field. Changing ϕ alters the maximum amplitude of the electric field, although the average intensity remains the same. Because ionization is a non-linear process, the probability of ionization is greatest for $\phi = \pi/2$ and $\phi = 3\pi/2$ where the maximum instantaneous field is greatest. It is found that the total ionization probability has a period of π when an even harmonic of ω is taken, while it has a period of 2π when an odd harmonic is taken. This observation of ours for the hydrogen atom is also compatible with that of Mercouris and Nicolaides [12] for the helium atom where they have found that for odd harmonics the ionization rate varies linearly with $\cos(\phi)$ while for even harmonics it varies linearly with $\cos(2\phi)$.

Acknowledgements

The authors are thankful to A T Stelbovics for useful discussions about L^2 programs. MM is thankful to UGC, DST and KB is thankful to CSIR India for financial support.

References

[1] T Brabec and F Krauz, *Rev. Mod. Phys.* **72**, 545 (2000)
[2] A D Bandrauk and S Chelkowski, *Current development in atomic, molecular and chemical physics with applications* edited by M Mohan (Kluwer Academic/Plenum Publishers, 2002) pp. 19–30
[3] A Baltuska, Th Udem, M Ulberacker, M Hentschel, E Goulielmakls, Ch Gohie, R Hoizwarth, V S Yakoviev, A Scrinzi, T W Hansch and F Krausz, *Nature (London)* **421**, 611 (2003)
[4] C Figueira de Morrison Faria and M L Du, *Phys. Rev.* **A64**, 23415 (2001)
[5] V C Reed and K Burnett, *Phy. Rev.* **A42**, 3152 (1990)
[6] Q Su, J H Eberly and J Javanainen, *Phys. Rev. Lett.* **64**, 862 (1990)
[7] K C Kulander and B W Shore, *J. Opt. Soc. Am.* **B7**, 502 (1990)
[8] M D Perry, A Szoke and K C Kulander, *Phys. Rev. Lett.* **63**, 1058 (1987)
[9] R Kundliya, V Prasad and Man Mohan, *J. Phys.* **B33**, 5263 (2000)
[10] R Kundliya, K Batra and Man Mohan, *Phys. Rev.* **A64**, 043404 (2001)
[11] R Kundliya, K Batra and Man Mohan, *J. Phys.* **B34**, 4083 (2001)
[12] T Mercouris and C A Nicolaides, *Euro. Phys. J.* **D14**, 241 (2001)