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Dust acoustic shock wave generation due to dust charge variation in a dusty plasma*

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Abstract. In a dusty plasma, the non-adiabaticity of the charge variation on a dust grain surface results in an anomalous dissipation. Analytical investigation shows that this results in a small but finite amplitude dust acoustic (DA) wave propagation which is described by the Korteweg–de Vries–Burger equation. Results of the numerical investigation of the propagation of large-amplitude dust acoustic stationary shock wave are presented here using the complete set of non-linear dust fluid equations coupled with the dust charging equation and Poisson equation. The DA waves are of compressional type showing considerable increase of dust density, which is of significant importance in astrophysical context as it leads to enhanced gravitational attraction considered as a viable process for star formation. The DA shock transition to its far downstream amplitude is oscillatory in nature due to dust charge fluctuations, the oscillation amplitude and shock width depending on the ratio $\omega_{\rm pd}/\nu_{\rm ch}$ and other plasma parameters.

Keywords. Non-adiabatic charge variation; shock wave.

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1. Introduction

The propagation of low-frequency dust acoustic (DA) or dust-ion acoustic (DIA) waves in dusty plasma is strongly influenced by the variation of the dust grain charge $q_{\rm d}$ determined by the grain charging equation in the non-dimensional form $\mathrm{d}q_{\rm d}/\mathrm{d}(t/\tau_{\rm d}) = (\tau_{\rm d}/\tau_{\rm ch})((I_e+I_i)/\nu_{\rm ch})$ where $\tau_{\rm d}$ is the dynamical time scale (: $\omega_{\rm pd}^{-1}/\omega_{\rm pi}^{-1}$ for DA/DIA waves) and $\tau_{\rm ch}(=\nu_{\rm ch}^{-1})$ is the dust charging time. For $\tau_{\rm ch}/\tau_{\rm d} \approx 0$ which applies for DA, we have the approximation $I_e + I_i \approx 0$ giving $q_{\rm d} = q_{\rm d}(\phi)$ as a function of the electrostatic

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potential ϕ. The dust charge varies but the response is instantaneous; such variation is termed adiabatic. For a non-negligible value of $\tau_{\mathrm{ch}}/\tau_{\mathrm{d}}$, there is an extra contribution from q_{d} through $(\tau_{\mathrm{ch}}/\tau_{\mathrm{d}})(\mathrm{d}q_{\mathrm{d}}/\mathrm{d}(t/\tau_{\mathrm{d}}))$, which will be known as non-adiabatic variation [1–3]. This causes an oscillatory decay of the dust charge magnitude and consequently plays the role of dissipative mechanism on the dust fluid by decreasing the driving force $(: q_{\mathrm{d}})$ causing dust motion. Apart from the dispersive effect, this anomalous dissipation tends to strike a balance with the wave breaking convective force and under suitable condition gives rise to generation of shock wave in the dusty plasma described as collisionless shock wave. When the dispersive effect is small and dissipation is strong, the shock transition layer becomes sufficiently thin and plays the role of an electrical double layer with electrostatic potential dropping sharply across thin shock front with consequent acceleration or reflection of charged particles (depending on the algebraic sign of the charge) by the resulting strong electric field. Further, the dust density enhanced by the DA shock wave, which is compressive in nature, may thus help initiate star formation in interstellar space through Jean's instability. The concomitant decrease in the charge q_{d} on the dust surface accentuates the gravitational condensation process by lessening the electrostatic repulsive force between the charged dust grains. Consequently, however large is the initial charge on the grain surface, the possibility of the linear theory predicted [4] saturation of the gravitational condensation instability by electrostatic levitation is annulled in the non-linear theory.

2. Large-amplitude collisionless dust acoustic shock waves

Assume that the dust fluid has a non-zero flow velocity for the upstream. The upstream boundary conditions on the normalized variables are

$$N_{\mathrm{d}} = \frac{n_{\mathrm{d}}}{n_{\mathrm{d}0}} = 1, \quad V = \frac{\upsilon_{\mathrm{d}}}{c_{\mathrm{d}}} = V_{\mathrm{dr}}, \quad \Phi = \frac{e\varphi}{T_e} = 0, \quad Q_{\mathrm{d}} = \frac{q_{\mathrm{d}}}{Z_{\mathrm{d}0}e} = -1. \tag{1}$$

Non-zero dust drift velocity V_{dr} causes modification of the plasma ion current to the dust grain surface by terms $O(V_{\mathrm{dr}}/V_{\mathrm{thi}})$. Neglect such contributions assuming $V_{\mathrm{dr}} = V_{\mathrm{thi}}$. Transforming to the frame of the wave with velocity λ

$$\zeta = \frac{x}{\lambda_{\mathrm{d}}} - \lambda\,\omega_{\mathrm{pd}} t = X - \lambda T \tag{2}$$

and using boundary conditions the equation of continuity yields

$$N_{\mathrm{d}}(V - \lambda) = V_{\mathrm{dr}} - \lambda = u. \tag{3}$$

To introduce the gravitational effect adjoin to the equation of motion of the dust fluid the gravitational force $\sim \partial\psi/\partial x$:

$$\frac{\partial \upsilon_{\mathrm{d}}}{\partial t} + \upsilon_{\mathrm{d}} \frac{\partial \upsilon_{\mathrm{d}}}{\partial x} = -\frac{\partial \psi}{\partial x} - \frac{q_{\mathrm{d}}}{m_{\mathrm{d}}} \frac{\partial \varphi}{\partial x} - \frac{1}{m_{\mathrm{d}} n_{\mathrm{d}}} \frac{\partial p_{\mathrm{d}}}{\partial x}, \tag{4}$$

where the gravitational potential ψ satisfies the Poisson's equation (with Jean's modification for avoiding infinities).

$$\frac{\partial^2 \psi}{\partial x^2} = 4\pi G m_{\mathrm{d}} (n_{\mathrm{d}} - n_{\mathrm{d}0}). \tag{5}$$

Assume that the dust pressure follows the adiabatic law $p_{\mathrm{d}} = \mathrm{const.}(n_{\mathrm{d}})^{\gamma}$. Next, eliminate gravitational potential ψ from the equation of motion and use the integral of the equation of continuity to get rid of υ_{d} to give the following equation for $\Delta N_{\mathrm{d}}(= N_{\mathrm{d}} - 1)$ in terms of the non-dimensional variables $(\delta = n_{i0}/n_{e0}; \sigma = T_i/T_e)$.

$$\left[\frac{u^2}{(1+\Delta N_{\mathrm{d}})^2} - \gamma\sigma_{\mathrm{d}}(1+\Delta N_{\mathrm{d}})^{\gamma-1}\right]\frac{1}{1+\Delta N_{\mathrm{d}}}\frac{\mathrm{d}^2\Delta N_{\mathrm{d}}}{\mathrm{d}\zeta^2}$$
$$= \frac{\omega_{\mathrm{Jd}}^2}{\omega_{\mathrm{pd}}^2}\Delta N_{\mathrm{d}} + \frac{1}{\delta-1}\left[\delta\exp\left(-\frac{\Phi}{\sigma}\right) - \mathrm{e}^{\Phi} - (\delta-1)(1+\Delta Q_{\mathrm{d}})(1+\Delta N_{\mathrm{d}})\right]$$
$$-\frac{1}{\delta-1}\frac{\mathrm{d}\Delta Q_{\mathrm{d}}}{\mathrm{d}\zeta}\frac{\mathrm{d}\Phi}{\mathrm{d}\zeta} + \left[3\frac{u^2}{(1+\Delta N_{\mathrm{d}})^2} + \gamma(\gamma-2)\sigma_{\mathrm{d}}(1+\Delta N_{\mathrm{d}})^{\gamma-1}\right]\left(\frac{1}{1+\Delta N_{\mathrm{d}}}\frac{\mathrm{d}\Delta N_{\mathrm{d}}}{\mathrm{d}\zeta}\right)^2. \quad (6)$$

Poisson's equation Φ for the electrostatic potential Φ is

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}\zeta^2} = -\left[\delta\exp\left(-\frac{\Phi}{\sigma}\right) - \mathrm{e}^{\Phi} - (\delta-1)(1+\Delta Q_{\mathrm{d}})(1+\Delta N_{\mathrm{d}})\right] \quad (7)$$

while the dust charging equation in the wave frame is

$$\frac{\mathrm{d}\Delta Q_{\mathrm{d}}}{\mathrm{d}\zeta} = \frac{(1+\Delta N_{\mathrm{d}})}{\mu}\left[\exp(\Phi - z\Delta Q_{\mathrm{d}}) - \exp\left(-\frac{\Phi}{\sigma}\right)\left(1+\frac{z}{\sigma+z}\Delta Q_{\mathrm{d}}\right)\right]$$
$$\sigma_{\mathrm{d}} = \left(\frac{T_{\mathrm{d0}}}{m_{\mathrm{d}}}\right)\Big/C_{\mathrm{d}}^2, \quad \omega_{\mathrm{Jd}}^2 = 4\pi G m_{\mathrm{d}} n_{\mathrm{d0}}, \quad (8)$$

where

$$z = \frac{Z_{\mathrm{d0}}e^2}{aT_e}; \quad \mu = \frac{\omega_{\mathrm{pd}}}{\nu_{\mathrm{ch}}}\frac{z(1+z+\sigma)}{\sigma+z}u. \quad (9)$$

For $\omega_{\mathrm{Jd}}^2 \neq 0$ the set of equations (6) through (8) possesses no steady state solution except of course the initial state (for upstream state) $\Delta N_{\mathrm{d}} = 0$, $\Delta Q_{\mathrm{d}} = 0$, $\Phi = 0$, $(\mathrm{d}\Delta N_{\mathrm{d}}/\mathrm{d}\zeta) = (\mathrm{d}\Phi/\mathrm{d}\zeta) = 0$. Starting from a small perturbation of the initial state and upon numerical integration of the above system of equations by Runge–Kutta–Fehlberg method of order five, ΔN_{d} is found to exhibit oscillatory increase while $|Q_{\mathrm{d}}|$ continues to decrease and so diminishes the magnitude of the electrostatic levitating force. For sufficiently small values of ω_{Jd}^2, the fluctuation of ΔN_{d} continues over a large distance before it tends to increase monotonically. However, for adiabatic dust charge variation $(q_{\mathrm{d}} = q_{\mathrm{d0}} = Z_{\mathrm{d0}}e; \Delta Q_{\mathrm{d}} = 0)$, the analysis of the fixed point $(\Delta N_{\mathrm{d}} = 0, \Phi = 0, (\mathrm{d}\Delta N_{\mathrm{d}}/\mathrm{d}\zeta) = (\mathrm{d}\Phi/\mathrm{d}\zeta) = 0)$ shows that the traveling wave generated by the perturbation of the initial state does not exhibit Jean's instability if

$$\frac{\omega_{\mathrm{Jd}}^2}{\omega_{\mathrm{pd}}^2} < 1 - (u^2 - \gamma\sigma_{\mathrm{d}})\left[\left(1+\frac{\delta}{\sigma}\right) + \frac{(\delta-1)(1+\sigma)(\sigma+z)}{z\sigma(1+z+\sigma)}\right]. \quad (10)$$

On the other hand, for $\omega_{\mathrm{Jd}}^2 = 0$, there exists steady state (apart from the initial state) defined for each

$$\Delta Q_{\mathrm{d}} = Z_0(-1 < Z_0 < 0) \quad (11)$$

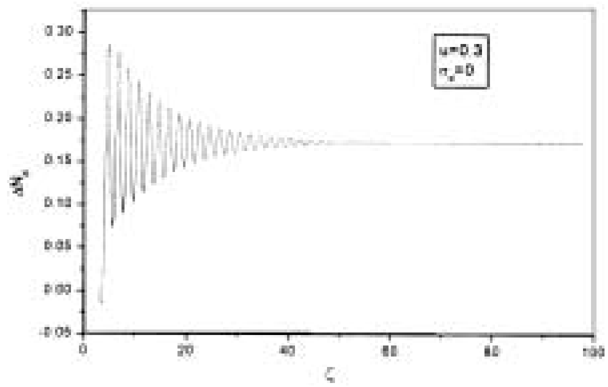

Figure 1.

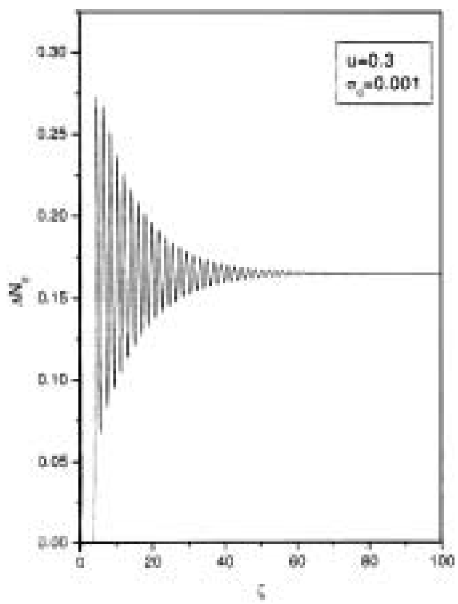

Figure 2.

by

$$\Phi_0 = Y_0 = \frac{\sigma}{\sigma+1}\left[zZ_0 + \ln\left(1+\frac{zZ_0}{z+\sigma}\right)\right], \tag{12}$$

$$1+\Delta N_{\rm d} = 1+X_0 = (\delta\exp(-Y_0/\sigma)-\exp(Y_0))/(\delta-1)(1+Z_0). \tag{13}$$

A steady state DA shock wave generated by perturbing the initial state $(\Delta N_{\rm d}=0, \Delta Q_{\rm d}=0, \Phi=0)$ on the far upstream will reach, on downstream side, the state with dust grain surface charge, electrostatic potential and dust density as given above provided the upstream state is unstable (unstable node or focus) while the downstream state is stable (stable node or focus). From fixed point analysis one is able to show that such shock exist for Mach number $M = V_{\mathrm{dr}}/\lambda$ lies between the two extreme values given by

$$\begin{aligned} \gamma\sigma_{\mathrm{d}} + \frac{\delta - 1}{(\delta/\sigma + 1) + (\delta - 1)(\sigma + 1)(\sigma + z)/\sigma z(1 + \sigma + z)} < (M - 1)^2 \\ < \frac{(\delta \mathrm{e}^{-Y_0/\delta} - \mathrm{e}^{Y_0})^3/(\delta - 1)^3(1 + Z_0)^2}{(1 + Z_0)(\delta \mathrm{e}^{-Y_0/\sigma}/\sigma + \mathrm{e}^{Y_0}) + \frac{1+\sigma}{\sigma}\frac{(\sigma+z)(\delta\mathrm{e}^{-Y_0/\sigma} - \mathrm{e}^{Y_0})}{1+\sigma+z(1+Z_0)}} \\ + \gamma\sigma_{\mathrm{d}}[(\delta \mathrm{e}^{-Y_0/\sigma} - \mathrm{e}^{Y_0})/(\delta - 1)(1 + Z_0)]^3. \end{aligned} \tag{14}$$

A high-dust density state with low-dust density charge $(N_{\mathrm{d}} = 1 + \Delta N_{\mathrm{d}} \gg 1$ and $Q_{\mathrm{d}} = 1 + \Delta Q_{\mathrm{d}} \to 0)$ may result in high Mach number. The dust pressure resists dust fluid compression and hence all other parameters remain unchanged and increase in σ_{d} lowers the asymptotic value of the dust density on the far downstream side (figures 1 and 2).

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