

## Hard X-ray bursts and DD microfusion neutrons from complex plasmas of vacuum discharge\*

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**Abstract.** We create the random complex media of high-power density in low-energy nanosecond vacuum discharges. Hard X-ray emission efficiency, generation of energetic ions ( $\sim 1$  MeV) and neutrons, trapping and releasing of fast ions and/or X-rays from interelectrode aerosol ensembles are the subject of our study. The neutrons from DD microfusion, as well as the modelling of some interstellar nuclear burning due to microexplosive nucleosynthesis are discussed. The value of neutron yield from DD fusion in interelectrode space varies and amounts to  $\sim 10^5 - 10^7 / 4\pi$  per shot under  $\approx 1$  J of total energy deposited to create all discharge processes.

**Keywords.** Hard X-rays; fast ions; microfusion; neutron yield; vacuum discharge; complex plasma; laboratory astrophysics.

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### 1. Introduction

The efficiency of ‘soft’ physics for studying the complex processes in plasma of extreme conditions have been recognised during the last decade. Some rather simple small-scale experiments carried out, provide much complimentary information to available large-scale studies of high-energy density matter physics (cf. [1]). For example, table-top experiments using femtosecond laser irradiation of clouds of clusters [2] have demonstrated how X-rays, fast ions and even neutrons can be generated. Some interesting phenomena have been observed near the cathode area in discharges involving laser irradiation of the cathode [3]. More generally, we may remark that different states of matter in unusual conditions and many intriguing physical problems have been raised by the study of vacuum discharges

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(see [4,5]). Indeed, vacuum discharges alone, even without laser irradiation of electrodes or special target ejection can be used to create foam-like ‘targets’ in high-power density matter studies.

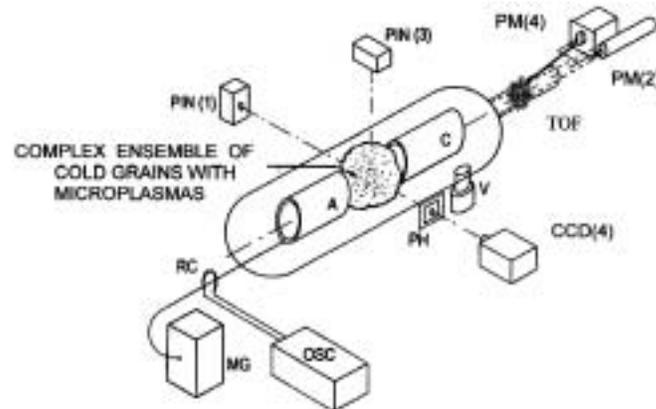
The present work concerns an attempt to incorporate some of the plasma density, coupling (correlation), dusty [6], collective and anomalous effects simultaneously in order to produce high-power density matter using a hollow cathode discharge [7]. We continue to study aerosol random media generated in vacuum discharges [7] with low energy  $\approx 1$  J (ensembles of cold grains possibly with a small fraction of hot microplasmas like from explosive centres, or ‘ectons’ [4]). These ensembles emitting X-rays are created by an intense energy deposition into the cold solid density, very low volume dusty ‘target’ specially nucleated and collected at interelectrode space (clusters, grains, microparticles of different size from anode material). The main anomalous deposition of external energy into the nucleated phase obtained in the vacuum starts immediately after the end of the breakdown (nanosecond-scale electron beams as well as Joule anomalous overheating of the particular solid density micrograins by post breakdown high-density current). The percolation of current through the stochastic media of quasiconductors (grains)–quasidielectrics (vacuum) may create a certain number of microplasmas (from a small part of the grains) with extreme temperatures and densities. Meanwhile, the majority of the clusters and grains remain ‘cold’ being the passive media of ensembles. Subsequently, the process of the explosive destruction of micrograins is accompanied by X-ray radiation (during hydrodynamic expansion, cooling and recombination of the dense hot microplasmas occurred) as well as by the ejection of hot electrons and fast ions. Their energy will be also deposited into the surrounding dusty media. The hot electrons bremsstrahlung on grains will produce additional X-ray photons and sublightning in the whole of dusty ensemble (X-rays ‘ball’), meanwhile, the fast ions stopped by the clusters and micrograins may provide another set of phenomena, including nuclear reactions.

Generally speaking, the arc phase of the discharge may potentially transform the near anode complex plasma into a specific amplifying media, if the gain through the volume exceeds the absorption and loses [7–10]. This feature of random interelectrode ensembles makes it possible to produce lasing in hard X-rays due to the effects of multiple scattering and non-resonant feedback by energy at stochastic resonators predicted earlier for visible spectra [8]. Furthermore, the generation of hot electrons and energetic ions with energies about 0.1–1 MeV or higher (estimated from time of flight (TOF) measurements [7]) is also going on simultaneously with X-rays, just as for laser irradiated clusters [2]. Both electrons and ions ultimately have to reach the speed of sound in hydrodynamically expanding microplasmas, i.e.,  $C_S \approx (\gamma_{ad} Z k T_e / m_i)^{1/2}$  ( $k T_e$  is the electron thermal energy,  $m_i$  and  $Z$  the ion mass and mean charge,  $\gamma_{ad}$  the adiabatic exponent) [11]. Most of the kinetic energy is contained in the ions since their mass is much greater. As a result, the interior of the interelectrode ensembles may resemble something like microreactors due to head-on collisions of fast ions and/or their stopping on the cold aerosol ‘target’. This should allow us to study the nucleosynthesis microevents, for example, DD microfusion with higher efficiency than for laser irradiated deuterium clusters [2].

## **2. Experimental set-up**

Few effects of high local power density were realised specially in our experiments, that allowed us to produce different ensembles of cold grains with a certain fraction of hot

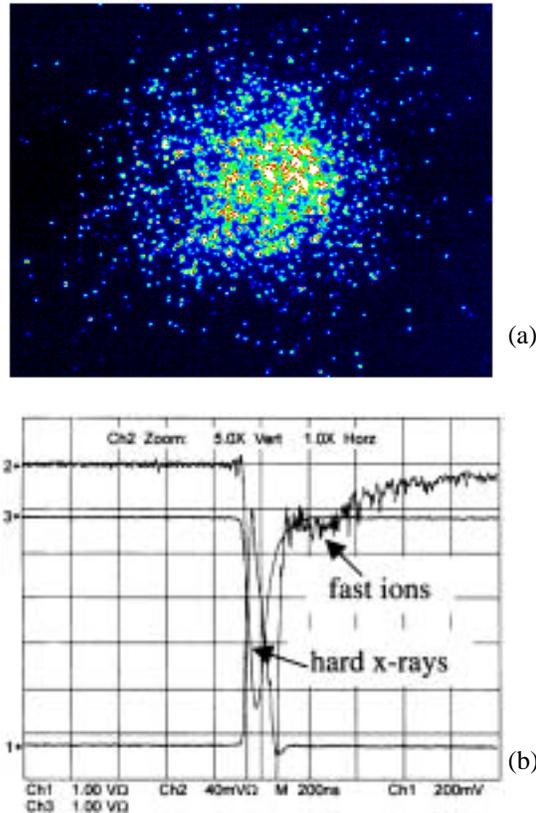
Complex plasmas of vacuum discharge



**Figure 1.** Schematic of the experiment for the generation of aerosol ensembles of cold micrograins with possible fraction of hot microplasmas: MG – Marx generator, R – Rogovskii coil, A and C – anode and cathode, PIN – PIN diodes, CCD – camera, PH – pinhole, PM(4) and PM(2) – photomultipliers, OSC – oscilloscope, TOF – time of flight tube, V – vacuum pump.

microplasmas ( $T \sim 1$  KeV and  $n_e \sim 10^{20} - 10^{22}$  cm $^{-3}$ ) [7]. The source (figure 1) consists of a cylindrical vacuum chamber (with diameter 50 mm) having three windows closed by mylar sheets 70  $\mu$ m thick. This cylinder is connected to a vacuum pump which is able to reach  $10^{-6}$  to  $10^{-7}$  mbar. Two electrodes are included on the cylinder axis: a hollow anode and a hollow cathode having different shapes. The anode is insulated from the ground cylinder by a teflon envelope. The distance between the electrodes can vary by 0.1 mm steps until a maximum of 6–7 mm is reached. The source is included in a coaxial high voltage cable having 50  $\Omega$  impedance; this cable is connected to a four-stage Marx generator delivering a 50 ns pulse of maximum voltage 70 kV in a 50  $\Omega$  load. Three mylar windows allow the X-ray intensity measurement in three perpendicular directions (side-on: right, left and upper) in the plane corresponding to the anode edge. Another mylar window and/or TOF tube allow the end-on measurement through the hollow cathode. Calibrated PIN diodes having a 1 to 2 ns rise-time are used to measure the X-ray yield.

The image of the multi X-ray sources is obtained by means of a pinhole with diameter 0.1 mm bored in a 1mm thick lead screen covered by 100  $\mu$ m thick Al foil. Thus, X-rays just with energies  $\geq 2$  keV are registered by a low noise and sensitive CCD camera, either from the end-on or the side-on right window. The entrance of the camera is protected against spurious light and covered by a scintillator sheet (NE102A) having a rise time of 2 ns. Detailed study of the TOF ion energy spectrum provides different, but important, information on the X-ray random media, including some indirect diagnostics. In particular, these data allow us to estimate independently some parameters of certain hot microplasmas. In fact, under shock wave expansion the microplasma temperature may be related to the front velocity,  $T_{e,i} \sim V_{ion}^2$  (radiation energy and pressure are vanished) [11]. For TOF measured values of ion velocity  $V_{ion} \sim 5 * 10^7 - 10^8$  cm/sec, the values of temperature in overheated dense microplasmas may be estimated as  $T_{e,i} \sim 1-5$  keV. The hard X-ray yield efficiency registered by calibrated PIN diodes and well-reproduced in the vacuum discharges, is about 0.1–0.3%. Time of flight measurements show that hard X-ray



**Figure 2.** (a) The CCD hard X-ray camera image of basic interelectrode complex plasma ensemble; (b) oscillograms from Channels 1–3 for the evolution of hard X-ray in time. Fast ion TOF signal has a maximum on Channel 2 at  $t > 200$  ns.

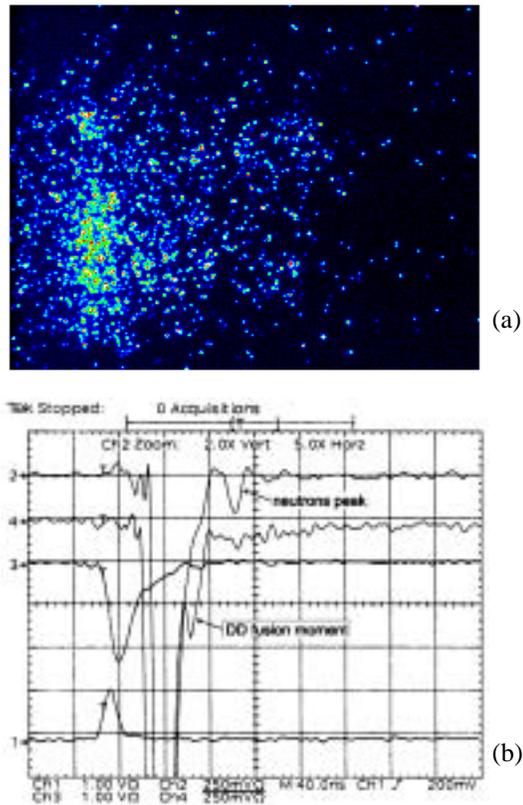
production is accompanied by energetic ions just as for irradiated clusters [2]. This study recognised also that fast ions (as well as X-rays, or both) may be trapped inside the X-ray aerosol ‘ball’ interior [7,12].

The CCD image of a typical transparent X-ray ensemble is shown in figure 2a. The corresponding set of oscillograms (figure 2b) illustrates the evolution in time for different hard X-rays from interelectrode media (bremsstrahlung, reflection of X-rays, hot microplasmas, X-ray ‘points’ at anode edge, etc.). Channels 1 and 3 represent the X-ray intensity (averaged for five shots) from PIN diodes which have the maximum sensitivity at 10–20 keV. Harder X-rays (with energy  $>50$ – $60$  keV) are registered by PM2 covered usually by 2 mm Cu absorber. The curve from Channel 2 at single-shot TOF regime contains two features. The first strong sharp peak is due to very hard X-rays followed by hot electrons from the ensemble (this signal is delayed artificially for  $\approx 35$  ns in comparison with PIN diode signal because of PM2 electronic scheme). The second smooth lower maximum with fluctuating signals is due to fast ion interaction with the target at the end of the TOF tube (this is single-shot TOF measure with the tube of length 30 cm, see figure 1). Last

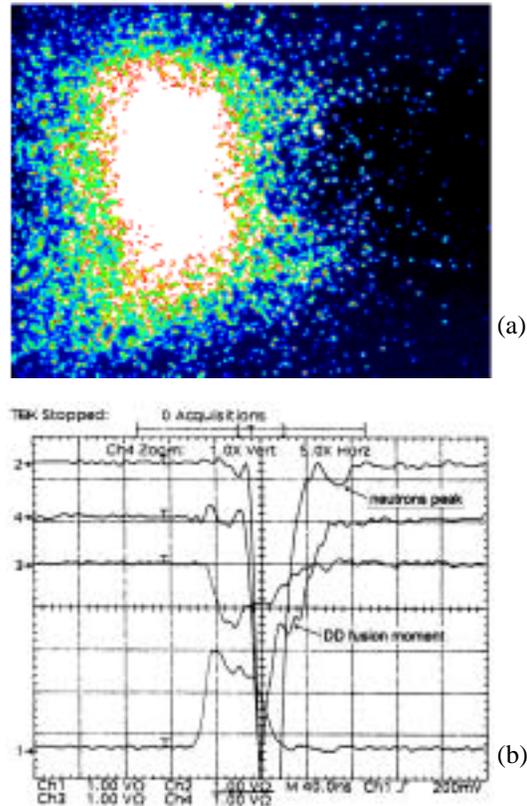
maximum around  $t \approx 300$  ns from the main strong peak of hard X-rays corresponds to Fe ions with energies  $\approx 300$  keV. Note, that in spite of the anode special training before TOF measurements, there may also be some mixture of ions of different elements at different stages of ionization in the total flux of particles. We may remark, that if we underestimate essentially the fast-particle energies and assume, for example, that TOF signal belongs just to protons, their energy will be as minimum as about  $\sim 10\text{--}20$  keV or higher.

### 3. Nuclear microfusion, neutrons yield

Fast-ion trapping and release from X-ray ‘ball’ as micro-reactor allow us to make further reasonable steps like generation of neutrons as well as some modelling of elements of interstellar burning. Neutron generation in X-ray ensembles (see figures 3a and 4a) has been registered using slightly modified experimental set-up in comparison with earlier X-



**Figure 3.** (a) Low intensity and transparent hard X-ray image of delute interelectrode ensemble with DD microfusion events accompanied by moderate neutron yield ( $\sim 10^6/4\pi$ ); (b) oscillograms with neutrons TOF peak (Channel 2). The extra hard X-rays at Channel 4 indicate the beginning of nuclear reactions (sensitivity of Channel 2 is 250 mV; grid is 40 ns).



**Figure 4.** (a) Dense complex plasma ensemble with higher number of DD microfusion events accompanied by higher neutron yield ( $\sim 10^7/4\pi$  per 1 J); (b) oscillograms. The delay of neutron peak corresponds to  $\approx 46.6$  ns/m (2.45 MeV neutrons from DD burning) (sensitivity of Channel 2 is 1V).

ray and fast ion studies [7,12]. Also, we have used specifically modified and deuterated anode from palladium. TOF measurements have been performed with photomultipliers PM4 and PM2 (figure 1), located along the electrode axes at the distances of 45 cm and 90 cm, respectively (Channels 4 and 2 on oscillograms in figures 3b and 4b; screened photomultipliers were covered by scintillators and located under different distances, but within 1 m from the source during all the shots). Beyond the usual hard X-rays (first strong peak at Channel 2), PM2 may indicate the well-reproducible signal (second small peak) with delay about 46.6 ns/m, typical signature for 2.45 MeV neutrons from DD synthesis reactions. Meanwhile, PM4 which is located usually between the source and the PM2 have to register specially the moment of time, when fusion events may take place (if the main X-ray peak itself is not too broad, fortunately, to screen natural extra hard X-rays 'reference point' as illustrated by oscillograms on figures 3b and 4b). The second small peak from PM2 corresponds to the neutron yield from  $D + D = n + He^3$  reaction. The changing of the distances between dusty hard X-ray source and PM2 is followed by the corresponding displacement of the second (neutron) peak. The plates CR39 and PN3 have been used

simultaneously with TOF scheme. Their development shows the variable number of tracks which have to be attributed to neutrons also. Remark, that the energy of deuterons  $E_D$  estimated from the spread of arrival times  $\Delta t \sim d_{SD}(E_D)^{1/2}$  of neutrons at the detector is about 20 keV ( $d_{SD}$  is the distance from source to detector).

The magnitude of particular TOF peaks measured by PMs, the number of tracks observed at CR39 plates and intensities of hard X-ray CCD images registered are in good and reproducible correlation. The number of CR39 tracks is growing with the number of shots also. The value of neutron yield from random interelectrode ‘dusty’ media is variable, and turns out to be about  $10^5-10^7/4\pi$  per shot under  $\approx 1$  J of total energy deposited to create all the discharge processes at a single shot (in assumption of isotropy yield). In fact, one of the typical example from X-ray data base of images of interelectrodes dusty matter with lower neutron yield (and low total X-ray ones) is shown in figure 3a. These conditions of rather low X-ray yield allow us to register more accurately the moment of microfusion events. The ensembles with more saturated X-ray images correspond to higher neutron yields, figure 4a [12]. We may remark, that in terms of [4] perhaps we study here the specifics of ensembles of microparticles, where the fraction of anode explosive centres (anode ‘ectons’) are related to the corresponding elements of microexplosive nucleosynthesis. If the well-known cathode ‘ectons’ produce the packages of electrons [4], from our study we may conclude that some of the anode ‘ectons’ (exploded clusters or microparticles) might be responsible for microexplosive nucleosynthesis due to head-on collisions in stochastic interelectrode current-carrying media.

The efficiency of neutron production from microfusion driven by anode ‘ectons’ at our low-energy discharge (as well as hard X-rays) may be two orders of magnitude higher than for fusion events driven by laser irradiated cluster explosions ( $10^4$  neutrons for 120 mJ of laser energy [2]). Partially, it has been realised due to the increase in the mean size of exploding grains in our experiments up to 0.1–1  $\mu\text{m}$  (instead of  $10^{-3}-10^{-2}$   $\mu\text{m}$  at laser focusing on clusters), and, correspondingly, due to longer confinement time as well as due to fast overheating of grains under anomalously low plasma conductivity. (The possible role of complexes like  $(\text{D}_2\text{O})_n$  still awaits clarification for our experimental conditions also.) Another reason for the higher efficiency of vacuum discharge is the fact that for experiments [2] with fusion driven by Coulomb explosion of clusters, the collisional free path  $l_D$  for ions  $\text{D}^+$  is much longer than the plasma dimension, which is equal to the laser focal diameter  $d_{\text{focal}} = 200$   $\mu\text{m}$  ( $l_D \gg d_{\text{focal}}$ ). In our case, we have the microfusion driven by anode ‘ectons’ that allows us to vary the relation between  $l_D$  and aerosol ensemble radius,  $R_{\text{ball}}$ . It includes even the trapping of all the deuterium fast ions generated inside the ensemble of cold grains ( $l_D < R_{\text{ball}}$ ), which may play the role of the additional cold foam-like dispersive target. As a result, the neutron yields in fact is higher for ensembles with brighter CCD camera images, which have a lower transparency for X-rays and fast ions [12].

#### **4. Concluding remarks**

Aerosol ensembles in interelectrode space in vacuum discharges are analogous partially to foam-like laser target systems suggested earlier for efficient neutron generation [13]. The stochastic nature of X-ray ensembles which may influence the reproducibility of X-rays and neutron yield [7,12] or their anisotropy requires further study. Nevertheless,

hopefully the simple nanosecond scale discharge scheme suggested here to produce X-rays and neutrons just from  $\approx 1$  J of energy deposited in the discharge, is complementary to novel approaches like laser irradiated clusters [2], laser excitation and hydrodynamic dissipation of foam-like targets [13]. Note, that the rather high neutron yield in our vacuum discharges corresponds to predictions from laser foam-like targets models, extrapolated to low-energy limit [13]. Remark the situations when laser-solid target experiments give much higher neutron yields than the results of advanced simulations [14]. Looking to the future, it is interesting to investigate in more detail the neutron yields from complex aerosol plasma ensembles with high symmetry, where X-rays and fast particles will be trapped almost completely as well as possible X-ray pulsating regimes with corresponding neutron yields [12]. We may recall, that potentially high energy density deposition in the limited number of exploding clusters ( $\sim 10^7$ – $10^8$  J/cm<sup>3</sup>) provides correspondingly the nuclear events due to head-on deuterium ion collisions and ions-clusters stopping just as in the rather limited number of microvolumes of interelectrode space or electrode surfaces.

The different mechanisms of ion acceleration and stopping [15] by collective fields of a different nature and different time-space scales need further analysis, including specifics of current-carrying [16] and complex [17] interelectrode plasmas. The proper incorporation of the features of collective modes [18,19] at strong plasma coupling is also required. Meanwhile, the well-known spherical drop model [20] allows us to make some simple estimations. In fact, this model suggests that ambipolar acceleration of ions by electrons in double layers is going on. The energy of fast ions is estimated as  $E_i \approx 4 \cdot 10^{-3} Z E_e^{1/2} n_{0e}^{1/2} r_0$ , where  $E_i$  and  $E_e$  are the mean ion and electron energies in eV,  $Z$  the mean ion charge,  $n_{0e}$  and  $r_0$  the initial electron concentration and drop radius [20]. Usually, this model has been applied to estimate the expansion of the plasma as a whole in a vacuum spark where  $r_0 \sim 1$  cm and rather low values of  $n_{0e}$ . At the same time, this result may be applied also to particular microcondensations in rarefied plasmas or vacuum. For example, for ensembles of cold micrograins with some ‘ectons’ ( $r_0 \sim 10^{-5}$  cm,  $n_{0e} \sim 10^{22}$  cm<sup>-3</sup>,  $E_e \approx 1$  keV and  $Z \approx 20$ – $25$ ) we obtain  $E_i \approx 250$ – $300$  keV (or  $E_i/Z \sim 10$ – $30$  keV), which is in agreement with estimations of deuteron or complex fast ion energies in our experiments. Generally speaking, the combination of different mechanisms of ion acceleration followed by their trapping may provide the available neutron yield. Direct interaction of accelerated deuterons with anode edge as well as the role of explosive cavitation of deuterium from the near surface area of deuterated anode have to be analysed also.

Last but not the least, it looks probable that the phenomena considered here in small scale low-energy vacuum discharge (with rather high rate of energy deposition obtained  $\sim 10^8$  J/s) can be scaled to larger systems, for example, for high energy ( $\sim$  kJ) discharges in the liquids, where the same mechanisms might be partially realised if the rates of energy deposition were similar. Another sort of scaling of our discharge phenomena probably may be attributed to astrophysics. Note, that besides DD microfusion, the accumulation of Si and S peaks have been observed systematically at the external surface of the anode and mylar windows during the discharges. The analysis shows that this accumulation is beyond the potential impurity effect in discharge. Perhaps, it has some relation to the nucleosynthesis processes like oxygen nuclear burning in the interiors of the stars at astrophysics, accompanied by the production of new elements (where Si and S do prevail). In fact, similar velocities for accelerated ions with different masses achieved at our experimental conditions may provoke some laboratory modelling of different ‘ignition temperatures’ for various shells of stars interiors. Generally speaking, the ‘laboratory astrophysics’ based

on interelectrode dusty X-ray media of discharge may include the broad list for experimental modelling: dense astrophysical plasma properties, dust formation and evolutions, self-organisation effects in dusty clouds, behaviour of clusters of stars, X-ray energy transport and X-ray bursts, comet tails, physics of interstellar medium, dynamics of particles in astrophysics like explosive conditions of supernova. For the last one, the explosive nucleosynthesis due to high temperatures associated with passage of the shock wave in large scale space has probably a similar nature like microexplosive centre effects in vacuum discharge at small scale experimental design. Note that proper nature of hard X-ray bursts [21], the feedback in complex ensembles for incoherent laser actions [8], self-organization effects for better ensembles symmetry and manifestations of Casimir-like forces, photonuclear processes, nuclear burning of complex elements are the subjects of further study and discussion [12].

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