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Implications of cosmic string-induced density fluctuations at the quark–hadron transition

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Abstract. We show that cosmic strings moving through the plasma at the time of a first-order quark–hadron transition in the early universe generate baryon inhomogeneities, which can survive till the nucleosynthesis epoch. We find out how these inhomogeneities actually affect the calculated values of the light element abundances. Recently a wealth of observational data from various experiments have helped to reduce the uncertainties in the values of these abundances. Using these we show that it is possible to derive constraints in the presence of cosmic strings during the quark–hadron transition.

Keywords. Cosmic string; structure formation; quark–hadron transition.

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1. Introduction

In a previous paper [1], we had shown that due to the presence of density fluctuations produced by cosmic strings, large scale baryon inhomogeneities at the quark–hadron phase transition may arise. Here, we determine [2] the detailed structure of these baryon inhomogeneities and find that the magnitude and length scale of these inhomogeneities are such that they should survive until the stage of nucleosynthesis, affecting the calculations of abundances of light elements. A comparison with observational data suggests that such baryon inhomogeneities should not have existed at the nucleosynthesis epoch. If this disagreement holds with more accurate observations then it will lead to the conclusion that cosmic string formation scales above 10^{14}–10^{15} GeV may not be consistent with nucleosynthesis and CMBR observations. Alternatively, some other input in our calculation, such as string velocity, should be constrained. Entire discussion of this paper is applicable only when quark–hadron transition is of first order.

2. Density fluctuations arising due to straight cosmic strings

In this section we briefly review the structure of density fluctuations produced by a cosmic string moving through a relativistic fluid. The space-time around a straight cosmic string (along the z-axis) is given by the following metric:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}z^2 - \mathrm{d}r^2 - (1-4G\mu)^2 r^2 \mathrm{d}\psi^2, \tag{1}$$

where μ is the string tension. This metric describes a conical space, with a deficit angle of $8\pi G\mu$. We have used the expressions from ref. [3] to study the density fluctuation in the wake of the moving string which is expressed in terms of fluid and sound four velocities

$$\frac{\delta\rho}{\rho} \simeq \frac{16\pi G\mu u_{\mathrm{f}}^2(1+u_{\mathrm{s}}^2)}{3u_{\mathrm{s}}\sqrt{u_{\mathrm{f}}^2 - u_{\mathrm{s}}^2}}, \qquad \sin\theta_w \simeq \frac{u_{\mathrm{s}}}{u_{\mathrm{f}}}, \tag{2}$$

where $u_{\mathrm{f}} = v_{\mathrm{f}}/\sqrt{1-v_{\mathrm{f}}^2}$ and $u_{\mathrm{s}} = v_{\mathrm{s}}/\sqrt{1-v_{\mathrm{s}}^2}$, with v_{s} being the sound speed. For our purpose we will use a sample value corresponding to string velocity of 0.9 ($v_{\mathrm{s}} = 1/\sqrt{3}$) for which we take $\theta_w \simeq 20°$ and $\delta\rho/\rho \simeq 3\times 10^{-5}$. In the next section, we will study the effects of such density fluctuations on the dynamics of a first-order quark–hadron transition.

3. Effect of string wakes on quark–hadron transition

In our previous paper [1] we have shown that cosmic string-produced density fluctuation can separate the quark gluon plasma phase in the wake region, while the region outside the wake converts to the hadronic phase. Moving interfaces then trap large baryon densities in sheet-like regions which can extend across the entire horizon. The detailed profile of the baryon inhomogeneities resulting from moving interfaces can be determined by calculating the evolution of baryon densities in the QGP phase and in the hadron phase as the transition proceeds. Let us first recall the effect of the expansion of the universe on the dynamics of the phase transition [1,4]. Using Einstein's equations, time evolution of the scale factor $R(t)$ can be written as [4],

$$\frac{\dot{R}(t)}{R(t)} = \left(\frac{8\pi GB}{3}\right)^{1/2}\left[4f + \frac{3}{x-1}\right]^{1/2}, \tag{3}$$

where B is the bag constant and $x = g_q/g_h$ is defined to be the ratio of degrees of freedom between the two phases. Here f denotes the fraction of the volume in the QGP phase.

Now, conservation of the energy–momentum tensor gives

$$\frac{\dot{R}(t)}{R(t)} = -\frac{\dot{f}(x-1)}{3f(x-1)+3}. \tag{4}$$

Equations (3) and (4) along with the following transport rate equations will give the evolution of baryon densities in the quark gluon plasma phase and in the hadron phase. If n_b^q and n_b^h are the net baryon densities in the QGP phase and the hadron phase respectively, then their evolution equations can be written as [4]

$$\dot{n}_b^q = -n_b^q\lambda_q + n_b^h\lambda_h - n_b^q\left[\frac{\dot{V}(t)}{V(t)} + \frac{\dot{f}}{f}\right], \tag{5}$$

$$\dot{n}_b^h = \left[\frac{f}{1-f}\right]\left[-n_b^h\lambda_h + n_b^q\lambda_q + n_b^h\frac{\dot{f}}{f}\right] - n_b^h\frac{\dot{V}(t)}{V(t)}, \tag{6}$$

where dot denotes the rate of change of the baryon density with time and λ_q, λ_h are the characteristic baryon transfer rates [4] from the QGP to hadron phase and hadron to QGP phase respectively. $V(t)$ is the volume of the region under consideration. The term $\dot{V}(t)/V(t)$ arises due to expansion of the universe.

Now in our model, each cosmic string forms wake-like over-density leading the trapping of the QGP region in between two planar interfaces. Collapse of these two interfaces towards each other leads to the concentration of baryons which is the subject of study here. Taking 15 long strings per horizon [5], initial volume relevant for each string is, $V_0 \approx (\frac{1}{15})r_{\rm H}^3$, where $r_{\rm H}(=2t)$ is the size of the horizon at the initial time t_0.

In our model the interface of the QGP region inside the string wake consists of two planar sheets. The area of each interface sheet is $A \sim V(t)^{2/3}$. Following ref. [4], $n_b^q\lambda_q$ is defined as

$$n_b^q\lambda_q = \frac{2A(\mathrm{d}z/\mathrm{d}t)Fn_b^q}{fV(t)} = \frac{2V(t)^{-1/3}(v_z)Fn_b^q}{f}. \tag{7}$$

Here, F is a filter factor which is defined in ref. [4] and $(\mathrm{d}z/\mathrm{d}t) \equiv v_z$ is the speed of the interfaces. Similarly, $n_b^h\lambda_h$ is defined as [4]

$$n_b^h\lambda_h = \left(\frac{1}{3}\right)\left(\frac{n_b^h v_b \Sigma_h}{f}\right)\left(\frac{2A}{V(t)}\right) = \left(\frac{2}{3}\right)\left(\frac{n_b^h v_b \Sigma_h}{f}\right)V(t)^{(-1/3)}. \tag{8}$$

Here, Σ_h is the baryon transmission probability across the phase boundary from the hadron phase to the QGP phase, and v_b is the typical thermal velocity of baryons in the hadron phase.

These two equations along with eqs (3) and (4) are solved simultaneously to get the detailed evolution of n_b^q and n_b^h. Baryon inhomogeneity will be produced as baryons are left behind in the hadronic phase as the interfaces collapse. To study the profile of the resulting baryon over-density after the interfaces collapse away let, $N_q(t)(=n_b^q(t)V(t)f(t))$ be the total baryon number in the QGP region at a particular time t. Taking center of the wake as the origin and considering motion of the interfaces along the z direction, we can write the evolution of z with time as $z(t) = \frac{f(t)}{2}V_0^{(1/3)}\frac{R(t)}{R_0}$.

Similarly, if $\rho(z)$ is the baryon density at position z, then we get the density profile of the baryon inhomogeneities as $\rho(z) = V_0^{(-2/3)}(\frac{R_0}{R(t)})^2(-\frac{\mathrm{d}N_q}{\mathrm{d}z})$.

4. Results

The profiles of baryon overdensity $\rho(z)$ are shown in figure 1. As we will discuss in the next section, relevant values of the over-density, $R \equiv n_b'/n_b$ for us is about 1000. Here n_b' and n_b are baryon densities in the over-dense and the background regions respectively. From the above plots we see that for $\Sigma_h = 10^{-1}$, the thickness of the region inside which $n_b'/n_b > 1000$ is about 5 m for $T_c = 150$ and about 4 m for $T_c = 170$ MeV. For $\Sigma_h = 10^{-3}$, this thickness varies from about 0.5 m to about 4 m as T_c changes from 170 to 150 MeV. As baryon density sharply rises for small z, it is more appropriate to calculate the largest value of the width ($W_{\rm max}$) of the inhomogeneity region within which the average value of baryon density is 1000 times larger than the asymptotic baryon density. We find that for

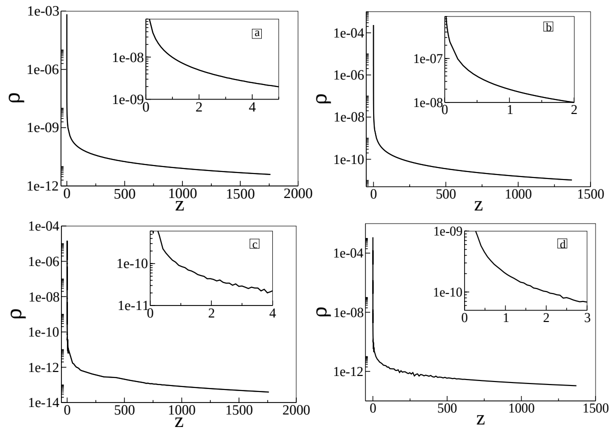

Figure 1. These figures show profiles of baryon inhomogeneities $\rho(z)$ generated by collapsing planar interfaces. Top figures are for $\Sigma_h = 10^{-1}$ and the bottom figures are for $\Sigma_h = 10^{-3}$. (**a**) and (**c**) are for $T_c = 100$ MeV, while (**b**) and (**d**) are for $T_c = 170$ MeV. Here, ρ is in units of fm^{-3} while z is given in meters. Insets show expanded plots of the region where ρ becomes larger than 1000 times the asymptotic value.

$\Sigma_h = 10^{-1}$, $W_{\max}$ is about 100 m for $T_c = 150$ MeV and is about 60 m for $T_c = 170$ MeV. For $\Sigma_h = 10^{-3}$, the values of $W_{\max}$ are about 120 m and 90 m for $T_c = 150$ MeV and 170 MeV respectively.

5. Nucleosynthesis constraints

To study the effects of these resulting inhomogeneities at the nucleosynthesis epoch, we use the results of IBBN calculations developed by Kainulainen *et al* [6]. The results in [6] for the SS geometry were given for a fixed value of $R = 1000$, with the volume fraction of the QGP region f_v varying from about 0.023 to 0.578. To use their results [6], we determine the thickness (and hence the value of f_v) of the baryon inhomogeneity regions from figures 1a–1d within which $R > 1000$. Since the baryon density is sharply peaked inside the over-dense region, we use the largest value of the width $W_{\max}$. Then the resulting value of f_v is about 0.03–0.05 for $\Sigma_h = 10^{-1}$ and 0.045–0.06 for $\Sigma_h = 10^{-3}$.

Next we note that the typical separation (r) between the inhomogeneities is about 1–2 km for our case. This corresponds to about 100–200 km length scale at the nucleosynthesis epoch. Importantly, this is precisely the range of values of r for having optimum effects on nucleosynthesis calculations in ref. [6].

We now apply observational constraints on the abundances of various elements. The most basic constraint is on the abundance of He^4 by mass, denoted by Y. If we take a liberal range of values of $Y = 0.238$–0.244, then using the results of IBBN calculations in ref. [6], we see that for inhomogeneities with optimum value of r, the corresponding value of η is around 4×10^{-10} to about 8×10^{-10}. These values are about a factor 2 larger than the allowed values of η for the case of SBBN.

An independent estimate of η comes from the cosmic microwave background (CMBR) anisotropy measurements. Constraints coming from various experiments seem to constrain

η to be less than 6×10^{-10}. If one takes large estimates of ^{4}He, then IBBN calculations suggest that the corresponding value of η will not be consistent with CMBR measurements. With this, we conclude that the baryon inhomogeneities of the type produced by cosmic strings are not consistent with the combined observations of ^{4}He abundance and CMBR anisotropy measurements. Therefore, some of the parameters of the cosmic string model may have to be constrained to avoid such inhomogeneities at the QCD epoch.

First, if the value of string scale is smaller, say 10^{14} GeV, then resulting excess temperature inside the wake will be even smaller than the nucleation temperature. It is extremely unlikely that in such a case any significant effect will be there on the dynamics of quark–hadron phase transition due to the presence of string wakes. Yet another possibility is that string velocity is either much smaller, or extremely close to the speed of light. In the first situation, resulting value of $\delta\rho/\rho$ is very small, so no effect will be there on the transition. For the second situation, the wake angle will be very small (of order $8\pi G\mu$). So, in this case string wake will cover a very small fraction of the total volume. Thus, resulting baryon inhomogeneities will contribute to negligible baryon fluctuation on the average.

6. Conclusion

We have calculated the detailed structure of the baryon inhomogeneities created by the cosmic string wakes [1]. We find that the magnitude and length scale of these inhomogeneities are such that they survive until the stage of nucleosynthesis, affecting the calculations of abundances of light elements. A comparison with observational data suggests that such baryon inhomogeneities should not have existed at the nucleosynthesis epoch. If this disagreement holds with more detailed calculations and more accurate observations, then it will lead to the conclusion that cosmic string formation scales above a value of about 10^{14}–10^{15} GeV are not consistent with nucleosynthesis and CMBR observations. Alternatively, the average string velocity has to be sufficiently small so that significant density perturbations are never produced at the QCD scale, or strings may move ultra-relativistically so that the resulting wakes are very thin, and trap a negligible amount of baryon number. Finally, all these considerations are valid only when quark–hadron transition is of first order.

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