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Effect of pre-existing baryon inhomogeneities on the dynamics of quark–hadron transition

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Abstract. Baryon number inhomogeneities may be generated during the epoch when the baryon asymmetry of the universe is produced, e.g. at the electroweak phase transition. These lumps will have a lower temperature than the background. Also the value of T_c will be different in these regions. Since a first-order quark–hadron (Q–H) transition is susceptible to small changes in temperature, we investigate the effect of the presence of such baryonic lumps on the dynamics of the Q–H transition. We find that the phase transition is delayed in these lumps for significant overdensities. Consequently, we argue that baryon concentration in these regions grows by the end of the transition. We mention some models which may give rise to such high baryon overdensities before the Q–H transition.

Keywords. Baryon inhomogeneities; quark–hadron transition.

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1. Introduction

Phase transitions may have occurred at different epochs in the early universe and one of their possible consequences is the generation of baryonic inhomogeneities (lumps). If these lumps survive till the nucleosynthesis era, they affect the calculated abundances of the light elements, leading to an inhomogeneous nucleosynthesis scenario. Inhomogeneities generated during the quark–hadron (Q–H) transition may survive until nucleosynthesis, but those generated earlier (through baryogenesis mechanisms) are likely to be wiped out by the effects of neutrino inflation and baryon diffusion.

However, even if these lumps do not survive until nucleosynthesis, they may survive till the Q–H phase transition. A detailed study by Jedamzik and Fuller [1], showed that lumps (e.g those generated at electroweak scale) with large amplitude and certain length-scales are dissipated very little by neutrino inflation. They may survive relatively undamped up to the Q–H transition.

We have shown [2] that if the Q–H transition is of first-order, proceeding via bubble nucleation, then the lumps which are already present during that time affect the dynamics of the phase transition. Since here the baryon lumps have to be present before the QCD epoch, we have considered only those lumps which are generated at earlier stages.

The presence of lumps cause temperature fluctuations throughout the universe. The temperature of the lumps being less than the surroundings, neutrinos passing through them deposit heat in them. The lumps then inflate to achieve pressure equilibrium, thereby reducing their amplitude. Inflation of the lump however decreases its temperature and so the whole process is repeated again until the lump is wiped out. The lumps have a higher baryon number density (n_b) than the background. So the value of the critical temperature (T_c) for the transition will be different there. We show that the combination of the effects of lower T_c and heat deposited by neutrinos is such that the phase transition is delayed in the lumps. The lump thus remains in the quark–gluon plasma (QGP) phase while the outside is in the hadronic phase. As baryon number tends to remain in the QGP phase rather than in the hadronic phase this will lead to an increase in the overdensity of the lump. This will again increase its pressure, thus leading to expansion in the size of the lump. So an initially small lump, will increase in size as the phase transition proceeds. Due to this, pre-existing baryon lumps will grow in size as well as in amplitude as transition proceeds. (It may happen that the bubble nucleation rate itself will be different in the two regions. However, here we have ignored this possibility.)

Section 2 gives the characteristics of these lumps. In §3, we discuss the phase transition and the effect of the lumps on the phase transition, and also mentions some models for generating these. Section 4 gives the conclusions.

2. Characteristics of the baryonic lumps

Small-scale density inhomogeneities in the early universe achieve pressure equilibrium rapidly with their surroundings. Before the Q–H phase transition, the universe is in the QGP phase, and the condition for pressure equilibrium between the outside and the inside of a lump is given by

$$\frac{1}{3}\varepsilon + 9N_q^{-1}\frac{n_b^2}{T^2} = \frac{1}{3}\varepsilon' + 9N_q^{-1}\frac{n_b'^2}{T'^2}. \tag{1}$$

Here $\varepsilon/3$ and $\varepsilon'/3$ are the radiation pressures of the background and lump respectively, due to relativistic particles (with $\varepsilon = g_{\text{eff}}aT^4$ and $g_{\text{eff}} \simeq 51$). For the relevant values of n_b, one can see from eq. (1) that among the two regions, whichever has a higher baryon number (n_b') has a lower T. Replacing $T' = T + \delta T$ in eq. (1) one calculates the temperature difference $\delta T/T$ between the baryon overdense and under-dense regions [1]. For $T \sim 170$ MeV and for $(n_b'/n_b)^2 \gg 1$ it is

$$\Delta T' \equiv \frac{\delta T}{T} = -3\times 10^{-19} \times \left(\frac{n_b'}{n_b}\right)^2. \tag{2}$$

Thus T in the lump is lower than the background. However, the baryon chemical potential μ inside the lump is larger than the corresponding value outside. As T_c depends on μ, the difference in the respective μs will cause a difference in the values of T_c (T_c' and T_c denote the values for the overdense and the background regions respectively). Unless a region supercools below T_c, bubble nucleation will not start. As T_c is different in the two regions, bubble nucleation also starts at different times, unless $\delta T/T$ exactly compensates for the effect of difference between their T_cs. Now, $\mu/T \sim 12\times\eta$, where μ is expressed in terms of $\eta = n_b/s$ with $s = (2\pi^2/45)g_{\text{eff}}T^3$.

For $\mu \neq 0$, using Gibb's criterion for a first-order phase transition, we determine the T_c corresponding to the particular μ from the following equation [3].

$$\frac{37}{90}\pi^2 T^4 + \left(\frac{\mu}{3}\right)^2 T^2 + \frac{1}{2\pi^2}\left(\frac{\mu}{3}\right)^4 - B = \left(\frac{3m^2T^2}{2\pi^2}\sum_{k=1}^{\infty}\frac{K_2(\frac{km}{T})}{k^2}\right) + \frac{2M^4}{3\pi^2}\int_0^1 \frac{u^4\mathrm{d}u}{(1-u^2)^3}[f(u;T,\mu)+f(u;T,-\mu)]. \quad (3)$$

$K_2(km/T)$ is the modified Bessel function of the second kind and $f(u;T,\mu) = \left(\exp\frac{[M/(1-u^2)^{1/2}-\mu]}{T}+1\right)^{-1}$. $M = 940$ MeV, $m = 140$ MeV (nucleon and pion mass).

3. Effect of baryon inhomogeneities on the first-order Q–H phase transition

During a first-order Q–H transition, the universe supercools below T_c and hadronic bubbles are nucleated which expand and convert the QGP to the hadronic phase. The supercooling required depends upon the latent heat L and the surface tension σ of the interface. Using the values, $L \sim 4T_c^4$ and $\sigma \sim 0.015T_c^3$ [4] we get the supercooling $\Delta T_{sc} \equiv ((T_c - T_{sc})/T_c) \sim 10^{-4}$.

After nucleation, the expanding bubbles reheat the universe back to T_c. The time-scale for this is given by $\Delta t_n \sim 10^{-5} t_H$ (where $t_H \sim 10^{-6}$ s is the Hubble time at Q–H transition) and the temperature interval is given by $\Delta T_n \sim 10^{-6}$. After this the bubble nucleation is shut off. The transition then proceeds by the slow expansion of the already nucleated bubbles as the universe expands and cools. For inhomogeneous nucleation of bubbles, some parts of the universe enter the slow combustion phase before other parts, and the latent heat generated prevents nucleation of bubbles elsewhere.

We consider the stage when the background reaches sufficient supercooling for bubble nucleation to start. For bubble nucleation to start in the lumps, they should also achieve sufficient supercooling. We consider the situation when the lumps have, $n_b'/n_b = 10^7$. With $T \sim 170$ MeV, the value of μ, for $n_b'/n_b = 10^7$ comes out to be $\sim$14 MeV. Compared to this, the value of μ outside is $\sim 10^{-6}$ MeV. We have taken the background value of $\eta \sim 7\times 10^{-10}$. The value of T_c at $\mu = 0$ is $\simeq$172 MeV (from eq. (3)).

Using the values of μ corresponding to the values of n_b and n_b', we calculate the corresponding values of the T_cs using eq. (3). As the background supercools to T_{sc}, the region inside the lump also cools by approximately the same factor. The difference between the background T and T' inside the lump is given by eq. (2). With the values of μ given above, and for values close to T_c, we get $(T'-T)/T \equiv \Delta T' \simeq -4\times 10^{-5}$. At the same time T_c' in the lump is given by $(T_c'-T_c)/T_c \equiv \Delta T_c' \simeq -4\times 10^{-5}$. The temperature difference between the lump and the background is the same as the difference in the values of T_cs.

Assuming similar supercooling factor of 10^{-4} for the lump, bubble nucleation cannot start until T drops to a value $T_{sc}' \simeq (1\text{–}4\times 10^{-5} - 10^{-4})\times T_c$. Using the relationship between T_c and T_{sc} we get, $T_{sc}' = (1\text{–}4\times 10^{-5})\times T_{sc}$. Difference between T_{sc}' and T_{sc} is of the same order as between the lump and the background. So, when the background T reaches the value T_{sc}, T in the lump may be close to the corresponding T_{sc}'.

Now, as the lump has lower T, neutrinos constantly pump in heat which temporarily raises T there. Pressure equilibrium then relaxes it, thereby decreasing T in accordance with eq. (1). But, the lump requires some time to regain its pressure equilibrium. When the outside reaches T_{sc} then even allowing for the possibility that the lump could also reach the corresponding T_{sc}', the lump will have to maintain pressure equilibrium with the surroundings. This implies that during the period until pressure equilibrium is re-established, T will be higher there and no nucleation of bubbles is possible.

The time-scale for the lump to attain pressure equilibrium will be $\Delta t_{\rm p} = R/c_{\rm s}$, where $c_{\rm s}$ is the sound velocity and R is its size. Taking $R \sim 1$ cm, with $c_{\rm s} = 1/\sqrt{3}$, $\Delta t_{\rm p} \sim 6 \times 10^{-11}$ s. This is larger than $\Delta t_{\rm n} \sim 10^{-5} t_{\rm H} = 10^{-11}$ s. Since T in the lump is temporarily increased during this time period, the bubble nucleation process may not start there. As we are interested in T being close to $T_{\rm c}$, the relevant $c_{\rm s}$ will be smaller, $\Delta t_{\rm p}$ is then longer.

We obtain the temperature rise due to the heat deposited in the lump for a time duration $\Delta t_{\rm n}$. The heat deposited by the neutrinos in a given volume depends on the ratio of R and λ (neutrino mean free path). As λ at the Q–H transition is a few cms, if we have $R \sim \lambda$, the neutrino radiation is perfectly absorbed [5]. So, $\mathrm{d}E/\mathrm{d}t \simeq 4\pi R^2 \Phi$, where Φ is the net energy flux into the lump. $\Phi = \sum_i \rho_i(T)(\delta T/T)$, $\rho_i(T)$ being the energy density of the neutrinos and $\delta T/T$ is given by eq. (2). Keeping volume constant, temperature rise is given by

$$V g_{\rm eff} a 4 T^3 \frac{\mathrm{d}T}{\mathrm{d}t} = 4\pi R^2 g_\nu a T^4 \times 3 \times 10^{-19} \left(\frac{n_b'}{n_b}\right)^2. \tag{4}$$

$g_\nu = 21/4$ is the neutrino degrees of freedom. Substituting the values we get,

$$\frac{\mathrm{d}T}{T} = 0.4 \times 10^{-19} \left(\frac{n_b'}{n_b}\right)^2 \times \frac{\mathrm{d}t}{R}. \tag{5}$$

For $R = 1$ cm and $\mathrm{d}t = \Delta t_{\rm n} = 10^{-11}$ s we get,

$$\frac{\mathrm{d}T}{T} \sim 10^{-20} \left(\frac{n_b'}{n_b}\right)^2 > 10^{-6} \quad \text{for } \frac{n_b'}{n_b} > 10^7. \tag{6}$$

Thus $\mathrm{d}T/T \sim \Delta T_{\rm n}$, where $\Delta T_{\rm n}$ is the temperature interval in which the nucleation process shuts off. This implies that while outside region reaches its lowest temperature $(\Delta T_{\rm sc} - \Delta T_{\rm n})$ the temperature inside the lump can barely reach down to $T_{\rm sc}'$. So we conclude for $R \simeq 1$ cm, and $n_b'/s \sim 10^{-3}$, no bubble nucleation is possible in the lump as it does not reach its respective nucleation temperature while nucleation starts and finishes off outside. Heat transport by neutrinos further raise the temperature inside the lumps. For smaller values of $c_{\rm s}$, even with smaller lumps these conditions hold. With $R \sim 0.1$ cm and $c_{\rm s} \sim 0.1$, $\Delta t_{\rm p}$ is of the same order and so the region does not achieve pressure equilibrium in the interval $\Delta t_{\rm n}$. For $R < \lambda$, eq. (5) will be modified by a factor of R/λ on the r.h.s. However this does not affect $\mathrm{d}T$ in eq. (6). Since for $T \sim 100$ MeV we have $\lambda \simeq 1$ cm. So we reach the same conclusion as before. For $R > \lambda$, neutrino heat conduction is small [5], but R being large, $\Delta t_{\rm p} > \Delta t_{\rm n}$ and so it cannot reach pressure equilibrium again. So for any size of lump, no nucleation of bubbles is possible during the time duration $\Delta t_{\rm n}$.

The lower limit for lumps to affect the Q–H phase transition (for $\Delta T_{\rm sc} \sim 10^{-4}$) is $n_b'/s \sim 10^{-3}$. Lumps with higher n_b'/s give a larger $\delta T/T$ (eq. (2)). Though $T_{\rm c}$ is smaller, due to larger δT, heat deposition by neutrinos increases. But for the background $\Delta T_{\rm sc}$ and $\Delta T_{\rm n}$ remain the same. So due to larger $\mathrm{d}T$ in eq. (6), bubble nucleation is even more difficult here. Thus the lumps remain in the QGP phase and baryon number concentrates there. So, the amplitude of the lump is not depleted as long as the phase transition continues.

An important part of our model is that the pre-existing lumps have a high n_b/s. Hence volume fraction occupied by these lumps must be small. Otherwise most of the baryon number would be concentrated there. To check whether we can have these and also focus a fairly large percentage of baryon number in them, we estimate the distance of separation l and radius R of these lumps. The volume fraction occupied by the lumps is given by $f_{\rm H} = (R/l)^3$. Thus, $f_{\rm H} n_{\rm H} + (1 - f_{\rm H}) n_{\rm L} = n_b$. $n_{\rm H}$ and $n_{\rm L}$ are the baryon densities in the high and low density regions, while n_b is the total average baryon density. For $R \ll L$, we have $f_{\rm H} \ll 1$. Hence, $n_{\rm L} \simeq n_b - f_{\rm H} n_{\rm H}$. Since, $n_b = 10^{-10}$ s and $n_{\rm H} = 10^{-3}$ s, to have $n_{\rm L} \sim 10^{-10}$ s we must have $f_{\rm H} < 10^{-7}$. So the condition for the lumps become $(R/l)^3 \leq 10^{-7}$. As discussed, $R \sim 0.1$ cm and thus $l \geq 20$ cm. So we can have lumps with radius 0.1 cm, separated by a distance of 20 cm before the Q–H transition. As focussing of baryon number in pre-existing lumps and generation of new overdensities in the inter-bubble spacings go on simultaneously, we compare the length-scales involved. In the latter case [4], the separation of new overdensities formed is a few cms. This is comparable to the length-scale $l \sim 20$ cm of the lumps. Further, with smaller c_s, we get smaller values of R and smaller l, and hence concentration of larger values of baryon number.

All this is possible if such lumps are present before the Q–H transition. We mention some processes which may generate these. However whether they exist or not is not clear. They may be generated by decaying topological defects (such as domain walls), vortons formed from superconducting strings, metastable electroweak strings and evaporation of black holes. Even a first-order electroweak transition seeded by impurities generates inhomogeneities. So it is possible that such lumps may be generated before the Q–H transition.

4. Conclusion

In conclusion, we have studied the effect of pre-existing baryon inhomogeneities on the dynamics of a first-order Q–H phase transition. Our studies show that though the temperature in the lumps is lower than the background, their nucleation temperature is also lower; so due to heat deposition by neutrinos, the process of phase transition is delayed in these lumps. They do not reach sufficient supercooling and the time-scale for them to inflate and achieve pressure equilibrium is of the same order of magnitude as the time required for nucleation to be completely shut off outside. Once the outside has reached the slow combustion phase the latent heat released by the expanding bubbles prevents further nucleation elsewhere, including the lumps. Bubble nucleation thus does not take place in the lumps while the transition gets completed in the outside region. As baryon number tends to stay in QGP phase, the baryon number in the lump increases, and since the size also increases due to neutrino inflation, we get larger lumps at the end of the Q–H transition. This is different from the conventional picture of the evolution of baryonic lumps where the lump increases in size while its amplitude decreases.

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