Low energy $K^+$ scattering on $N = Z$ nuclei

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Abstract. The data for the total cross-section of $K^+$ scattering on various nuclei have been analysed in the Glauber multiple scattering theory. Energy-dependent $K^+$-nucleus optical potential is generated using the forward $K^+$-nucleon scattering amplitude and the nuclear density distribution. Along with this, the calculated total $K^+$-nucleus cross-sections using the effective $K^+$-nucleon cross-section inside the nucleus are also presented.

Keywords. $K^+$-nucleus scattering; nucleon–nucleon correlation; effective $K^+$-nucleon cross-section.

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1. Introduction

Amongst the various hadrons and hyperons, the $K^+$-meson has some peculiar characteristics. It has a larger mean free path ($\sim 5$ fm) in the nucleus indicating that the interaction of $K^+$-meson with the nucleon is weaker as compared to many other hadronic probes. This weaker interaction makes the $K^+$-nucleus optical potential such that the scattering cross-section is more forward peaked. The higher transparency of this potential makes the $K^+$ a good probe to study the behaviour of nucleons inside a nucleus.

Up till now, the measurements for the $K^+$-nucleus cross-sections are very scarce. Experiments, to measure these cross-sections, were carried out at Brookhaven National Laboratory. Whereas the total $K^+–^{12}$C scattering cross-sections had been measured for a large number of beam momenta [1], the data for total cross-sections of $K^+$ scattering on various other nuclei, such as $^6$Li, $^{28}$Si, $^{40}$Ca, were taken for the four beam momenta only [2]. To reduce the systematic error appearing in the experimental set-up, the data for total $K^+$-nucleus scattering cross-section is expressed by the ratio: $R = \frac{\sigma_i(K^+\text{-nucleus})}{(A/2)\sigma_i(K^+\text{-deuteron})}$. In this ratio the systematic errors get cancelled significantly.

Using the conventional nuclear physics mechanism, several authors have attempted to reproduce the data, but their calculations predict the cross-section ratio $R$ smaller than the measured $R$ at higher $K^+$ beam momenta. For example, the calculation due to Siegel et al [3], which incorporates nucleon binding, Pauli blocking, off-shell effects within a consistent theoretical framework, shows an unexplainable discrepancy with the data. Another calculation due to Jiang et al [4], which incorporate fully covariant kinematics, invariant amplitude, Fermi averaging, etc. also fails to reproduce the data.
In order to disentangle the above issue, several other mechanisms have been proposed. Brown et al [5] describe the \( K^+ \)-nucleus scattering to occur due to the exchange of vector meson between the \( K^+ \)-meson and nucleus. Their calculations reproduce the data at higher momenta only. Siegel et al [3] have shown that the increase in nucleon size in the nucleus results in an enhancement for the \( K^+ \)-nucleus cross-section. This calculation is also consistent with the measured spectrum at higher energies. Akulinichev [6] and Jiang and Koltun [7] have incorporated the contribution of \( K^+ \)-scattering on the virtual pion present in the nucleus. This additional contribution lifts the cross-section towards the data. Caillon and Labarsouque [8] have calculated microscopically the \( K^+ \)-nucleus total cross-section incorporating the medium effect on the nucleon and various mesons exchanged between the \( K^+ \)-meson and the target nucleus. Their calculations reproduce the data for all target nuclei except for the \(^6\text{Li}\) nucleus.

In the present work, the total cross-sections for the \( K^+ \)-scattering on various nuclei have been calculated using the Glauber model. In \( \S 2 \), the formalism for this work has been described and in \( \S 3 \), the results and discussions have been presented.

2. Formalism

According to Glauber theory [9] for nuclear reactions, the total cross-section \( \sigma(K^+ A) \) for the scattering of \( K^+ \) on a nucleus \( A \) is given by

\[
\sigma(K^+ A) = 2 \int [1 - \text{Re}\{i \chi(b)\}] \, db, \tag{1}
\]

where \( \chi(b) \) is the phase-shift function for this scattering at the impact parameter \( b \). In the optical model approach, it is written as: \( \chi(b) = -\frac{1}{v} \int V(b, z) \, dz \), where \( v \) is the velocity of the projectile and \( V(b, z) \) denotes the optical potential at \( r(=\sqrt{b^2 + z^2}) \) for \( K^+ \) scattering on the nucleus. Up to second order in scattering amplitude and nuclear density [9], \( V(r) \) can be expressed as

\[
-\frac{1}{v} V(r) = \frac{2\pi A}{k} f_{K^+ N}(0) \varrho(r) + i \left[ \frac{2\pi A}{k} f_{K^+ N}(0) \varrho(r) \right]^2 L_c, \tag{2}
\]

where \( A \) is the mass number of the target nucleus and \( k \) is the momentum of the projectile. \( f_{K^+ N}(0) \) denotes the \( K^+ \)-nucleon scattering amplitude, \( \varrho(r) \) describes the nucleon density distribution (normalised to unity) in the nucleus and \( L_c \) denotes the average two-nucleon correlation length in the target nucleus. According to Fermi–Gas model [9], the form for \( L_c \) is: \( L_c = -3\pi/5k_F \approx -1.4 \text{ fm} \), since the typical value of Fermi momentum \( k_F \) of a nucleon in the nucleus is about 270 MeV/c.

3. Results and discussion

In the present study the total cross-sections \( \sigma(K^+ A) \) for the \( K^+ \)-meson scattering on various nuclei have been calculated. To evaluate \( \sigma(K^+ A) \) of eq. (1), one is required to calculate the optical potential \( V(b, z) \) of eq. (2) for the \( K^+ \)-nucleus scattering. The ingredients required to generate \( V(b, z) \) are the forward \( K^+ \)-nucleon scattering amplitude \( f_{K^+ N}(0) \) and
Low energy $K^+$ scattering on $N = Z$ nuclei

Figure 1. The calculated cross-section ratios $R$ for $K^+$ scattering on various nuclei are compared with the corresponding data. The dashed and solid curves are due to the nucleon–nucleon correlation length $L_c$ taken equal to 0 and $-1.4$ fm respectively in the optical potential (see eq. (2) in the text).

As mentioned earlier, there are significant systematic errors in the measured total cross-sections $\sigma_t(K^+A)$ for the $K^+$-nucleus scattering. Therefore, these systematic uncertainties are eliminated in the ratio with the deuteron results, i.e., $R = (\sigma_t(K^+A)/A)\sigma_t(K^+d))$. The calculated $R$ values for the $K^+$ scattering on $^6$Li, $^{12}$C, $^{28}$Si and $^{40}$Ca are compared in figure 1 with the data [1,2]. In this figure, the calculated results are shown for the correlation length $L_c$, appearing in $V(r)$ of eq. (2), taken equal to 0 and $-1.4$ fm. The latter value of $L_c$ has been obtained in the Fermi–Gas model for the nucleus [9]. The calculated results corresponding to $L_c = 0$ fm are shown by the dashed lines. These results reproduce the measured spectra at lower momenta for all nuclei except the $^6$Li nucleus. The calculated results corresponding to $L_c = -1.4$ fm are shown by solid lines. These results show that the incorporation of nucleon–nucleon correlation improves the agreement between the calculated results and the data in the higher momentum region.

The variation of the measured cross-section ratio $R$ with the momentum, however, is not properly reproduced. In order to improve this feature, one has to modify the $K^+$-nucleon

the nucleon density distribution of the nucleus $\varrho(r)$. The energy dependent values for $f_{K^+N}(0)$ have been obtained from ref. [10], and the form of $\varrho(r)$ is taken to be the same as that extracted from the electron scattering on various nuclei [11].
Swapan Das and Arun K Jain

Figure 2. The solid curves show the calculated cross-section ratios \( R \) for \( K^+ \) scattering on various nuclei, where the effective \( K^+ \)-nucleon cross-section \( \sigma_{t}^{\text{eff}}(K^+N) \) in eq. (3) has been used to generate the \( K^+ \)-Li optical potential (see eq. (2) in the text). The comparison of the calculations with the data is also shown.

cross-section in the nucleus other than the deuteron. The nucleons in the deuteron behave almost freely unlike those in the nucleus, since they spend much more time outside the range of their strong interaction. In this study, the \( K^+ \)-nucleon cross-section in the nucleus has been modified as follows:

\[
\sigma_{t}^{\text{eff}}(K^+N) = \sigma_{t}(K^+N) \left[ 1 + x \frac{P - P_0}{P_0} \right],
\]

where \( x = 0.15 \), \( P_0 = 0.7 \) GeV/c, and \( P \) is the momentum of the \( K^+ \)-meson. \( \sigma_{t}(K^+N) \) and \( \sigma_{t}^{\text{eff}}(K^+N) \) denote the free and the in-medium effective \( K^+ \)-nucleon cross-section respectively.

The effective cross-section \( \sigma_{t}^{\text{eff}}(K^+N) \) of eq. (3) modifies the imaginary part of the \( K^+ \)-nucleon scattering amplitude. Therefore, this modification, as shown in eq. (2), generates an effective \( K^+ \)-nucleus optical potential. Using this effective potential, the total cross-sections for the \( K^+ \) scattering on various nuclei have been calculated. In figure 2, the calculated cross-section ratios \( R \) are shown by solid lines. In this figure, it has been shown that the present calculations reproduce the measured distributions very well.
4. Conclusions

The calculated cross-section ratio, using the $K^+$-nucleus optical potential linear in $K^+$-nucleon scattering amplitude and nuclear density distribution, reproduces better the data in the region of lower beam momentum ($<0.6$ GeV/c). At higher beam momentum region, calculated results fall below the measured distributions. The incorporation of the nucleon–nucleon correlation in the $K^+$-nucleus optical potential lifts the calculated cross-sections throughout the spectrum. Therefore, at higher momentum ($\geq 0.6$ GeV/c) the calculated results have better agreement with the data. The calculated results, using the effective $K^+$-nucleon total cross-section in the $K^+$-nucleus optical potential, reproduce the measured distributions reasonably well.

References