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Nuclear effects in the structure functions

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Abstract. By using a relativistic framework and accurate nuclear spectral function the structure functions F_{2A} and F_{3A} of deep inelastic charged lepton and neutrino scattering are calculated in nuclei and results are presented.

Keywords. Structure functions; self-energy; parton distribution function.

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1. Introduction

The deep inelastic scattering (DIS) experiments done with charged lepton and neutrino beams provide information about the structure functions of the nucleons [1,2]. Most of these experiments are done on nuclei. The nuclear effects modify the nucleon structure functions. In this paper we report a calculation of the modification of the nucleon structure functions F_2 and F_3 in nuclei. In this paper a relativistic many-body approach developed for calculating the nucleon and meson cloud contribution earlier for electron scattering has been applied to neutrino scattering.

The differential cross-section for $\nu(\bar{\nu})$ scattering is given by

$$\frac{\mathrm{d}^2\sigma^{\nu(\bar{\nu})}}{\mathrm{d}\Omega'\mathrm{d}E'} = \frac{G^2}{(2\pi)^2}\,\frac{|k'|}{|k|}\,\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 [L^{\alpha\beta} \pm L_5^{\alpha\beta}]\,W_{\alpha\beta}^{\nu}\,, \tag{1}$$

where the lepton tensors, $L^{\alpha\beta}$ and $L_5^{\alpha\beta}$, are given by

$$L^{\alpha\beta} = k^{\alpha}k'^{\beta} + k^{\beta}k'^{\alpha} - (kk')g^{\alpha\beta}, \quad L_5^{\alpha\beta} = -i\varepsilon^{\alpha\beta\rho\sigma}k_{\rho}k'_{\sigma} \tag{2}$$

and the hadronic tensor $W_{\alpha\beta}^{\nu}$ is defined as

$$W_{\alpha\beta}^{\nu} = \frac{1}{2\pi}\bar{\sum_{s_p}}\sum_X\sum_{s_i}\prod_{i=1}^{N}\int\frac{\mathrm{d}^3p_i'}{(2\pi)^3}\prod_{l\in f}\left(\frac{2M_l'}{2E_l'}\right)\prod_{j\in b}\left(\frac{1}{2\omega_j'}\right) \times\langle X|J_{\alpha}|N\rangle\langle X|J_{\beta}|N\rangle^*(2\pi)^4\delta^4\left(p+q-\sum_{i=1}^{N}p_i'\right), \tag{3}$$

where q is the momentum of the intermediate vector boson, s_p the spin of the nucleon and s_i the spin of the fermions in X.

The usual convention is to express the hadronic tensor as [3]

$$W_{\alpha\beta}^{\nu(\bar{\nu})} = \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^{\nu(\bar{\nu})} + \frac{1}{M^2}\left(p_\alpha - \frac{pq}{q^2}\, q_\alpha\right)\left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W_2^{\nu(\bar{\nu})}$$
$$-(+)\frac{i}{2M^2}\varepsilon_{\alpha\beta\rho\sigma}p^\rho q^\sigma W_3^{\nu(\bar{\nu})} \tag{4}$$

where $W_i^{\nu(\bar{\nu})}$ are the structure functions, which depend on the scalars q^2 and pq. Using Bjorken variables x and y defined as [3]

$$\nu = \frac{pq}{M}, \quad x = \frac{Q^2}{2M\nu}, \quad y = \frac{\nu}{E_\nu(k)}\,, \tag{5}$$

the differential cross-section is written as [3]

$$\frac{\mathrm{d}^2\sigma^{\nu(\bar{\nu})}}{\mathrm{d}x\mathrm{d}y} = \frac{G^2 M E_\nu(k)}{\pi}$$
$$\times\left\{xy^2 F_1^{\nu(\bar{\nu})}(x) + \left(1 - y - \frac{xyM}{2E_\nu(k)}\right) F_2^{\nu(\bar{\nu})}(x) \pm xy(1-y/2)F_3^{\nu(\bar{\nu})}(x)\right\}. \tag{6}$$

$F_i^{\nu(\bar{\nu})}(x)$ are dimensionless structure functions defined as

$$F_1^{\nu(\bar{\nu})} = MW_1^{\nu(\bar{\nu})}, \quad F_2^{\nu(\bar{\nu})} = \nu W_2^{\nu(\bar{\nu})}, \quad F_3^{\nu(\bar{\nu})} = \nu W_3^{\nu(\bar{\nu})}\,. \tag{7}$$

In the quark parton model, the structure functions are expressed in terms of the quark distributions, $q(x)$ and $\bar{q}(x)$. Using Callan-Gross relation, these structure functions are given as

$$2xF_{1N}^{\nu} = 2xF_{1N}^{\bar{\nu}} = F_{2N}^{\nu} = F_{2N}^{\bar{\nu}} = x(q(x) + \bar{q}(x)),$$

$$F_{3N}^{\nu} = q(x) - \bar{q}(x) + 2s(x) - 2c(x),\;\; F_{3N}^{\bar{\nu}} = q(x) - \bar{q}(x) - 2s(x) + 2c(x),$$

$$F_3(x) = \frac{1}{2}[F_{3N}^{\nu}(x) + F_{3N}^{\bar{\nu}}(x)] = q(x) - \bar{q}(x) = u_\nu(x) + d_\nu(x)\,. \tag{8}$$

Here

$$q(x) = u(x) + d(x) + s(x) + c(x), \quad \bar{q}(x) = \bar{u}(x) + \bar{d}(x) + \bar{s}(x) + \bar{c}(x)\,. \tag{9}$$

The structure functions for neutrino and charged lepton are related by

$$\frac{F_{2N}^{l}}{F_{2N}^{\nu}} = \frac{5}{18}\left(1 - \frac{3}{5}\,\frac{s(x) + \bar{s}(x) - c(x) - \bar{c}(x)}{q(x) + \bar{q}(x)}\right)\,. \tag{10}$$

2. Nuclear effects in neutrino scattering

In order to calculate the neutrino nucleus cross-section we first evaluate the related neutrino self-energy in the medium (figure 1). The self-energy is given by

$$\Sigma(k) = \frac{iG}{\sqrt{2}}\frac{4}{m_\nu}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\,\frac{L^{\alpha\beta}+L_5^{\alpha\beta}}{k'^2-m_\mu^2+i\varepsilon}\left(\frac{m_W}{q^2-m_W^2}\right)^2\,\Pi_{\alpha\beta}(q)\,, \tag{11}$$

where

$$\begin{aligned}\Pi^{\alpha\beta}(q) = (-i)\int \frac{\mathrm{d}^4 p}{(2\pi)^4} iG(p)\sum_X \sum_{s_p,s_i}\prod_{i=1}^{N}\int \frac{\mathrm{d}^4 p'_i}{(2\pi)^4}\\ \times\prod_l iG_l(p'_l)\prod_j iD_j(p'_j)\left(\frac{-Gm_W^2}{\sqrt{2}}\right)\\ \times\langle X|J^\alpha|N\rangle\langle X|J^\beta|N\rangle^*(2\pi)^4\delta^4(q+p-\Sigma_{i=1}^N p'_i).\end{aligned} \tag{12}$$

The probability per unit time for the neutrino to collide with nucleons when travelling through nuclear matter is [4]

$$\Gamma(k) = -\frac{2m_\nu}{E_\nu(k)}\,\mathrm{Im}\,\Sigma(k)\,, \tag{13}$$

Im Σ is evaluated from eq. (11) by means of the Cutkosky rules [5]. The differential cross-section is then given by [4]

$$\frac{\mathrm{d}^2\sigma^{\nu(\bar\nu)}}{\mathrm{d}\Omega'\mathrm{d}E'} = -\frac{G}{\sqrt{2}}\frac{4}{(2\pi)^3}\frac{|k'|}{|k|}\left(\frac{m_W}{q^2-m_W^2}\right)^2[L^{\alpha\beta}\pm L_5^{\alpha\beta}]\int \mathrm{d}^3 r\,\mathrm{Im}\,\Pi_{\alpha\beta}(q)\,. \tag{14}$$

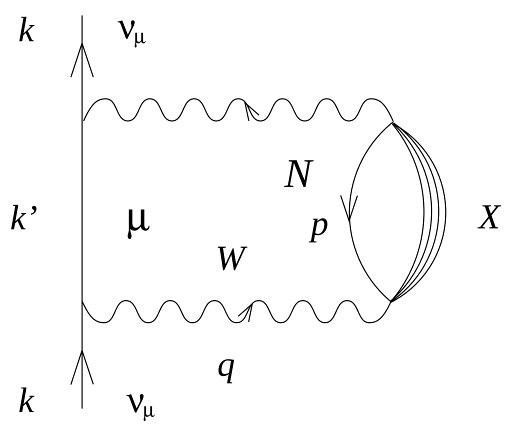

Figure 1. Self-energy diagram of the neutrino in the nuclear medium associated with the process of deep inelastic neutrino–nucleon scattering.

Comparing eqs (1) and (14) we see that

$$W_A^{\alpha\beta}(q) = -\frac{\sqrt{2}}{\pi}\,\frac{1}{Gm_W^2}\int \mathrm{d}^3r\,\mathrm{Im}\,\Pi^{\alpha\beta}(q)\,. \tag{15}$$

In the antineutrino case the expressions obtained are very similar, with a minus sign in the W self-energy, eq. (12), we have $\langle X|J_\alpha^\dagger|N\rangle$, instead of $\langle X|J_\alpha|N\rangle$. From now on we will always speak of the average of neutrino and antineutrino structure functions and will omit the superscripts ν and $\bar{\nu}$. For the nucleon propagator in the medium, $G(p)$, we take a relativistic version [4], which is written as

$$G(p^0,p) = \frac{M}{E(p)}\sum_r u_r(p)\bar{u}_r(p)\left[\int_{-\infty}^{\mu}\mathrm{d}\omega\frac{S_h(\omega,p)}{p^0-\omega-i\eta} + \int_{\mu}^{\infty}\mathrm{d}\omega\frac{S_p(\omega,p)}{p^0-\omega+i\eta}\right], \tag{16}$$

$S_h(\omega,p)$ and $S_p(\omega,p)$ being the hole and particle spectral functions respectively, which are taken from the work of [4,6].

We use the local density approximation in which the spectral functions depend on the density of the nucleus at the point at which they are evaluated. In our formalism we use spectral functions for symmetric nuclear matter. The normalization of the hole spectral function is given by

$$4\int \mathrm{d}^3r\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega,p,k_\mathrm{F}(r))\,\mathrm{d}\omega = A\,, \tag{17}$$

where $k_\mathrm{F}(r) = [3\pi^2\rho(r)/2]^{1/3}$ is the local Fermi momentum at the point r.

We now calculate $\mathrm{Im}\,\Pi^{\alpha\beta}(q)$ by using Cutkosky rules, the expression of eq. (16) for the nucleon propagator in the medium, free propagators for particles in the final state and by means of eq. (15) we have the hadronic tensor

$$W_A^{\alpha\beta} = 4\int \mathrm{d}^3r\int\frac{\mathrm{d}^3p}{(2\pi)^3}\,\frac{M}{E(p)}\int_{-\infty}^{\mu}\mathrm{d}p^0 S_h(p^0,p)W_N^{\alpha\beta}(p,q)\,. \tag{18}$$

3. Calculation of F_{2A} and F_{3A}

In order to calculate F_{2A}^{ν}, we calculate the transverse components xx from eq. (18) on both sides. Taking $\mathbf{q}$ along the z-axis we get:

$$W_{xx}^A = W_{1A} + \frac{(p_x)^2}{M^2}W_{2A} = \frac{F_{1A}(x)}{M} \tag{19}$$

in the Bjorken limit.

This component has the virtue that the coefficient of W_1 is independent of p and hence it is the same for on-shell or off-shell nucleons, or pions, or the nucleus.

Hence we can write

$$\frac{F_{1A,N}(x)}{M_A} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}) \frac{F_{1N(x_N)}}{M}. \tag{20}$$

Since usually one compares the ratio of the F_2 structure functions this is easily accomplished by making use of the Callan Gross relation and we find [4,6]

$$F_{2A,N}(x) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \times \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}) \frac{x}{x_N} F_{2N}(x_N)\theta(x_N)\theta(1-x_N). \tag{21}$$

As explained in ref. [4], similar considerations for pionic contribution lead to

$$\frac{F_{2A,\pi}(x)}{M_A} = -6\int d^3r \times \int \frac{d^4p}{(2\pi)^4}\theta(p^0)(-2)\delta\,\mathrm{Im}\,D_\pi(p)\frac{x}{x_\pi} 2MF_{2\pi}(x_\pi)\theta(x_\pi)\theta(1-x_\pi), \tag{22}$$

where $x_\pi = x^2/2p.q$, Im $D_\pi(p)$ is the imaginary part of the pion propagator and $\delta\,\mathrm{Im}\,D(p)$ is the corrected propagator for the pion in the medium. Similar contribution is obtained from the ρ meson with $D_\pi(p)$ replaced by $D_\rho(p)$ as explained in ref. [4].

Similarly considering xy component in eq. (18), we get

$$W_A^{xy} = -\frac{i}{2M_A} q_z W_{3A}\,, \tag{23}$$

and for the right-hand side we will have for the moving nucleon

$$W_N^{xy} = \frac{p_x p_y}{M^2} W_{2N}(p,q) + \frac{i}{2M^2} W_{3N}[p_z q_0 - p_0 q_z]\,. \tag{24}$$

We have

$$q_0 W_{3A} = F_{3A}(x), \quad \frac{pq}{M} W_{3N}(p,q) = F_{3N}(x_N) \tag{25}$$

with x as defined in eq. (3) and x_N is the Bjorken variable expressed in terms of the nucleon variables, (p^0, p), in the nucleus.

We obtain the expression for $F_{3A}(x)$ in the Bjorken limit

$$\frac{F_{3A}(x)}{A} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \times \int_{-\infty}^{\mu} dp^0 S_h(p^0, p) \frac{x_N}{x} \left[\frac{p_0 q_z - p_z q_0}{M q_z}\right] F_{3N}(x_N). \tag{26}$$

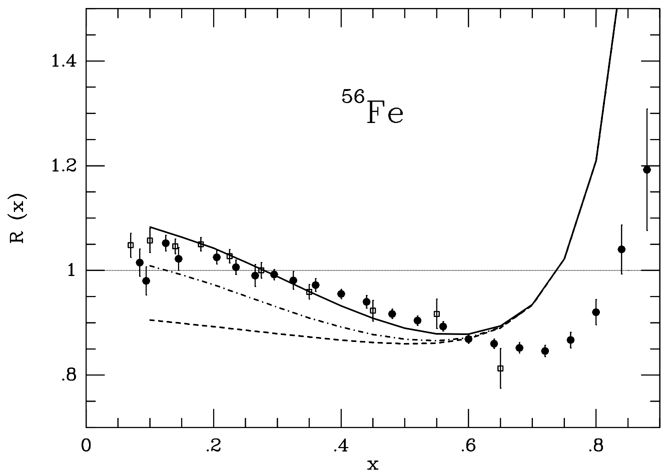

Figure 2. Results for $R_2(x)$ for ^{56}Fe. (—) Whole calculation including the nucleon and the mesons; (- - -) contribution of the nucleons; (· - · -) contribution of nucleons plus pions. Experimental points from ref. [9] (•), ref. [10] (□).

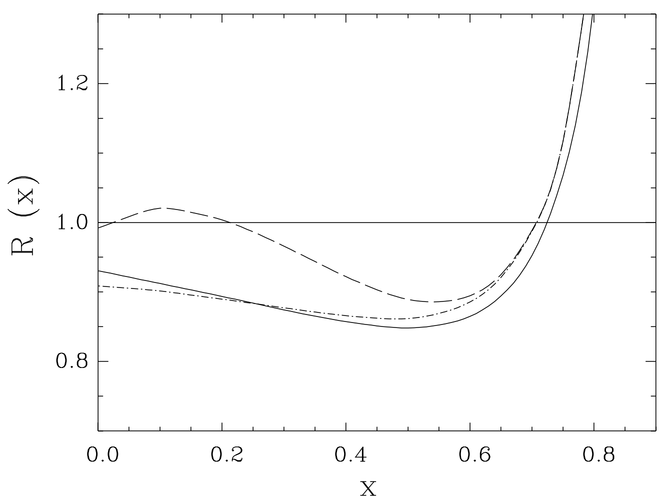

Figure 3. (—) Results for the ratio $F_{3A}(x)/AF_{3N}(x)$; (- - -) results for the ratio $F^l_{2A}(x)/AF^l_{2N}(x)$ including the contribution of the nucleons and the mesons; (· - · -) results for the ratio $F^l_{2A}(x)/AF^l_{2N}(x)$ including only the contribution of the nucleons. All curves are evaluated at $Q^2 = 50$ GeV^2.

4. Results

Numerical calculations for F_{2A} and F_{3A} are done using eqs (21), (22) and (26). We show the results for $R_2 = F^l_{2A}/AF^l_{2N}$ in figure 2 and for $R_3 = F_{3A}/AF_{3N}$ in figure 3. For F_{3N} and F^l_{2N} we have taken the parametrization of [7]. For the meson structure functions that contribute to R_2 we have used the parametrization given in [8].

The theoretical results for R_2 are in reasonable agreement with the experimental data [9,10]. In figure 3 we present the results for R_3 and compare this with R_2. We can see some similarities in the region around $x = 0.5$–0.6 where the ratio is smaller than unity and which is mostly due to nuclear binding effects, as discussed in detail in [4]. Similarities in the region of $x > 0.6$, where the ratio shows a fast increase, are also apparent, and they are mostly due to the effect of Fermi motion [4]. The differences between the neutrino and charged lepton ratios are more important at values of $x < 0.6$. These differences are mostly due to the lack of meson renormalization effects in the neutrino structure function, as can be seen in the figure, where we also show the results for R_2 without the meson renormalization effects.

We should note that the nuclear effects are sizeable, with values of R_3 around 0.8 in the region of $x \simeq 0.6$ and around 0.9 for low values of x. These nuclear corrections are considerably larger than those found for the deuteron [11], as it was also the case for R_2 in the charged lepton case.

References

[1] M Conrad, M H Shaevitz and T Bolton, hep-ex/9707015
[2] W G Seligman *et al*, *Phys. Rev. Lett.* **79**, 1213 (1997)
[3] A W Thomas and W Weise, *The structure of the nucleon* (Wiley-VCH, Berlin, 2001)
[4] E Marco, E Oset and P Fernandez de Cordoba, *Nucl. Phys.* **A611**, 484 (1996)
[5] C Itzykson and J B Zuber, *Quantum field theory* (McGraw-Hill, New York, 1980)
[6] P Fernandez de Cordoba and E Oset, *Phys. Rev.* **C46**,1697 (1992)
[7] A D Martin, W J Stirling and R G Roberts, *Phys. Rev.* **D51**, 4756 (1995)
[8] M Gluck, E Reya and A Vogt, *Z. Phys.* **C53**, 651 (1992)
[9] J Gomez *et al*, *Phys. Rev.* **D49**, 2221 (1994)
[10] A C Benvenuti *et al*, *Phys. Lett.* **B189**, 483 (1987)
[11] A V Sidorov and M V Tokarev, *Phys. Lett.* **B358**, 353 (1995)