The proton electromagnetic form factor F_2 and quark orbital angular momentum

PANKAJ JAIN[1] and JOHN P RALSTON[2]

[1]Physics Department, Indian Institute of Technology, Kanpur 208 016, India
[2]Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, USA

Abstract. We analyse the proton electromagnetic form factor ratio $R(Q^2) = QF_2(Q^2)/F_1(Q^2)$ as a function of momentum transfer Q^2 within perturbative QCD. We find that the prediction for $R(Q^2)$ at large momentum transfer Q depends on the exclusive quark wave functions, which are unknown. For a wide range of wave functions we find that $QF_2/F_1 \sim$ const. at large momentum transfer, which is in agreement with recent JLAB data.

Keywords. Protein; electromagnetic form factors; perturbative QCD; quark orbital angular momentum.

PACS Nos 13.40.Gp; 12.38.Bx; 13.88.+e; 14.20.Dh

The recent JLAB measurement of the proton electromagnetic form factor ratio [1–3] shows the puzzling behavior $R(Q^2) = QF_2(Q^2)/F_1(Q^2) \sim$ const. at large Q. The form factors F_1 and F_2 are defined by the standard relation

$$\langle p', s'|J^\mu|p, s\rangle = \bar{N}(P',s') \left(\gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu}q_\nu \frac{F_2(Q^2)}{2M} \right) N(P,s), \tag{1}$$

where $q = p' - p$, $Q^2 = -q^2$, J^μ is the electromagnetic current of proton and s and s' refer to the spins of the initial and final protons. F_1 and F_2 are referred to as the Dirac and Pauli form factors respectively.

The amplitude $i\bar{N}(p', s')F_2(Q^2)\sigma^{\mu\nu}q_\nu N(p,s)$ represents the amplitude for *chirality* of the proton to flip under momentum transfer Q. This flip can occur due to flip in the chirality of the proton's constituents. However, at large momentum transfer $Q \gg m_q$, where $m_q \sim$ few MeV is the mass of the quarks, the amplitude for a quark chirality flip is negligible. Alternatively the chirality of the proton can flip due to quark orbital angular momentum [4–7]. General principles allow the existence of substantial quark orbital angular momentum (OAM), but a body of belief accumulated from the non-relativistic quark model (NRQM) has not favored this possibility. Note that the three-dimensional OAM of non-relativistic physics has awkward Lorentz transformation properties, and we are concerned with the OAM of an $SO(2)$ subgroup of rotations preserving particle momenta. Here we analyse the non-zero quark OAM contribution to F_2 in order to determine pQCD predictions for the scaling behavior of the ratio $R(Q)$.
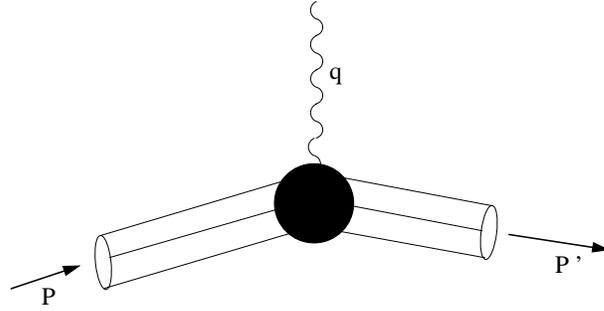
Figure 1. Schematic illustration of the elastic scattering of a proton from a virtual photon, $p(P) + \gamma^*(q) \to p(P')$.

The large Q^2 behavior of form factors is often discussed within the Brodsky–Lepage [8] factorization scheme. In this scheme the dominant contribution to amplitudes is assumed to come from the asymptotically short-distance (ASD) region $b \sim 1/Q$, where b is the transverse separation between quarks. Non-zero OAM is excluded in the first step, and should not contribute at large Q^2. The ASD formalism is tested by the hadron helicity conservation (HHC) rule [9] $\lambda_A + \lambda_B = \lambda_C + \lambda_D$ for the reaction $A + B \to C + D$, where λ_i is the helicity of the particle i. Failures of the ASD approach to correctly predict experimental results are well-known. These include many observed violations of the helicity conservation rule and the observation of oscillations in fixed-angle proton–proton elastic scattering [10]. The success of the formalism is the correct prediction of the Q^2 scaling behavior of many exclusive processes. Yet scaling can also be obtained from the quark counting model of Brodsky and Farrar and Matveev *et al* [11], without assuming the details of the ASD formalism [8]. Thus it is not possible to conclude whether or not quarks exist with non-zero OAM by appealing to ASD models.

The elastic scattering of a proton from a virtual photon is shown schematically in figure 1. The three-quark contribution to the proton form factor can be written in pQCD as

$$\bar{N}(P',s')\left(\gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu}q_\nu \frac{F_2(Q^2)}{2M}\right) N(P,s) = \int (\mathrm{d}\mathbf{k_T})(\mathrm{d}x)(\mathrm{d}\mathbf{k_T'})(\mathrm{d}x')$$
$$\times \bar{Y}_{\alpha'\beta'\gamma'}(P',\mathbf{k_{Ti}'},s')\Gamma^\mu_{\alpha'\beta'\gamma'\alpha\beta\gamma}(q,\mathbf{k_{Ti}},\mathbf{k_{Ti}'})Y_{\alpha\beta\gamma}(P,\mathbf{k_{Ti}},s) \, . \tag{2}$$

We have factored the amplitude into products of a hard scattering kernel Γ^μ and soft initial and final state wave functions Y and Y' respectively. The argument $\mathbf{k_{Ti}}$ of the initial wave function and the hard scattering refers to the momenta of the three quarks, $\mathbf{k_{T1}}, \mathbf{k_{T2}}$ and $\mathbf{k_{T3}}$ with similar definition of the argument $\mathbf{k_{Ti}'}$ of the final state wave function. We use the 'brick-wall' frame for our calculations. The integration measures are given by, $(\mathrm{d}\mathbf{k_T}) = \mathrm{d}^2k_{T1}\mathrm{d}^2k_{T2}\mathrm{d}^2k_{T3}\delta^2(\mathbf{k_{T1}} + \mathbf{k_{T2}} + \mathbf{k_{T3}})$, $(\mathrm{d}x) = \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\delta(x_1 + x_2 + x_3)$. Note that eq. (2) postpones any assumptions about the dominance of any particular integration region of $\mathbf{k_T}$ or x. If care is not taken, limit interchange errors leading to the ASD results can occur.

To extract the contribution due to quark OAM, it is convenient to work directly with the coordinates $\mathbf{b_i}$ conjugate to the transverse quark momenta $\mathbf{k_{Ti}}$. We choose coordinates so that the third quark (down) lies at the origin [12], i.e. $\mathbf{b_3} = 0$. The wave function $\tilde{Y}_{\alpha\beta\gamma}$ can be decomposed as a sum of terms [12]:

$$\tilde{Y}_{\alpha\beta\gamma}(P,\mathbf{b_i},s) = \frac{f_N}{8\sqrt{2}N_c}(\mathscr{C}^1_{\alpha\beta\gamma}V(P,\mathbf{b_i}) + \mathscr{C}^2_{\alpha\beta\gamma}A(P,\mathbf{b_i})$$
$$+\mathscr{C}^3_{\alpha\beta\gamma}T(P,\mathbf{b_i}) + \mathscr{C}^4_{\alpha\beta\gamma}X(P,\mathbf{b_i}) + \cdots). \tag{3}$$

We are concerned with terms which lead by power counting in large momenta. Under a Lorentz transformation along the momentum axes the variables b are invariant. We therefore keep leading wave functions whether or not a power of b occurs. Each power of $b_T \rightarrow b_x \pm ib_y$ can be further decomposed into combinations of quark OAM. (At this point, ASD would reject powers of b and OAM $\neq 0$.) The first few operators $\mathscr{C}$ are given by

$$\mathscr{C}^1_{\alpha\beta\gamma} = (\slashed{P}C)_{\alpha\beta}(\gamma_5 N)_\gamma,$$
$$\mathscr{C}^2_{\alpha\beta\gamma} = (\slashed{P}\gamma_5 C)_{\alpha\beta}N_\gamma,$$
$$\mathscr{C}^3_{\alpha\beta\gamma} = -(\sigma_{\mu\nu}P^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5 N)_\gamma,$$
$$\mathscr{C}^4_{\alpha\beta\gamma} = i(\slashed{P}\gamma_5 C)_{\alpha\beta}(\slashed{b}_1 N)_\gamma. \tag{4}$$

Here C is the charge-conjugation matrix. Note the operator $\mathscr{C}^4$ which depends on b_1. Here b_i are four vectors with transverse components equal to $\mathbf{b_i}$ and all other components zero. Similar operators exist for other transverse coordinates.

The Dirac and Pauli form factors can now be obtained from eq. (2). F_1 can be schematically written as

$$F_1(Q) = \int (\mathrm{d}\mathbf{b})(\mathrm{d}\mathbf{b}')(\mathrm{d}x)(\mathrm{d}x')\psi^*(b,x)\tilde{H}(b_i,b'_i\,x,\,x',\,Q)\psi(b,x'). \tag{5}$$

Here $\tilde{H}(b_i,b'_i,\,x,\,x',\,Q)$ is the Fourier transform of the hard scattering kernel after projecting out the Dirac bilinear covariant $\bar{N}\gamma^\mu N$, and ψ is a linear combination of the wave functions V, A and T defined in eq. (3). A detailed calculation of F_1 at leading order in perturbation series, neglecting the transverse momenta in the Dirac propagators, is given in ref. [12].

We focus on the b integrations to obtain F_2 in the limit of zero quark masses. Contributing are wave functions such as X (eq. (3)), as well as the transverse momenta in the Dirac propagators. The scaling behavior can be determined by considering the leading order hard scattering kernel. Since we are working in the impact parameter space, we need to take the Fourier transform of this kernel. In this case the transverse momentum factors such as $(k_{T1})_i$, which occur in the Dirac propagators, turn into derivatives $i\partial/\partial b_{1i}$. The remaining Fourier transform has a form similar to what is obtained for the form factor F_1. The dependence on the impact parameter in this kernel arises through the Bessel functions, such as $K_0(\sqrt{x_1 x'_1}Q\tilde{b}_{12})$, where $\tilde{b}_{12} = |\mathbf{b_1} - \mathbf{b_2}|$. Taking the derivative of this kernel with respect to b_{1i} gives factors of the form,

$$\frac{(\mathbf{b_1} - \mathbf{b_2})_i}{\tilde{b}_{12}}\,Q\sqrt{x_1 x'_1}K'_0(\sqrt{x_1 x'_1 Q^2 \tilde{b}_{12}^2})\,.$$

Besides this transverse separation dependence, an additional power of $\mathbf{b}$ arises from the operator $\mathscr{C}^4$ in the wave function.

The scaling of $F_2(Q^2)$, and the form factor ratio, hinges on scaling of the effective transverse separation b at large Q. Counting powers of Q, including one for the prefactor $q^\mu\sigma_{\mu\nu}$,

we find that if $b \sim 1/Q$ at large Q, then $F_2/F_1 \sim 1/Q^2$ in this limit. This result has recently been confirmed in [13] which adopts the ASD formalism from the start. Alternatively if $b \sim$ const. then $QF_2/F_1 \sim$ const. and $F_2 \sim 1/Q^5$ in the same limit. Such scaling was predicted in [5] and is also seen in a relativistic quark model calculation [14].

We turn to how the dominant region of b actually scales with Q. The simpler case of the pion form factor [15] is instructive. The one gluon exchange kernel in this case can be written as

$$\tilde{H}(b, x, \ x', \ Q) = 8\pi^2 C_F \alpha_s K_0(\sqrt{xx'Q^2b^2}), \tag{6}$$

where α_s is the strong coupling and C_F is the color factor. At large Q the dominant contribution is obtained from the region

$$\sqrt{xx'}Qb < 1. \tag{7}$$

In order to reproduce the ASD assumptions, average values of $x \sim 1/2$ for pion and $x \sim 1/3$ for proton can be assigned, converting $\sqrt{xx'}Qb < 1 \rightarrow Qb < 1$. Such estimates are not the same thing as actually computing the result! Indeed one cannot rule out the possibility that in the limit of very large Q, $b \sim$ const. and $\sqrt{xx'} \sim 1/Q$, or any intermediate combinations of integration regions, given the complexity of the problem. Elsewhere [7] we have shown that it is not possible, in principle, to determine the scaling of a power of b without specifying the interplay of x and b-dependence in the wave functions. Indeed the 'rules of pQCD' require one to objectively treat wave functions as unknown non-perturbative objects, to be determined from the data. Constructing the wave functions to get the ASD results is possible, but we find such logic circular.

Suppose one uses a Gaussian model for the wave function $\Phi(b, x, x') = b^A e^{-b^2/(2a^2)} \phi(x, x')$ to represent the wave functions cutting off large b. The factor of b^A is the phase-space to find A quarks close together from naive quark-counting. We probe effects of quark OAM by calculating the moment $\langle b(Q) \rangle_\pi$ defined by

$$\langle b(Q) \rangle_\pi = \frac{\int \mathrm{d}x\mathrm{d}x'\mathrm{d}^2 b\, b \mathscr{F}_\pi(Q, x, x', b)}{F_\pi(Q^2)}. \tag{8}$$

Then the pion form factor $F_\pi(Q^2)$ is given by

$$F_\pi(Q^2) = \int \mathrm{d}x\mathrm{d}x'\mathrm{d}^2 b \mathscr{F}_\pi(Q, x, x', b) \tag{9}$$

and $\mathscr{F}_\pi = \tilde{H}\Phi$. We change variables for the longitudinal fractions to $\xi = \sqrt{xx'}$, $\zeta = x/x'$ and parametrize $\phi(\xi, \zeta) \sim \xi^{r+1}(1 - \xi)^{r+1}\phi(\zeta)$, as $\xi \rightarrow 1$: the ζ dependence can be left unspecified. By substituting this into eq. (8), when $r < A$, the dominant contribution to the numerator in eq. (8) is obtained from $x \sim 1/Q$, and the moment $\langle b(Q) \rangle_\pi \sim$ const. Due to the inherent unknowns of the wave function's dependence on both x and b, it is not possible to establish suppression of the contributions of quark OAM in pQCD in a model-independent way.

Similar results are obtained for the proton. In figure 2 we plot one moment of the transverse separation b_2 for the proton using the COZ model for the x-dependence of wave functions [16], both including and not including the Sudakov form factor [15]. It is clear that over a very large range the moment has a very weak dependence on Q. Hence the
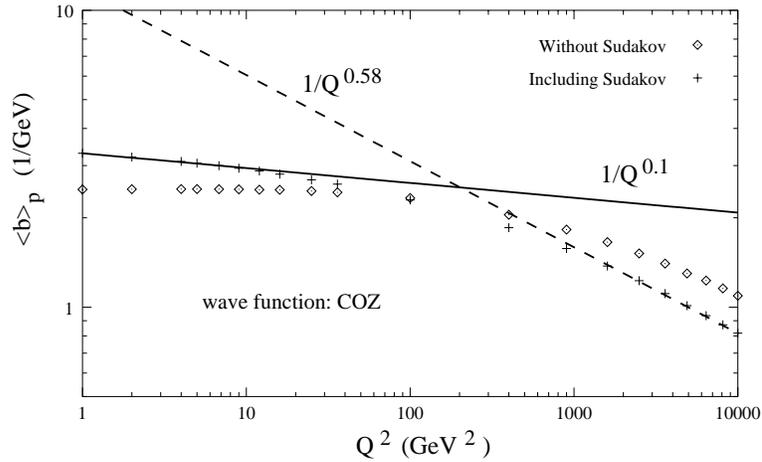
Figure 2. The moment $\langle b_2 \rangle$ of the proton form factor kernel by including and not including the Sudakov form factor. The COZ model is used for the *x*-dependence of wave functions.

form factor ratio $R(Q) = QF_2/F_1$ scales like a constant in this region, in agreement with the recent JLAB experimental result. Asymptotic analysis [7] shows that for a wide range of *x*-dependence in wave functions, the form factor ratio $R(Q)$ scales like a constant, modulo logarithms.

Acknowledgements

References

[1] M K Jones *et al*, *Phys. Rev. Lett.* **84**, 1398 (2000)
[2] O Gayou *et al*, *Phys. Rev. Lett.* **88**, 092301 (2002)
[3] C F Perdrisat, *Pramana – J. Phys.* **61**, 827 (2003)
[4] P Jain and J P Ralston, in *Future directions in particle and nuclear physics at multi-GeV hadron beam facilities*, Proceedings of the Workshop held at BNL, 4–6 March 1993, hep-ph/9305250
[5] R Buniy, J P Ralston and P Jain, in *VII International Conference on the Intersections of Particle and Nuclear Physics*, *Quebec City, 2000* edited by Z Parseh and W Marciano (AIP, NY, 2000), hep-ph/0206074
[6] J P Ralston, R V Buniy and P Jain, *Proceedings of DIS 2001, 9th International Workshop on Deep Inelastic Scattering* (Bologna, 27 April–1 May 2001) hep-ph/0206063
[7] J P Ralston and P Jain, hep-ph/0302043
[8] G P Lepage and S J Brodsky, *Phys. Rev.* **D22**, 2157 (1980)
[9] S J Brodsky and G P Lepage, *Phys. Rev.* **D24**, 2848 (1981)
[10] B Pire and J P Ralston, *Phys. Lett.* **B117**, 233 (1982)

[11] S J Brodsky and G R Farrar, *Phys. Rev.* **D11**, 1309 (1975)
      V A Matveev, R M Muradian and A N Tavkhelidze, *Lett. Nuovo Cimento* **7**, 719 (1973)
[12] H-N Li, *Phys. Rev.* **D48**, 4243 (1993)
[13] A V Belitsky, X Ji and F Yuan, hep-ph/0212351
[14] M R Frank, B K Jennings and G A Miller, *Phys. Rev.* **C54**, 920 (1996)
      G A Miller and M R Frank, nucl-th/0201021
[15] H-N Li and G Sterman, *Nucl. Phys.* **B381**, 129 (1992)
[16] V L Chernyak, A A Ogloblin and I R Zhitnitsky, *Sov. J. Nucl. Phys.* **48**, 536 (1988)
      V L Chernyak, A A Ogloblin and I R Zhitnitsky, *Yad. Fiz.* **48**, 841 (1988)