

## Strong interaction at finite temperature

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**Abstract.** We review two methods discussed in the literature to determine the effective parameters of strongly interacting particles as they move through a heat bath. The first one is the general method of chiral perturbation theory, which may be readily applied to this problem. The other is the method of thermal QCD sum rules. We show that, when the spectral sides of the sum rules are calculated correctly, they do not lead to any new results, but reproduce those of the vacuum sum rules.

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### 1. Introduction

In conventional statistical physics, one calculates the partition function of a physical system, from which its energy density, pressure etc. can be obtained. But to describe the heavy-ion accelerator experiments and the physics of the early Universe, one is also in need of other particle specific properties, like the effective masses and couplings of strongly interacting particles in a medium. These quantities differ from their values in vacuum due to the additional interactions these particles have with the particles of the medium. Although the methods we are reviewing here should apply to any medium, we shall restrict our discussion to one maintained at temperature  $T$  with zero chemical potentials.

At low energies the strong interaction as implied by QCD is best described by an effective field theory, called the chiral perturbation theory ( $\chi$ PT) [1]. It is based on the flavour symmetry group  $SU(3)_R \times SU(3)_L$  of the massless QCD Lagrangian of three flavours, which breaks spontaneously to its diagonal subgroup  $SU(3)_V$ , explaining the existence of the octet of low mass pseudoscalar mesons, the (pseudo-) Goldstone bosons. The broken symmetry implies the presence of derivatives on Goldstone fields in the interaction Lagrangian, so that their interactions amongst themselves and with other particles are weak at low momenta.

$\chi$ PT can be readily applied to problems of strong interaction at low temperatures, when the pions dominate the heat bath, the presence of other hadrons being exponentially suppressed [2]. One is thus led to consider the reduced chiral symmetry group  $G = SU(2)_R \times SU(2)_L$ , broken spontaneously to its isospin subgroup  $H = SU(2)_V$ . Since the pion momenta are low, the interaction can be treated as perturbation.

The extension of all conventional (vacuum) quantum field theoretic techniques to the thermal field theory is straightforward [3]. All we need is to replace the vacuum expectation values by the corresponding ensemble averages. The perturbative evaluation of any quantity follows the same steps as for its vacuum counterpart, with the replacement of free vacuum propagators by free thermal ones. In our calculations below, we shall use the real-time formulation of the thermal field theory [4,5].

Here we shall review the application of  $\chi$ PT to determine the effective masses and couplings of particles. The original work is that of Leutwyler and Smilga [6], who calculated the nucleon pole term in the thermal two-point function of the nucleon current. For lack of space, we only consider the vector and the axial-vector mesons [7]. Their effective parameters are obtained from the respective meson pole terms in the thermal two-point functions of the vector and the axial-vector currents.

The method of QCD sum rules [8], also based on two-point functions, was extended to finite temperature by Bochkarev and Shaposhnikov [9] and subsequently by many others [10] to determine the effective parameters of particles. These works were based on an ansatz for the pole term in the spectral function, which also included branch point contributions from low-lying two-particle intermediate states. Though quite suggestive and extensively used in the past, such an ansatz lacks a theoretical basis. Indeed, when we calculate the complete spectral function using  $\chi$ PT [11], we find it to be incorrect.

We begin with a brief description of kinematics in §2, mainly to introduce the notations and the quantities we shall work with. We then review  $\chi$ PT in §3, obtaining, in particular, the Lagrangian for the vector and the axial-vector mesons in the presence of external fields. The mass and coupling shifts are calculated in §4. In §5 we calculate the spectral representation for the two-point functions, anticipating the QCD sum rules. The latter are obtained in §6. Finally we conclude briefly in §7.

## 2. Kinematics

As already stated, we are concerned here with the evaluation of the thermal two-point functions of vector and axial-vector currents of QCD:

$$V_\mu^a(x) = \bar{q}(x)\gamma_\mu \frac{\tau^a}{2} q(x), \quad A_\mu^a(x) = \bar{q}(x)\gamma_\mu \gamma_5 \frac{\tau^a}{2} q(x), \quad (2.1)$$

where  $q(x)$  is the quark field and  $\tau^a$  are the Pauli matrices. For the vector current, it is

$$T_{\mu\nu}^{ab} = i \int d^4x e^{iq \cdot x} \text{Tr} \rho T V_\mu^a(x) V_\nu^b(0), \quad (2.2)$$

and a similar one for the axial-vector current. Here  $\rho = e^{-\beta H} / \text{Tr} e^{-\beta H}$  is the thermal density matrix at temperature  $T = 1/\beta$  and  $H$  is the QCD Hamiltonian. Note that in the limit of chiral symmetry, in which we shall work, the axial-vector current is also conserved and the kinematics, in particular, the invariant decomposition is the same for both the correlation functions.

The current conservation leads to the invariant decomposition

$$T_{\mu\nu}^{ab}(q) = \delta^{ab} (P_{\mu\nu} T_t + Q_{\mu\nu} T_l), \quad (2.3)$$

where the gauge invariant tensors are chosen as

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$$P_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} - \frac{q^2}{\bar{q}^2} \tilde{u}_\mu \tilde{u}_\nu, \quad Q_{\mu\nu} = \frac{q^4}{\bar{q}^2} \tilde{u}_\mu \tilde{u}_\nu,$$

with  $\tilde{u}_\mu = u_\mu - \omega q_\mu/q^2$ , where  $u_\mu$  is the four-velocity of the medium and  $\omega$  and  $\bar{q}$  are Lorentz invariant scalars,  $\omega = u \cdot q$  and  $\bar{q} = \sqrt{\omega^2 - q^2}$ , representing the time and space components of  $q_\mu$  in the rest frame of the heat bath ( $u_0 = 1, \vec{u} = 0$ ). The invariant amplitudes are functions of the scalar variables, say,  $q^2$  and  $\omega$ . They can be conveniently extracted from the Feynman diagrams by forming the scalars

$$T_1 = g^{\mu\nu} T_{\mu\nu}, \quad T_2 = u^\mu u^\nu T_{\mu\nu}, \quad (2.4)$$

which are simply related to the invariant amplitudes. For details see [7].

In the real-time thermal field theory we are going to use here, each of the above amplitudes stands for a  $2 \times 2$  matrix, whose components depend on a single analytic function [3]. This function, in turn, is determined completely by the 11-component function itself: their real parts are equal and the imaginary part of the former equals  $\pi^{-1} \tanh(\beta q_0/2)$  times that of the latter. (The factor  $\pi^{-1}$  is inserted for convenience.) To avoid further symbols, we henceforth redefine  $T$  to denote this analytic function. Its spectral representation at fixed  $\bar{q}$  is given by

$$T_{l,t}(q_0^2, \bar{q}) = \int \frac{dq_0'^2 \text{Im } T_{l,t}(q_0', \bar{q})}{q_0^2 - q_0'^2 - i\varepsilon}. \quad (2.5)$$

Our task is to calculate these amplitudes in the effective theory.

### 3. Chiral perturbation theory

From the transformation laws for the quark fields under  $G = SU(2)_R \times SU(2)_L$ , one can derive the corresponding laws for fields describing the physical particles. The Goldstone fields  $\pi^a(x)$  can be collected in the form of a unitary matrix

$$U(x) = e^{i\pi(x)/F_\pi}, \quad \pi(x) = \sum_{a=1}^3 \pi^a(x) \tau^a,$$

where the constant  $F_\pi$  is the pion decay constant,  $F_\pi = 93$  MeV. The matrix  $U$  transforms under  $G$  according to the rule,  $U \rightarrow g_R U g_L^\dagger$ , where  $g_{R,L} \in SU(2)_{R,L}$ . But any multiplet  $\psi(x)$  of non-Goldstone fields transforms as  $\psi \rightarrow h\psi$ , where  $h(\pi)$  is an appropriate representation of  $SU(2)_V$ . In particular, if they belong to the adjoint (triplet) representation, such as the vector and the axial-vector mesons we are considering here, we may use the same  $h$  as for the fundamental representation: Denoting the triplet fields by  $R_\mu(x)$ ,

$$R_\mu(x) = \frac{1}{\sqrt{2}} \sum_{a=1}^3 R_\mu^a(x) \tau^a,$$

it transforms as  $R_\mu \rightarrow h R_\mu h^\dagger$ . Any singlet field  $S_\mu(x)$ , of course, remains unchanged.

The evaluation of the two-point functions is most conveniently carried out in the external field method, where one introduces external fields  $v_\mu^a(x)$  and  $a_\mu^a(x)$  coupled to the currents

$V_\mu^a(x)$  and  $A_\mu^a(x)$  [1]. The original QCD Lagrangian  $\mathcal{L}_{\text{QCD}}^{(0)}$  of massless quarks is then augmented to

$$\begin{aligned} & \mathcal{L}_{\text{QCD}}^{(0)} + v_\mu^a(x)V_\mu^a(x) + a_\mu^a(x)A_\mu^a(x) \\ & = i\bar{q}_R\gamma^\mu\{\partial_\mu - i(v_\mu + a_\mu)\}q_R + i\bar{q}_L\gamma^\mu\{\partial_\mu - i(v_\mu - a_\mu)\}q_L + \dots, \end{aligned} \quad (3.1)$$

where  $q_R(q_L)$  is the right (left) handed component of  $q$ ,  $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$ . The ellipsis denote the (flavour neutral) gauge field term, and  $v_\mu(x)$  and  $a_\mu(x)$  are matrices in flavour space.

$$v_\mu(x) = \sum_{a=1}^3 v_\mu^a(x) \frac{\tau^a}{2}, \quad a_\mu(x) = \sum_{a=1}^3 a_\mu^a(x) \frac{\tau^a}{2}. \quad (3.2)$$

The global invariance of  $\mathcal{L}_{\text{QCD}}^{(0)}$  is now raised to local invariance under  $G$  with  $x$ -dependent group elements  $g_{R,L}(x)$ .

One can now construct the field strengths for the external potentials and the covariant derivatives of  $U$  and  $R_\mu$ , which transform as proper tensors under the local transformations of  $G$  [12,13]. The effective Lagrangian is given by all the terms that can be constructed out of these elements and are invariant under the chiral symmetry group and the Lorentz group as well as the discrete transformations of  $P, C$  and  $T$ . The leading terms correspond to the least number of derivatives on  $U$ .

Having obtained the chirally symmetric Lagrangian in this way [12,13], we may expand  $U$  and its derivatives in powers of the pion field and read off the vertices to appear in the Feynman diagrams. The relevant pieces of chiral couplings of pions with themselves and the external fields are given by

$$\mathcal{L}_{\text{int}}(\boldsymbol{\pi}, v, a) = \mathcal{L}(\boldsymbol{\pi}) + \mathcal{L}_v(\boldsymbol{\pi}) + \mathcal{L}_a(\boldsymbol{\pi}), \quad (3.3)$$

where

$$\begin{aligned} \mathcal{L}(\boldsymbol{\pi}) &= -\frac{1}{6F_\pi^2}(\boldsymbol{\pi} \cdot \boldsymbol{\pi} \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi} \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}), \\ \mathcal{L}_v(\boldsymbol{\pi}) &= \mathbf{v}_\mu \cdot \boldsymbol{\pi} \times \partial^\mu \boldsymbol{\pi} + \frac{1}{2}(\boldsymbol{\pi} \cdot \boldsymbol{\pi} \mathbf{v}_\mu \cdot \mathbf{v}^\mu - \mathbf{v}_\mu \cdot \boldsymbol{\pi} \mathbf{v}^\mu \cdot \boldsymbol{\pi}), \\ \mathcal{L}_a(\boldsymbol{\pi}) &= \frac{1}{2}F_\pi^2 \mathbf{a}_\mu \cdot \mathbf{a}^\mu - F_\pi \mathbf{a}_\mu \cdot \partial^\mu \boldsymbol{\pi} - \frac{1}{2}(\boldsymbol{\pi} \cdot \boldsymbol{\pi} \mathbf{a}_\mu \cdot \mathbf{a}^\mu - \mathbf{a}_\mu \cdot \boldsymbol{\pi} \mathbf{a}^\mu \cdot \boldsymbol{\pi}) \\ & \quad + \frac{2}{3F_\pi}(\boldsymbol{\pi} \cdot \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi} \cdot \mathbf{a}_\mu - \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} \boldsymbol{\pi} \cdot \mathbf{a}_\mu), \end{aligned} \quad (3.4)$$

the letters in bold face denoting isospin vectors. Since we are considering thermal corrections to one loop only, we may already evaluate here the single pion loops generated by the above interaction vertices. Such a loop is given by the thermal pion propagator,

$$\Delta_{11}^{(\boldsymbol{\pi})}(k) = \frac{i}{k^2 - m_\pi^2} + 2\pi\delta(k^2 - m_\pi^2)n(k), \quad (3.5)$$

formed by contracting the two pion fields without derivatives at such vertices. Here  $n(k) = (e^{\beta|k_0|} - 1)^{-1}$  is the Bose distribution function. Then the thermal part of the loop is

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$$\text{Tr } \rho T \pi^a(x) \pi^b(x) |_{11} \rightarrow \delta^{ab} \int \frac{d^4k}{(2\pi)^3} n(k_0) \delta(k^2 - m_\pi^2) = \delta^{ab} \frac{T^2}{12}, \quad (3.6)$$

in the chiral limit. Thus we can write  $\mathcal{L}(\pi)$  and  $\mathcal{L}_a(\pi)$  as the effective two-point vertices,

$$\begin{aligned} \mathcal{L}(\pi) &= -\frac{T^2}{36F_\pi^2} \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}, \\ \mathcal{L}_a(\pi) &= \frac{F_\pi^2}{2} \left(1 - \frac{T^2}{6F_\pi^2}\right) \mathbf{a}_\mu \cdot \mathbf{a}^\mu - F_\pi \left(1 - \frac{T^2}{9F_\pi^2}\right) \mathbf{a}_\mu \cdot \partial^\mu \boldsymbol{\pi}. \end{aligned} \quad (3.7)$$

Next, we write the couplings of the isotriplets  $[\rho(770), a_1(1230)]$  and isosinglets  $[\omega(782), f_1(1282)]$  of vector ( $1^{--}$ ) and axial vector ( $1^{++}$ ) mesons respectively [12,13]. Although we take their fields to transform according to  $SU(2)$  in constructing their interactions, we take the physical (zero temperature) masses of the multiplets of each of the  $SU(3)$  octets to be degenerate. Then the couplings linear in the vector meson fields are given by [7].

$$\begin{aligned} \mathcal{L}(V) &= \frac{F_\rho}{m_V} \left\{ \left(1 - \frac{T^2}{12F_\pi^2}\right) \partial^\mu \mathbf{v}^\nu \cdot (\partial_\mu \boldsymbol{\rho}_\nu - \partial_\nu \boldsymbol{\rho}_\mu) \right. \\ &\quad \left. + \frac{1}{F_\pi} \partial^\mu \mathbf{a}^\nu \cdot (\partial_\mu \boldsymbol{\rho}_\nu - \partial_\nu \boldsymbol{\rho}_\mu) \times \boldsymbol{\pi} \right\} \\ &\quad - \frac{2G_\rho}{m_V F_\pi^2} \partial_\mu \boldsymbol{\rho}_\nu \cdot \partial^\mu \boldsymbol{\pi} \times \partial^\nu \boldsymbol{\pi} - \frac{\sqrt{2}H_\omega}{m_V F_\pi} \varepsilon_{\mu\nu\lambda\sigma} \omega^\mu \partial^\nu \boldsymbol{\pi} \cdot \partial^\lambda \mathbf{v}^\sigma, \end{aligned} \quad (3.8)$$

while those linear in the axial-vector meson fields are

$$\begin{aligned} \mathcal{L}(A) &= -\frac{F_{a_1}}{m_A} \left\{ \left(1 - \frac{T^2}{12F_\pi^2}\right) \partial^\mu \mathbf{a}^\nu \cdot (\partial_\mu \mathbf{a}_{1\nu} - \partial_\nu \mathbf{a}_{1\mu}) \right. \\ &\quad \left. + \frac{1}{F_\pi} \partial^\mu \mathbf{v}^\nu \cdot (\partial_\mu \mathbf{a}_{1\nu} - \partial_\nu \mathbf{a}_{1\mu}) \times \boldsymbol{\pi} \right\} + \frac{\sqrt{2}H_{f_1}}{m_A F_\pi} \varepsilon_{\mu\nu\lambda\sigma} f_1^\mu \partial^\nu \boldsymbol{\pi} \cdot \partial^\lambda \mathbf{a}^\sigma, \end{aligned} \quad (3.9)$$

where we have again contracted the two pion fields at the vertices forming single pion loops. Finally the quadratic couplings of the triplets with the singlets and between themselves are given by

$$\begin{aligned} \mathcal{L}(V,A) &= -2\varepsilon_{\mu\nu\lambda\sigma} \left( \frac{g_1}{F_\pi} \partial^\mu \omega^\nu \boldsymbol{\rho}^\lambda \cdot \partial^\sigma \boldsymbol{\pi} + \frac{g_2}{F_\pi} \partial^\mu f_1^\nu \mathbf{a}_1^\lambda \cdot \partial^\sigma \boldsymbol{\pi} \right) \\ &\quad + \frac{g_3}{F_\pi} \partial^\mu \boldsymbol{\rho}^\nu \cdot (\mathbf{a}_{1\mu} \times \partial_\nu \boldsymbol{\pi} - \mathbf{a}_{1\nu} \times \partial_\mu \boldsymbol{\pi}). \end{aligned} \quad (3.10)$$

The coupling constants in the above interaction terms can be determined from the decay rates of the particles [7,12]. One gets  $F_\rho = 154$  MeV,  $F_{a_1} = 135$  MeV and the dimensionless constant  $g_1 = 0.87$ . There does not appear any data relating to the decay of  $f_1$  to determine  $g_2$ . The remaining coupling constants in eqs (3.8)–(3.10) will not appear in our results to  $O(T^2)$ . Also it is possible to argue that other particles not included in the above Lagrangian cannot contribute to our results, calculated to this order.

#### 4. Mass and coupling shifts

We are now in a position to evaluate the current correlation functions to one loop. For the vector current, the Feynman diagrams can be grouped into four types (figures 1–4), namely, the free  $\rho$  propagation with constant vertex corrections, the self-energy corrections, the non-constant loop corrections at the vertices and finally the two-particle intermediate states. Then the invariant amplitude  $T_l$  given by the free pole term of figure 1a in the variable  $E \equiv q_0$  for  $\vec{q} = 0$  is

$$-E^4 \cdot \frac{(F_\rho/m_V)^2}{E^2 - m_V^2 + i\epsilon}.$$

We now determine how this pole position and the residue are modified by interactions at finite temperature. The self-energy diagrams of figure 2 modify it to

$$-E^4 \cdot \frac{(F_\rho/m_V)^2}{E^2 - m_V^2 - \Pi_l},$$

where we use a decomposition like eq. (2.3) to construct  $\Pi_l$  from the finite temperature part of the polarisation tensor  $\Pi_{\alpha\beta}$ ,

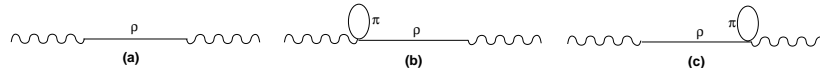


Figure 1.  $\rho$  meson pole and constant vertex correction diagrams.

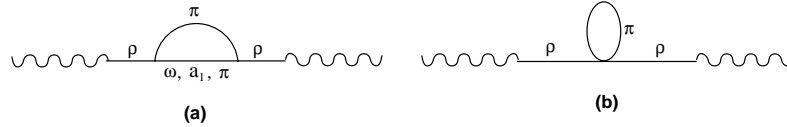


Figure 2. Self-energy diagrams.

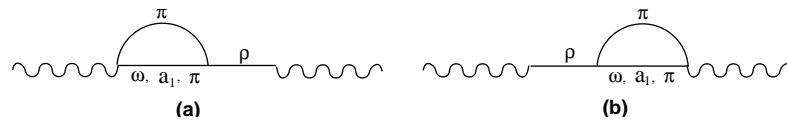


Figure 3. Vertex correction diagrams.

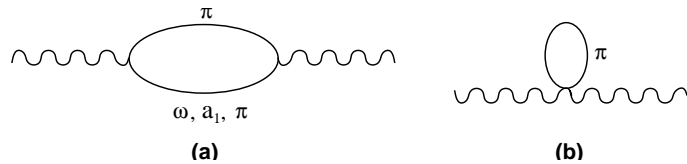


Figure 4. Intermediate state diagrams.

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$$\Pi_{\alpha\beta}(q) = \sum_{c=\omega,a_1,\pi} \Pi_{\alpha\beta}^{(c)}(q), \quad (4.1)$$

the sum running over the diagrams of figure 2a. (We have left out figure 2b, which is clearly of order  $T^4$ .) The respective contributions to  $\Pi_t$  for  $E^2$  near  $M_V^2$  can be worked out in the chiral limit to leading order in  $T^2$ ,

$$\Pi_t^{(\omega)}(E) = -\frac{g_1^2 T^2}{18F_\pi^2}(E^2 - m_V^2), \quad \Pi_t^{(a_1)} \sim \Pi_t^{(\pi)} \sim O(T^4). \quad (4.2)$$

Taking into account the self-energy correction given by eq. (4.1) and the constant vertex corrections from figures 1b,c, we get for the complete pole term

$$-E^4 \frac{(F_\rho^T/m_V)^2}{E^2 - m_V^2}, \quad (4.3)$$

with

$$F_\rho^T = F_\rho \left\{ 1 - \left( 1 + \frac{g_1^2}{3} \right) \frac{T^2}{12F_\pi^2} \right\}. \quad (4.4)$$

One can show that neither the remaining diagrams nor any other particle with a mass other than  $m_V$  can contribute to the shift to order  $T^2$ . Note the absence of mass-shift to this order.

One can also calculate the axial-vector meson pole term in the axial-vector current correlation function in an entirely analogous manner. Again one finds that the pole position does not shift from  $E^2 = m_A^2$ , and the residue  $F_{a_1}^T$  is given by eq. (4.4) with  $g_1$  replaced by  $g_2$ .

## 5. Spectral representation

We now turn to study the QCD sum rules following from the thermal two-point functions of the vector and the axial-vector currents. Instead of assuming any ansatz for the spectral sides, we construct it directly from the same Feynman diagrams, that we considered in the last section to determine the effective parameters of the vector mesons. However, their evaluation follows a different line: While we evaluated them earlier in the neighbourhood of the respective poles, we now have to do it at large space-like momenta.

The free  $\rho$  propagation with its constant vertex corrections (figure 1) is given by the tensor amplitude

$$T_{\mu\nu}^{(\rho)}(q) = -\left(\frac{F_\rho}{m_V}\right)^2 \left(1 - \frac{T^2}{6F_\pi^2}\right) \frac{q^4}{q^2 - m_V^2} A_{\mu\nu}, \quad (5.1)$$

where

$$A_{\mu\nu}(q) = -g_{\mu\nu} + q_\mu q_\nu / q^2 = P_{\mu\nu} + Q_{\mu\nu}.$$

The amplitudes for the  $\pi\pi$  intermediate state together with the tadpole (to ensure current conservation) has the thermal part

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$$T_{\mu\nu}^{(\pi\pi)}(q) \rightarrow -\frac{T^2}{6} A_{\mu\nu}, \quad (5.2)$$

at large space-like momenta. For the  $\pi a_1$  intermediate state, we obtain the thermal part of the invariant amplitudes as

$$T_{l,t}^{(\pi a_1)} \rightarrow \left(\frac{F_{a_1}}{m_A}\right)^2 \frac{(-Q^2, Q^4)}{Q^2 + m_A^2} \cdot \frac{T^2}{6F_\pi^2}. \quad (5.3)$$

One can show that the remaining diagrams do not contribute to  $O(T^2)$  at large  $Q^2$ .

Observe that the self-energy diagram with  $\pi\omega$  intermediate state, which produced the term proportional to  $g_1^2$  in the residue of the vector meson pole, brings no contribution to the amplitude at large momenta. To see how it happens, we note that the amplitude for this diagram,

$$-\left(\frac{F_\rho}{m_V}\right)^2 \cdot \frac{q^4}{(q^2 - m_V^2)^2} \Pi_{\mu\nu}(q), \quad (5.4)$$

is the product of a double pole with a branch point given by the self-energy tensor  $\Pi_{\mu\nu}$ . The latter can be decomposed into its invariant amplitudes, each of which has a spectral representation of the form

$$\Pi(E^2) = \int \frac{\text{Im}\Pi(E') dE'^2}{E'^2 - E^2}.$$

Then the *product* structure can be written in a partial fraction

$$\begin{aligned} \frac{E^4}{(E^2 - m_V^2)^2} \Pi(E) &= \frac{m_V^4}{(E^2 - m_V^2)^2} \int \frac{dE'^2 \text{Im}\Pi(E')}{E'^2 - m_V^2} \\ &+ \frac{m_V^2}{E^2 - m_V^2} \int \frac{dE'^2 (2E'^2 - m_V^2) \text{Im}\Pi(E')}{(E'^2 - m_V^2)^2} \\ &+ \int \frac{dE'^2 E'^4 \text{Im}\Pi(E')}{(E'^2 - m_V^2)^2 (E'^2 - E^2)}, \end{aligned} \quad (5.5)$$

getting the *sum* of three terms, the first, second and third having the double pole, the simple pole and the branch point respectively. Clearly to determine the pole parameters the last term is not relevant, the first and the second terms giving respectively the shifts in its position and the residue. Since  $\text{Im}\Pi \sim (E^2 - m_V^2)$ , for this diagram, we get only a shift in the residue to order  $O(T^2)$ , in conformity with the earlier result (eq. (4.4)).

On the other hand, to get the amplitude at large  $Q^2$  we must take all the terms in eq. (5.5), they being of the same order. It is simple to find the Borel transform, which with the leading order approximation, simplifies to

$$e^{-m_V^2/M^2} \int dE^2 \left(1 - \frac{E^2 + m_V^2}{M^2} + \frac{E^4}{2M^4}\right) \text{Im}\Pi(E).$$

It is clearly of order  $T^4$ .

We thus have the result that the self-energy diagram with the  $\pi\omega$  loop gives a correction to the pole residue to order  $T^2$ , but the Borel transform of the amplitude has no contribution



to this order. It disproves the ansatz by earlier authors for the pole term in the spectral representation, which retains this correction to the residue at large space-like  $Q^2$ . It amounts to the neglect of the third, regular term in eq. (5.5), which is of course justified near the pole, but not for large space-like momenta, where the pole and the regular terms are not only of comparable magnitudes, but actually cancel each other in the leading order. A similar ansatz for the nucleon pole is also disproved in the context of the nucleon sum rules [14].

## 6. Sum rules

Having calculated the spectral sides of the sum rules in the last section, we now need their operator sides. The latter is given by the well-known expansion of the product of current operators into a series of appropriate local operators with calculable c-number coefficients. Since we are really interested in the thermal sum rules from which the corresponding vacuum sum rules have been subtracted out, the operators, whose ensemble average contribute to the lowest order in  $T$ , turn out to be the isospin non-scalar four-quark operators,

$$O_{V,A} = \alpha_s \bar{q} \gamma_\mu (1, \gamma_5) \frac{\tau^c t^i}{2} q \bar{q} \gamma^\mu (1, \gamma_5) \frac{\tau^c t^i}{2} q,$$

$\alpha_s = g_s^2/4\pi$  being the strong interaction fine structure constant. Their Wilson coefficients are known [8],

$$\begin{aligned} i \int d^4x e^{iq \cdot x} \langle T V_\mu^a(x) V_\nu^b(0) \rangle &\rightarrow \delta^{ab} \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{8\pi}{3Q^4} \langle O_A \rangle, \\ i \int d^4x e^{iq \cdot x} \langle T A_\mu^a(x) A_\nu^b(0) \rangle &\rightarrow \delta^{ab} \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{8\pi}{3Q^4} \langle O_V \rangle. \end{aligned} \quad (6.1)$$

If we now retain only the vacuum and the one-pion states in the expansion of the thermal average of the operators and use the familiar techniques of PCAC and Current Algebra to evaluate the latter, we get

$$\langle O_{V,A} \rangle = \langle 0 | O_{V,A} | 0 \rangle \mp \frac{T^2}{6F_\pi^2} \langle 0 | O_V - O_A | 0 \rangle.$$

It is now simple to write the sum rules for a correlation function by equating the Borel transform of the spectral representation to that of the operator product expansion. It turns out that both the vector and the axial-vector *thermal* correlation functions give rise to the same sum rules,

$$\begin{aligned} -F_\rho^2 e^{-m_\rho^2/M^2} + F_{a_1}^2 e^{-m_{a_1}^2/M^2} + F_\pi^2 &= -\frac{4\pi}{3M^4} \langle 0 | O_V - O_A | 0 \rangle, \\ -m_\rho^2 F_\rho^2 e^{-m_\rho^2/M^2} + m_{a_1}^2 F_{a_1}^2 e^{-m_{a_1}^2/M^2} &= \frac{8\pi}{3M^2} \langle 0 | O_V - O_A | 0 \rangle. \end{aligned} \quad (6.2)$$

It is interesting to observe that these sum rules are nothing but the vacuum sum rules derived from the difference of the correlation functions of the vector and the axial-vector currents, whose spectral side consists of the pole terms due to  $\pi$ ,  $\rho$  and  $a_1$  exchanges. Also as  $M^2$  tends to infinity, we recover the well-known sum rules

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$$F_\rho^2 - F_{a_1}^2 - F_\pi^2 = 0, \quad m_V^2 F_\rho^2 - m_A^2 F_{a_1}^2 = 0, \quad (6.3)$$

originally derived by Weinberg [15] as superconvergence sum rules for the difference of the spectral functions.

## 7. Conclusion

We have reviewed here two methods that have been used in the literature to determine the shifts in the masses and couplings, the strongly interacting particles suffer while moving through a medium. One is the effective Lagrangian method of  $\chi$ PT, which can be readily extended to such problems at finite temperature (and density). We wish to point out that although there are other Lagrangians [16] that have been used for such determinations, it is only the Lagrangian of the  $\chi$ PT which is derived directly from QCD without any extraneous assumptions.

The other is the method of QCD sum rules extended to finite temperature. We use the effective Lagrangian of  $\chi$ PT to calculate the spectral side of the sum rules. It is found that an ansatz often used in the literature disagrees with this calculation. Further when the sum rules are evaluated using our calculation for the spectral side, one does not obtain any new results, but simply reproduces the old results derived from the vacuum sum rules.

Although the QCD sum rules do not lead to any new prediction at finite temperature, they are expected to yield new results in a medium with non-zero chemical potential, e.g., the nuclear medium. Here the spectral side can again be calculated using  $\chi$ PT. The sum rules would then predict the condensates in the medium, for which we have no reliable method of determination at present.

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## References

- [1] J Gasser and H Leutwyler, *Ann. Phys.* **142** (1984); *Nucl. Phys.* **B250**, 465 (1985)
- [2] J Gasser and H Leutwyler, *Phys. Lett.* **B184**, 83 (1987)
- [3] P Gerber and H Leutwyler, *Nucl. Phys.* **B231**, 387 (1989)
- [4] N P Landsmann and Ch G van Weert, *Phys. Rep.* **145**, 141 (1987)
- [5] A J Niemi and G W Semenoff, *Ann. Phys. (NY)* **152**, 105 (1984)
- [6] H Leutwyler and A V Smilga, *Nucl. Phys.* **B342**, 302 (1990)
- [7] S Mallik and S Sarkar, *Euro. Phys. J.* **C25**, 445 (2002)
- [8] M A Shifman, A I Vainshtein and V I Zakharov, *Nucl. Phys.* **B147**, 385 (1979)
- [9] A I Bochkarev and M E Shaposhnikov, *Nucl. Phys.* **B268**, 220 (1986)
- [10] T Hatsuda, Y Koike and S H Lee, *Nucl. Phys.* **B394**, 221 (1993). The earlier references may be traced from this paper

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- [11] S Mallik and S Sarkar, *Phys. Rev.* **D66**, 056008 (2002)
- [12] G Ecker, J Gasser, A Pich and E de Rafael, *Nucl. Phys.* **B321**, 311 (1989)
- [13] G Ecker, J Gasser, H Leutwyler, A Pich and E de Rafael, *Phys. Lett.* **B223**, 425 (1989)
- [14] S Mallik and S Sarkar, *Phys. Rev.* **D65**, 016002 (2002)  
See also Y Koike, *Phys. Rev.* **D48**, 2313 (1993)
- [15] S Weinberg, *Phys. Rev. Lett.* **18**, 507 (1967)
- [16] U-G Meissner, *Phys. Rep.* **161**, 213 (1988)