Review of chiral perturbation theory

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Abstract. A review of chiral perturbation theory and recent developments on the comparison of its predictions with experiment is presented. Some interesting topics with scope for further elaboration are touched upon.

1. Introduction

Quantum chromodynamics (QCD) (for a review, see, e.g. [1]), is the microscopic theory of the strong interactions. It is formulated for quark and gluon degrees of freedom, and its fundamental coupling constant is the strong coupling constant α_s. The theory is asymptotically free, in that as a renormalizable field theory, the coupling constant becomes small at large momenta, due to the non-abelian nature of the gauge interactions and due to the fact that the number of quark flavours is relatively small. On the other hand, a Landau singularity at low energies renders the theory non-perturbative. Furthermore, the asymptotic spectrum of the theory consists of mesons and baryons, which implies that a full solution to QCD must include the phenomenon of confinement of quarks and gluons and ensure that these emerge as the asymptotic states of the theory. When the energies under consideration are small enough for the t, b, c quark degrees to have frozen out, striking features of the spectrum include the fact that some of the mesons are very light compared to the baryons. One way to understand this, due to Nambu, is that of spontaneous breakdown of approximate chiral symmetries of the strong interaction Lagrangian. The members of the pseudo-scalar octet are the Goldstone bosons associated with these spontaneously broken symmetries, and η' would be the ninth Goldstone boson associated with the additional $U(1)_A$ symmetry that would be realized when N_c the number of colours tends to infinity. In the real world where $N_c = 3$, this symmetry is anomalous and therefore η' is very far from being a Goldstone boson.

Chiral perturbation theory is the effective low-energy theory of the strong interactions and is an expansion of the Green functions of its currents associated with the near masslessness of the three lightest quarks, and with the spontaneously broken approximate axial vector symmetries, in powers of momentum and quark masses. The modern formulation of the subject was presented in an influential series of papers for the case of 2 flavours in [2] and for the case of 3 flavours in [3]. For a pedagogical introduction, see [4], and for

detailed reviews, see [5]. It is also possible to include the electromagnetic and weak interactions into the framework, in order to describe the effects of virtual photons and in order to study semi-leptonic decays of mesons. The nucleons and the baryon octet may also be included in the theory that results in baryon chiral perturbation theory. The relativistic formulation was first presented in [6]. Other versions include heavy baryon chiral perturbation theory [7], and the recent infra-red regularized baryon chiral perturbation theory [8].

In the mesonic sector, at leading order one starts out with the non-linear sigma model, and employing the external field technique, one arrives at the next to leading order in the momentum expansion with an effective Lagrangian whose coupling constants absorb the divergences that are generated by the non-linear sigma model Lagrangian at one-loop order. This procedure has also been carried out to the next to next to leading order, and can in principle be carried out to arbitrary order. At the formal level, an important invariance theorem has been proved by Leutwyler [9], where it is shown that in order to arrive at a consistent formulation for the generating functional, one must necessarily consider the symmetries of QCD (the underlying level) at the local level, and establish the gauge invariance of the generating functional for non-anomalous symmetries. Note that the coupling constants at each order have been fixed by comparison with experiment. Recently there has been some interest in trying to measure some of these constants on the lattice (see [10]).

For instance, at the lowest order, the effective Lagrangian in the 2-flavour case with mesons alone, which involves only the pion mass and the pion decay constant, reads:

$$\mathscr{L}_M = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U + M^2 (U + U^\dagger) \rangle,$$

where $U \in SU(2)$ and $\langle A \rangle$ denotes the trace. The matrix U contains the pion fields and F is the pion-decay constant. The mass of the pion is such that $M^2 = 2\hat{m}B$, where $\hat{m}$ is the average of the u- and d-quark masses, and B is the value of the quark condensate, the leading order parameter that determines the spontaneous symmetry breaking of the chiral symmetry. Today the accurate measurements of pion scattering lengths, to be reviewed later, have confirmed this to be the case, something that had been called into question [11]. At a purely theoretical level, if the quark condensate had indeed vanished, one would have to find other order parameters which would have led to spontaneous symmetry breaking. These would have higher dimension operators such as a mixed condensate of dimension five involving quark and gluon field tensors, or those of higher dimensions, which are less and less theoretically appealing, but distinctly allowed by the theory.

In the following sections, we will review some of the important and interesting advances that have been made in the formalism, computation and comparison of the theory with experiment.

2. Meson physics

Chiral perturbation theory achieves its best results in the purely mesonic sector. Furthermore, when one confines oneself to the three light mesons, an unprecedented level of accuracy in the low-energy sector can be reached, which provides a great challenge also to experiment. As far as the formalism is concerned the complete enumeration of the terms of the effective Lagrangian at two-loop or $O(p^6)$ order is now available [12]. The formidable task of renormalizing the theory at this order is also complete [13]. This is

achieved by starting out with the lowest order Lagrangian and expanding around its classical solutions to generate the loop-expansion of the generating functional. The divergences of the one-particle irreducible diagrams are always local and can be renormalized. The determination of the divergences turns out to be the evaluation of Seeley–DeWitt coefficients and the reduction of the result to a standard basis of operators. The result is checked by the requirement of the Weinberg consistency conditions, which state that the residues of the poles in $(d-4)$, in dimensional regularization, should be polynomials in external momenta and masses. The anomalous sector has also been considered (see [14]). The role of the scalar, vector, axial-vector, tensor resonances is also well-understood in the modern framework, and in general the bulk of the contribution to the low-energy constants come from resonance saturation, and has been elaborated in [15].

2.1 $\pi\pi$ Scattering

Pion–pion scattering is perhaps the simplest of all hadronic scattering problems, since the pions are the lightest hadrons. In the absence of electromagnetic interactions, the pions do not have a bound state, and as a result, dispersion relations can be written in a simple and straightforward manner for the system and are of great importance for the analysis of experimental information. On the other hand, this setting is also the one where chiral perturbation theory is expected to achieve maximum accuracy. This interplay of pion–pion scattering amplitudes in chiral perturbation theory and their dispersion relation representation have now been studied in great detail [16].

The starting point of this recent work is the computation of the amplitudes to two-loop precision [17,18]. The scattering amplitudes at this order involve two additional low-energy constants over those that enter the expressions at one-loop order. A systematic analysis of pion–pion scattering information available at medium and higher energies, far away from the threshold region relies on a system of integral equations for the partial waves, the Roy equations [19]. Since dispersion relations for pion–pion scattering amplitudes in the s-channel converge with as many as two-subtractions, fixed-t dispersion relations bring in unknown t-dependent functions. Using crossing symmetry effectively, these functions may be eliminated in favour of the two S-wave scattering lengths a_0^0 and a_0^2 (note that there are three iso-spin amplitudes with $I = 0, 1, 2$, generalized Bose statistics imply that for even I, $l = 0, 2, 4, ...$, and for $I = 1$, $l = 1, 3, 5, ...$; and the partial wave expansions for the iso-spin amplitudes given by

$$T^I(s,t,u) = 32\pi \sum (2l+1) f_l^I(s) P_l \left(\frac{t-u}{s-4m_\pi^2} \right),$$

where s, t, u are the conventional Mandelstam variables and $f_l^I(s)$ are the partial wave amplitudes, such that we may introduce the threshold expansion in the physical region $\text{Re } f_l^I(q^2) = q^{2l}(a_l^I + b_l^I q^2 + \cdots), q^2 = (s - 4m_\pi^2)/4)$. At leading order, the predictions for the two S-wave scattering lengths is the one given by Weinberg from current algebra and these read $a_0^0 = 7m_\pi^2/32\pi F_\pi^2 (\simeq 0.16)$ and $a_0^2 = -m_\pi^2/16\pi F_\pi^2$ respectively. Once the Roy representation for the amplitudes is so obtained, the Roy equations for the partial waves are obtained by expanding the amplitude and the absorptive part of the amplitude in terms of partial waves. Thus the Roy equations for each partial wave involves the scattering lengths (for $l \leq 2$), and the imaginary parts alone of all the partial waves in the physical

region and therefore allows a reconstruction of the entire amplitude from knowing the scattering lengths and the imaginary parts of the partial waves in the physical region. We note that in the *t*-channel, the $I = 1$ amplitude is such that a convergent dispersion relation with one subtraction can be written down, which renders the combination $2a_0^0 - 5a_0^2$ into a tightly constrained quantity. By assuming that in the low-energy region, the scattering is dominated by the *S*- and *P*-waves, one may turn the Roy equations for these into a closed system, absorbing the high energy and higher angular momentum state contributions into inhomogeneous terms for these. (It may also be noted that up to and including two-loops in chiral perturbation theory, the amplitudes have cuts generated only by the *S*- and *P*-waves.) The medium energy data in the form of phase shifts come from a variety of sources including the reaction $\pi N \rightarrow \pi\pi N$, the accurate measurement of the ρ-shape from the electromagnetic form factor of the pion from the reaction $e^+e^- \rightarrow \pi^+\pi^-$ performed by the CLEO Collaboration. Other sources could be the decay $\tau \rightarrow \pi\pi\nu$.

In the modern context, a fresh Roy equation analysis with the view of combining dispersion relations with chiral perturbation theory has been carried out [20]. The evaluation of the inhomogeneous terms, the so-called 'driving terms' for the Roy equations requires a detailed analysis of the *D*- and *F*-waves from threshold up to the beginning of the asymptotic region, of all the waves in the asymptotic region from Regge dynamics and from Pomeron contributions, and the treatment of crossing constraints on absorptive parts of the amplitude, which leads to their determination to rather small uncertainties.

This Roy equation analysis from which certain low-energy constants of the chiral expansion are accurately determined, and together with computations in chiral perturbation theory both for the pion scattering amplitudes and for its scalar form-factor yields a precise two-loop prediction for the scattering lengths, giving $a_0^0 = 0.220 \pm 0.005$ [21]. One of the most significant experiments is the one where the phase shift difference $\delta_0^0 - \delta_1^1$ in the near threshold region is probed by the rare decay K_{l4}. The E-865 experiment at Brookhaven National Laboratory has reconstructed 400,000 events and reported a value $a_0^0 = 0.229 \pm 0.015$ [22], which is to be compared with the results from the Geneva-Saclay experiment of 1977 based on 30,000 reconstructed events, which yielded $a_0^0 = 0.26 \pm 0.05$.

It may therefore be concluded that, the problem studied probably in greatest detail in dispersion relation theory, based on axiomatic field theory, is now at a stage where experiment and theory are in full agreement after decades of analysis. This remarkable agreement also rules out a theoretical possibility that the leading order parameter of spontaneous symmetry breaking is not the quark condensate which would have implied a reordering of the chiral expansion, and a significantly larger value for a_0^0, perhaps close to even 0.30. In other words, the E-865 data leads to a precise measurement of the quark condensate [23], and is therefore a sensitive probe of the ground state of QCD.

2.2 *Pionium lifetime computations*

Since it has always been a challenge to find experimental settings in which pion scattering phenomena can be studied, due to the absence of pion targets, it is important that the results from the K_{l4} decays be independently checked. One system that affords such a test is that of pionium, where the decay of the atom made up of π^+ and π^- bound by the electromagnetic interaction, decays into a final state with $2\pi^0$s. The lifetime of the atom gives a direct measurement of the combination of scattering lengths: $a_0^0 - a_0^2$. This

was first considered in [24], and is referred to as Deser's theorem. In order to estimate the lifetime at the desired level of precision, a bound state non-relativistic theory has been formulated [25], which has developed further a formalism first proposed by Caswell and Lepage. On the experimental side, it is expected that the DIRAC experiment at CERN will reach the desired level of precision.

2.3 πK Scattering

The process in the full $SU(3)$ theory, which also entails an expansion in the mass of the s-quark, that probes the structure of the theory is that of pion–kaon scattering. At the formal level, the problem is more complicated than that of the pion–pion case, since the process is inelastic due to the $\pi\pi \to KK$ process in the t-channel. The one-loop chiral amplitude was first presented in [26].

Indeed, as in the pion–pion scattering case, here too the amplitudes which now number six, up to two-loop accuracy have cuts generated only by the S- and P-waves. By considering a system of amplitudes that are linear combinations of the iso-spin amplitudes, that are even and odd under the interchange of the Mandelstam variables s and u, the dispersive representations for the amplitudes have been written in a manner whereby this can be compared with the chiral representations for the same amplitudes. The corresponding dispersion relation representation is technically more complicated, and the economical Roy representation that was available for the pion–pion case which exploited three-channel crossing symmetry is no longer available. Instead, one turns to the Roy–Steiner representation for the dispersion relations which employs both fixed-t and hyperbolic dispersion relations in order to be able to perform a comparison with the chiral representation [27].

On the experimental side, precise data available is significantly less. For a review of the experimental and theoretical scenario till 1970, see [28]. While there are more recent measurements of the S- and P-waves and some higher wave phase shifts for the $I = 1/2, 3/2$ amplitudes, in the t-channel the $I = 0, 1$ waves are also known in the physical region to varying degrees of accuracy. In the unphysical region $4m_\pi^2 \leq t \leq 4m_K^2$, the waves have to be reconstructed from generalized unitarity using Omnès techniques. While a full Roy equation analysis is in progress [29], an analysis based on existing information has led to an estimate for the large N_c suppressed low-energy constant L_4 [30], which had been estimated in the past only from Zweig rule arguments. A complete analysis combined with accurate experimental information would also shed light on the important question of possible flavour dependence of the quark condensate, and would be a sensitive probe of the QCD ground state in the $SU(3)$ theory. On the experimental side, it is hoped that pion–kaon atoms can be studied which will lead to a measurement of the scattering length $a_0^- (\neq a_0^{1/2} - a_0^{3/2})/3)$, and that the S- and P-waves s-channel can be better measured at, e.g., the COMPASS experiment at CERN. The t-channel waves are discussed in some detail in ref. [31] whose considerations must be included in future analyses.

3. Inclusion of weak and electromagnetic interactions

From the earliest days of chiral perturbation theory, the form factors for various semi-leptonic decays have been considered, in order to study the agreement between theory and

experiment (for a review see, [32]). There has been considerable amount of work for the measurements to be performed at DAFNE, and for the KLOE detector.

Recently at the formal level there has been interest in including the effects of virtual photons, which would correct the masses for the pseudo-scalars and would also test well-known theorems established during the days of current algebra calculations such as Dashen's theorem, which says that in the chiral limit, $m_{\pi^+}^2 - m_{\pi^0}^2 = m_{K^+}^2 - m_{K^0}^2$. Introducing a chiral power counting in this sector leads to additional terms in the effective Lagrangian at $O(e^2 p^2)$. The corresponding Lagrangian has been evaluated [33], and phenomenological implications have been studied (see [34,35]). There are now additional low-energy constants associated with this effective Lagrangian, and there is very little knowledge on the magnitudes of these quantities, and their determination is a matter of current research (see [36,37]). In order to reach higher precision to account for virtual leptons in semi-leptonic decays, the theory has to be further extended and the corresponding effective Lagrangian evaluated. This was achieved in ref. [38], and the decays π_{l2} and K_{l2} were studied in great detail. Another interesting process that has been considered is the beta decay of the charged pion [39], which would lead to the determination of the Kobayashi–Maskawa matrix element V_{ud} at an unprecedented level of accuracy, provided sufficiently accurate experimental information is available.

The presence of the anomaly in QCD is accounted for in the effective theory by the inclusion of the Wess–Zumino–Witten term which arises at $O(p^4)$. By including electromagnetism as an external source, this accounts for the observed neutral pion decay into two photons. This is often referred to as the anomalous or the odd intrinsic parity sector. Recent developments in this field include the evaluation of the generating functional in this sector at $O(p^6)$ [14] and also the inclusion of virtual photons to yield the generating functional at $O(e^2 p^4)$ [40]. Implications to the neutral pion lifetime have been considered in some detail [40,41] which will be measured to high accuracy at Jefferson Laboratory at the experiment PrimEx.

4. Baryon chiral perturbation theory

Baryon chiral perturbation theory in the modern era was first formulated in [6]. This was a relativistic formulation, which faced the technical difficulty that no chiral power counting was possible when the baryon was included in this framework. Inspired by developments in the field of heavy-quark physics, a version called heavy baryon chiral perturbation theory was proposed, a formalism within which it was possible to restore chiral order [7]. This suffered from the deficiency that it was no longer manifestly Lorentz invariant. This showed up in the analysis of certain form factors where analyticity properties were destroyed. A recent variant which simultaneously accounts for manifest Lorentz invariance and chiral order has been developed [8]. For related work that preceded this, see [42]. The main feature that is addressed is that in the chiral limit, the baryon still remains massive.

This infra-red regularized baryon chiral perturbation theory is formally very interesting, and is also the basis of computations of observables and much work is being carried out in comparing the consistency of this framework and experimental information. For instance, the pion–nucleon scattering amplitudes have been studied in [43,44]. The corner stones of these studies include the corrections to the Goldberger–Treiman relation and estimation of the sigma term. Predictions for the masses of the baryon octet in heavy baryon chiral

perturbation theory were presented in [45] and a comparison between the heavy baryon and infra-red versions of the theory may be found in [46]. Magnetic moments have been considered in the heavy baryon case in ref. [47], and work is in progress for the infra-red case [48]. For results on form factors, see [49]. Despite many successes, a coherent picture of the nature of convergence of baryon chiral perturbation theory, especially in the full $SU(3)$ is yet to emerge. Difficulties associated with iso-spin breaking in pion–nucleon scattering need to be dealt with in a systematic manner, especially when fresh data from pion photo-production is made available.

5. Additional topics

It has been pointed out earlier that the chiral prediction for pion scattering lengths is a sensitive probe of the ground state of QCD and chiral symmetry breaking therein. Furthermore, the properties of the ground state and the non-perturbative sector of QCD have important implications for the chiral expansion, and in particular for the values of some of the low-energy constants, and for those processes which involve the axial anomaly. Indeed, it follows that the study of the global properties of the QCD Lagrangian alone will not suffice for pinning down the chiral expansion precisely. One of the additional topics we consider is the N_c^{-1} expansion. We also briefly mention the topic of simulations on the lattice that might allow one to derive estimates for some low-energy constants. Finally, we comment on the thermodynamics of QCD and the limits in which chiral symmetry is able to essentially determine its structure.

5.1 *Large N_c*

An extremely interesting and subtle aspect of chiral perturbation theory arises when one is trying to address the issue of the N_c^{-1} expansion. In particular, the role of η' is modified in this approach, and one must understand the dynamics behind the Okubo–Zweig–Izuka rule. Right from the original work in modern chiral perturbation theory, this has been highlighted. Recent work in the applications of the expansion may be found in [50,51]. Indeed, we had pointed out earlier that one of the low-energy constants L_4 which is suppressed in the N_c^{-1} expansion has now been estimated from πK sum rules, and supports earlier estimates from the Zweig rule.

5.2 *Chiral perturbation theory and the lattice*

A relatively new area of research has been the one in which one may try to evaluate the low-energy constants using lattice simulations. For instance, one may wish to evaluate the ratio of decay constants F_K/F_π on the lattice which would give an estimate for one of the low-energy constants. Furthermore, in chiral perturbation theory one may study the quark mass dependence of physical observables at a given order, which can be done independently on the lattice. However, problems remain in trying to reach small quark masses on the lattice and one would have to wait some more years to have a reliable estimate of the low-energy constants using this approach. There is also more recent work in the field which

is not discussed here. However, it may be noted that the lattice simulations might be able to test theoretical ideas regarding the large N_c expansion in a novel manner. Of special interest would be the possibility of testing the mixed expansion proposed in ref. [50]. Note that there has also been an effort to determine the $I = 2$ pion scattering length a_0^2 on the lattice [52], for which we now have precise chiral predictions and measurements from E-865.

5.3 *Chiral perturbation theory and thermodynamics*

Another rich field is that of the behaviour of chiral perturbation at finite temperature. The foundations of the subject were laid in [53]. There it has been shown that for small quark masses, low temperatures and large volumes, the strong interaction partition function is essentially fixed by chiral symmetry. In addition, the law governing the temperature dependence of the quark condensate has been determined. Its dependence on the quark mass has also been computed. The formal similarities between the finite temperature and the lattice simulations provide useful checks for the latter. The nature of pions at finite temperature [54], in particular the expansions for masses and decay constants have also been studied. Of related interest is the $\pi^0 \to 2\gamma$ rate at finite temperature [55].

6. Outlook

In this talk I have outlined what I consider to be some specially interesting topics. The list of references provided in this written version of the talk is not a complete list by any means in this very active field of research, but a list of representative papers that might give the reader a flavour of the subject. What is specially striking is the fact that effective field theories can be as predictive as they have been in this context. The criterion of renormalizability which is associated with predictivity cannot be applied to this field. I have also tried to bring the reader's attention to the areas where there is much scope for active research in the near future. I have not discussed some important topics, e.g., non-leptonic weak decays, $\eta \to 3\pi$ decay [56], etc., which represent important challenges. The *NN* system being significantly more involved has not been addressed in this talk, although there is considerable amount of work in the field.

In addition to computing more interesting processes to two-loop accuracy in chiral perturbation theory, and accounting for electromagnetic corrections, it might also be useful to look into techniques based on axiomatic field theory and disperion relations which are completely general and are independent of the underlying dynamics and of effective field theory. These techniques could yield additional constraints and provide consistency checks both on the effective Lagrangian computations and on the analysis of experimental information. The successful application of these methods has already been demonstrated in the pion–pion and pion–kaon scattering systems, and could definitely be extended to other settings. There has also been a call for improving the experimental picture in several sectors that would test chiral predictions, including photon-induced reactions, and those in which kaons are involved in a significant manner [57].

It might also be worthwhile to revisit several arguments that are available in the literature for the inevitability of spontaneous symmetry breaking of the axial-vector symmetries of

QCD, including results due to Vafa–Witten and Banks–Casher (reviewed in ref. [58]) and to see if any fresh insights can be found. Finally, we note that after this talk was given, a series of comprehensive discussions has appeared on the archives [59].

Acknowledgements

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