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α_s Measurements

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Abstract. The strong coupling constant, α_s, has been determined from many pure inclusive and semi-inclusive measurements. All these measurements, measured at different scales, are consistent among each other and the measurements can be combined to give $\alpha_s(m_Z) = 0.118 \pm 0.003$.

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1. Introduction

The theory of strong interaction QCD [1] has only one free parameter, the coupling constant, α_s. QCD has a well-defined prediction for the energy dependence of α_s – large at low energies (large distances leading to confinement of quarks) and small at high energies (small distances leading to asymptotic freedom). α_s appears with colour factors in all basic couplings of quarks and gluons: (a) $C_F \cdot \alpha_s$ for quark bremsstrahlung ($C_F = \frac{N_C^2-1}{2N_C} = \frac{4}{3}$), (b) $T_R \cdot \alpha_s$ in gluon splitting ($T_R = \frac{1}{2}$), (c) $C_A \cdot \alpha_s$ in triple gluon vertex ($C_A = N_C = 3$).

The value of α_s is obtained from a variety of pure inclusive and semi-inclusive measurements and the theoretical calculations for these processes are complete to different order of perturbation theory. The processes with the order of QCD calculation are summarised below:

Total hadronic cross-section in Z decays	$\mathcal{O}(\alpha_s^3)$
Hadronic decays of τ-branching ratio, spectra	$\mathcal{O}(\alpha_s^3)$
Kinematic distributions of hadronic final states in e^+e^- interactions:	
Jet rates	$\mathcal{O}(\alpha_s^2)$
All event shape variables	$\mathcal{O}(\alpha_s^2)$
Selective event shape variables (1–T, $\rho_H, \ldots$)	$\mathcal{O}(\alpha_s^2)$ + resummation
Scaling violation in deep-inelastic scattering and in fragmentation functions	$\mathcal{O}(\alpha_s^2)$
Jet rates in high energy hadron–hadron or lepton–hadron scattering	$\mathcal{O}(\alpha_s^2)$
Hadronic decays of heavy quarkonia	$\mathcal{O}(\alpha_s^2)$

In the following sections, we describe the measurements of α_s from each of these processes and we summarise in §8.

2. Scaling violation

The first quantitative test of perturbative QCD has been carried out in the scaling violation in deep inelastic scattering (observed in ep and νN scattering). Now large amount of data exist on structure functions from $ep/\mu p/\cdots$ experiments. Most recent data come from the H1 and ZEUS experiments [2] at the HERA collider. QCD provides Q^2 dependence of structure functions in terms of parton density function (P) and α_s:

$$\frac{\mathrm{d}F_2^{\mathrm{em}}}{\mathrm{d}\ln Q^2} = \frac{\alpha_s}{2\pi}\{P_{qq}\otimes F_2^{\mathrm{em}} + [P_{qG}\otimes xG]\}.$$

Fits have been performed to determine parton density function and α_s. Tevatron jet data have been used to constrain gluon density at large x. Treating correlated errors properly, one obtains

$$\alpha_s(m_Z) = 0.119 \pm 0.002\,(\mathrm{expt}) \pm 0.003\,(\mathrm{theory}).$$

QCD evolution of non-singlet structure function is known to $\mathscr{O}(\alpha_s^3)$:

$$\int_0^1 \mathrm{d}x(F_3^{\bar{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2)) = 3\left[1 - \left(\frac{\alpha_s}{\pi}\right) - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 \right. \\ \left. -19.0\left(\frac{\alpha_s}{\pi}\right)^3 - \Delta\mathrm{HT}\right].$$

The higher twist term (ΔHT) has been estimated and a fit to the existing ν data [3] yields

$$\alpha_s(m_Z) = 0.118 \pm 0.011.$$

The spin dependent structure functions measured in polarised lepton–hadron scattering [4] have been used to determine α_s:

$$\alpha_s(m_Z) = 0.114\,^{+\,0.004}_{-\,0.005}\,(\mathrm{expt}) \pm 0.009\,(\mathrm{theory}).$$

Fragmentation functions ($\mathrm{d}_i(z,E)$) have been measured [5] in e^+e^- collisions for a variety of hadrons (i) with energy fraction (z) of the initial parton of energy E. Flavour tag and 3-jet analysis have been used to dis-entangle quark and gluon fragmentation. Global analysis of the energy evolution measures α_s:

$$\alpha_s(m_Z) = 0.117\,^{+\,0.006}_{-\,0.007}\,(\mathrm{expt})\,^{+\,0.002}_{-\,0.003}\,(\mathrm{theory}).$$

3. Inclusive jet production

Inclusive cross-section of jet production has been measured in hadron–hadron [6] and lepton–hadron [7] scattering. CDF has defined jets using the cone algorithm with cone angle of 0.7. The inclusive cross-section has been measured as a function of transverse jet

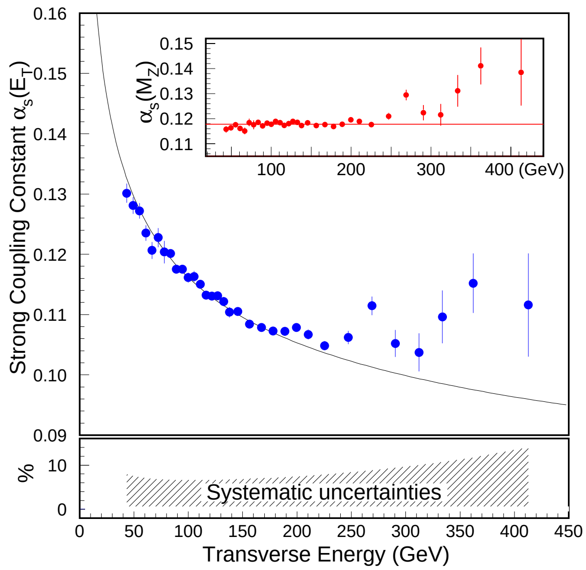

Figure 1. Strong coupling constant α_s measured by the CDF experiment from inclusive jet cross-section as a function of energy scale.

energy (E_T) between 50 GeV and 400 GeV. In that range, the cross-section drops by more than seven orders of magnitude. This cross-section has been calculated to next-to-leading order:

$$\frac{d\sigma}{dE_T} = \alpha_s^2 \times F_{LO} + \alpha_s^3 \times F_{NLO}.$$

Using these measurements, evolution of α_s has been tested over a large energy range (see figure 1).

ZEUS Collaboration [7] has studied inclusive jet production by reconstructing jets using the $k_\perp$ algorithm [8] in the Breit frame. Measurements have been made over a large range of Q^2 as well as E_T. NLO $(\mathscr{O}(\alpha_s^2))$ calculation has provided a reasonable description of data over the entire range of Q^2 and E_T giving

$$\alpha_s(m_Z) = 0.1212 \pm 0.0017\,(\text{stat.})\,^{+0.0023}_{-0.0031}\,(\text{syst.})\,^{+0.0028}_{-0.0027}\,(\text{theory}).$$

4. Quarkonium decays

Decay branching ratio of heavy quarkonia can be used to determine α_s. In these determinations one assumes the hadronic and leptonic decay widths to factorise into a perturbative and a non-perturbative part. Three sets of measurements have been used to measure α_s:

- The ratio of partial widths to hadrons and to muon pair is measured: $R_\mu(\Upsilon) = \frac{\Gamma(\Upsilon\to\text{hadrons})}{\Gamma(\Upsilon\to\mu^+\mu^-)}$.

The ratio is corrected for relativistic nature of $Q\overline{Q}$ system and for non-perturbative correction due to colour octet contribution.

- The ratio of partial widths:

  $R_\gamma(\Upsilon) \; = \; \frac{\Gamma(\Upsilon\to\gamma gg)}{\Gamma(\Upsilon\to ggg)}$

  is measured and higher order calculation for radiative decays has been used.
- The moments of the ratio of cross-section:

$$M_n = \int_0^\infty \mathrm{d}s \frac{R_b(s)}{s^{n+1}}$$

$$R_b(s) = \frac{\sigma(e^+e^- \to b\overline{b})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

are used.

Bulk of these measurements have been done at Υ(1S) [9] and these measurements can be combined to provide

$$\alpha_\mathrm{s}(m_Z) = 0.109 \, \pm \, 0.004.$$

5. *Z* lineshape

The ratio (R_Z) of partial width of Z to hadrons and Z to lepton pairs:

$$R_Z = \frac{\Gamma(Z \to q\overline{q})}{\Gamma(Z \to e^+e^-)} = \frac{\Gamma_q}{\Gamma_\ell}$$

has been determined [10] at LEP and SLC from measurements of inclusive cross-sections as a function of centre-of-mass energy. This measurement has been compared with theory in improved Born approximation and assuming that the QCD correction will factorise out:

$$R_Z = R_Z^0 \cdot (1 + \delta_\mathrm{QCD})$$

$\delta_\mathrm{QCD} \simeq 4\%$ and is used to measure α_s.

Here no assumption has been made on the hadronisation mechanism and the use of a ratio helps partial cancellation of electroweak radiative corrections. So one expects to measure α_s with small theoretical uncertainty.

The effect of lepton mass has to be taken care of – otherwise this will alter the α_s value by 2%. δ_QCD has been calculated to $\mathscr{O}(\alpha_\mathrm{s}^3)$:

$$\delta_\mathrm{QCD} = a_1 \left(\frac{\alpha_\mathrm{s}}{\pi}\right) + a_2 \left(\frac{\alpha_\mathrm{s}}{\pi}\right)^2 + a_3 \left(\frac{\alpha_\mathrm{s}}{\pi}\right)^3.$$

One should note that QCD correction affects R_Z^0 through its contribution to Z self energy and to $Zb\overline{b}$ vertex. QCD corrections to the vector and the axial vector components are also different. So factorisation is not exact. Some higher order corrections exist due to recent calculations:

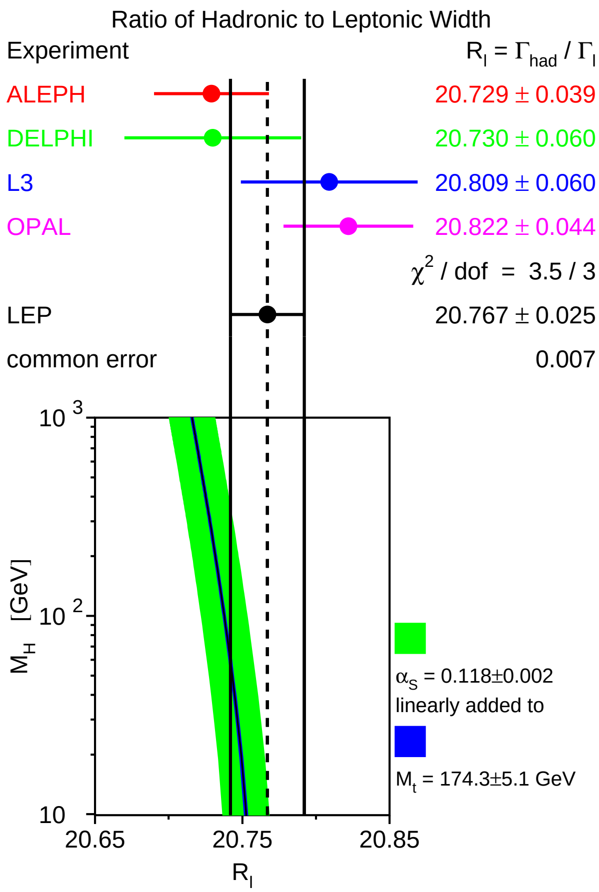

Figure 2. Combination of ratio of Z partial width to hadrons to that to leptons from the four LEP experiments.

- $\mathscr{O}(G_F^2 m_t^4)$ correction for ρ and $Zb\bar{b}$ vertex,
- effect of heavy top on Z self energies and $Zb\bar{b}$ vertex in $\mathscr{O}(\alpha_s G_F m_t^2)$,
- $\mathscr{O}(\alpha\alpha_s)$ correction for Z self energy,
- $\mathscr{O}(\alpha_s^3)$ correction to Z decay rates to hadrons for both vector and axial vector components,
- $\mathscr{O}(\alpha_s^2)$ m_t dependent corrections,
- complete mass corrections of $\mathscr{O}(\alpha_s^2 \frac{m_b^2}{m_Z^2})$ to the axial coupling of Z boson,

and these have been used during the extraction of α_s.

The measurements from the four LEP experiments have been combined with a χ^2 of 3.5 for three degrees of freedom (see figure 2). This measurement provides one of the cleanest determination of α_s:

$$\alpha_s(m_Z) = 0.121 \pm 0.004\,(\text{expt}) \pm 0.004\,(\text{theory}).$$

Estimation of theoretical uncertainties include: (i) uncertainties in the electroweak corrections and their implementations (studied using different electroweak libraries); (ii) uncertainty due to light quark contribution in radiative corrections for α_{QCD}; (iii) uncertainty due to error in b quark mass; (iv) uncertainty due to unknown mass corrections; (v) missing higher orders in QCD calculations; (vi) non-perturbative corrections; (vii) unknown Higgs mass.

LEP EW Working Group has performed a standard model fit to all EW measurements. This fit yields

$$\alpha_{\mathrm{s}}(m_Z) = 0.1199 \pm 0.0030.$$

6. τ Decays

The ratio (R_τ) of τ decay branching ratio to hadrons and that for leptons:

$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau + \mathrm{hadron})}{\Gamma(\tau \to \ell \bar{\nu}_\ell \nu_\tau)} = \frac{1 - B_e - B_\mu}{B_\ell}$$

is also a pure inclusive measurement and has been used to determine α_{s}. Here B is the branching ratio of τ. Mass of the hadronic system from semi-leptonic decays of τ varies between m_π and m_τ. R_τ is measured in e^+e^- colliders from leptonic branching ratio and lifetime measurements. The measurements are then compared with theory:

$$R_\tau = 12\pi \int_0^{m_\tau^2} \frac{\mathrm{d}s}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + \frac{2s}{m_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

$$\mathrm{Im}\Pi^{(J)} = \text{Hadronic spectral function.}$$

The spectral functions have been calculated and R_τ can be expressed in terms of CKM matrix elements and correction terms:

$$R_\tau = 3\left[|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2\right] S_{\mathrm{EW}}\left[1 + \delta_{\mathrm{EW}} + \delta_{\mathrm{QCD}}\right].$$

In quark-parton model, $R_\tau = 3$. The overall correction factor is $\sim$20%. S_{EW} is the sum of leading logs and has been estimated using RGE (= 1.0194). δ_{EW} is the EW correction term and has been calculated to NLL order (= 0.0010). δ_{QCD} has perturbative as well as non-perturbative components. The perturbative component has been calculated to $\mathcal{O}(\alpha_{\mathrm{s}}^3)$:

$$\delta_{\mathrm{QCD}}^{\mathrm{pert}} = \left(\frac{\alpha_{\mathrm{s}}(m_\tau)}{\pi}\right) + 5.2023\left(\frac{\alpha_{\mathrm{s}}(m_\tau)}{\pi}\right)^2 + 26.366\left(\frac{\alpha_{\mathrm{s}}(m_\tau)}{\pi}\right)^3.$$

The non-perturbative part has been estimated using operator product expansion and QCD sum-rule. Effect of quark mass effect has also been estimated and used in the extraction of α_{s}.

B_e and B_μ are directly measured in e^+e^- experiments by identifying τ pair events and then looking for τ decays to one charged particle where the charged particle is identified as an electron or a muon. Most precise measurements come from LEP experiments. Taking world average [11] of all existing measurements

$$B_e = 0.1784 \pm 0.0006, \quad B_\mu = 0.1737 \pm 0.0006$$

and correcting for mass effects (phase space suppression), one obtains:

$$R_\tau = 3.629 \pm 0.015.$$

Alternately B_ℓ is extracted from τ lifetime measurement:

$$B_\ell = \frac{\tau_\tau}{\tau_\mu}\left(\frac{m_\tau}{m_\mu}\right)^5.$$

Again using world average values for m_τ and τ_τ [11]:

$$m_\tau = (1.7770 \pm 0.0003)\ \mathrm{GeV},$$
$$\tau_\tau = (0.2906 \pm 0.0011)\ \mathrm{ps}$$

one obtains

$$R_\tau = 3.645 \pm 0.020.$$

Combining these results from the two independent measurements, one obtains a more precise value for R_τ and hence of α_s:

$$R_\tau = 3.635 \pm 0.012,$$
$$\alpha_s(m_\tau) = 0.35 \pm 0.03.$$

Propagating to a scale m_Z with five quark-flavours and taking care of the threshold effects suitably, this gives:

$$\alpha_s(m_Z) = 0.121 \pm 0.003.$$

ALEPH Collaboration [12] has measured the spectral function for the spin 1 and 0 states of the hadronic system and also separately for the vector and axial-vector components:

$$v_1(s)/a_1(s) = \frac{m_\tau^2}{6|V_{\mathrm{ud}}|^2 S_{\mathrm{EW}}} \cdot \frac{B(\tau \to V/A\nu_\tau)}{B(\tau \to e\bar{\nu}_e\nu_\tau)} \frac{1}{N_{V/A}} \cdot \frac{\mathrm{d}N_{V/A}}{\mathrm{d}s} \cdot \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_0(s) = \frac{m_\tau^2}{6|V_{\mathrm{ud}}|^2 S_{\mathrm{EW}}} \cdot \frac{B(\tau \to \pi\nu_\tau)}{B(\tau \to e\bar{\nu}_e\nu_\tau)} \cdot \frac{1}{N_A} \cdot \frac{\mathrm{d}N_A}{\mathrm{d}s}\left(1 - \frac{s}{m_\tau^2}\right)^{-2}.$$

Events belonging to the τ-pair final state are selected. Each τ-decay is then identified from the number of reconstructed charged and neutral pions. The measured invariant mass spectrum is corrected using a regularised inversion matrix. Constraints from isospin symmetry is used to extract the branching ratios. From the corrected mass spectrum (see figure 3), the spectral moments are extracted:

$$R_{\tau,V/A}^{kl} = \int_0^{m_\tau^2} \mathrm{d}s \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{\mathrm{d}R_{\tau,V/A}}{\mathrm{d}s}.$$

These are fitted simultaneously to extract $\alpha_s(m_\tau)$ and phenomenological operators from the non-perturbative components. The fit yields

$$\alpha_s(m_Z) = 0.120 \pm 0.003.$$

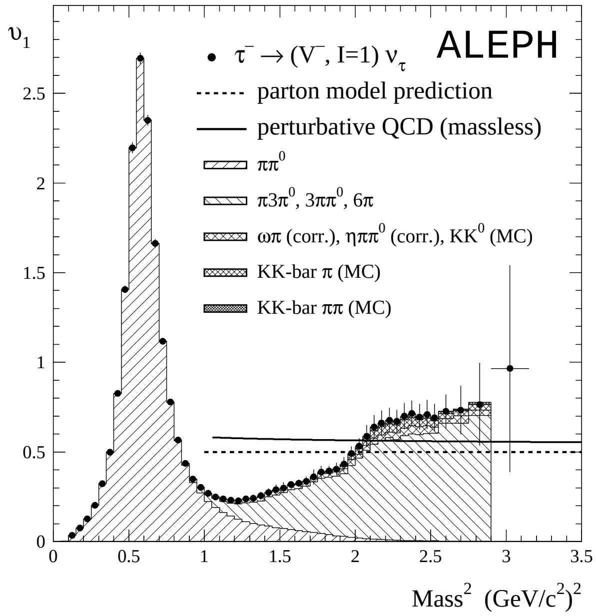

Figure 3. Corrected mass distribution of the vector component of the hadronic system from τ decays.

7. Event shapes

There are several global event shape variables from the final state $e^+e^- \to$ hadrons which are sensitive to the value of α_s. The four LEP experiments, ALEPH, DELPHI, L3 and OPAL, measured these variables at many centre-of-mass energies. At high energies, the background to the hadronic sample is large and one needs to worry for controlling backgrounds due to initial state radiation (ISR) and 4-fermion processes (WW, ZZ production). There were too many energy points each with a small number of events and the LEP experiments combined some of these energy points to make a measurement (e.g. $\sqrt{s}$ in the range 204–209 GeV have been combined to give an effective measurement at $\sqrt{s} = 206$ GeV). Typical number of events and level of background in the event sample per experiment are summarised in table 1.

Event shape distributions have been measured using charged and neutral particles. Measurements have been made for six event shape variables for which improved analytical calculations are available: thrust (T), scaled heavy jet mass (ρ_H), jet broadening variables (B_T, B_W), C- and D-parameters. These distributions are corrected for residual contamination, detector resolution and acceptance.

7.1 *Analysis of moments*

The moments of the event shape variables have been described [13] as a sum of the perturbative contribution and a power law dependence due to non-perturbative contribution.

Table 1. Statistics and level of background in the hadronic event sample in a typical LEP experiment.

$\sqrt{s}$ (GeV)	$\int L\ \mathrm{d}t$ (pb^{-1})	No. of events	Background
91.2	100	$> 3 \times 10^6$	Negligible
133	12	800	2%
161	11	300	5%
172	10	250	10%
183	60	1300	12%
189	170	3500	13%
200	200	3500	14%
206	210	3500	15%

These two contributions have different energy dependence. The first moment of an event shape variable f is written as

$$\langle f \rangle = \langle f_{\mathrm{pert}} \rangle + \langle f_{\mathrm{pow}} \rangle ,$$

where the perturbative contribution $\langle f_{\mathrm{pert}} \rangle$ has been determined to $\mathscr{O}(\alpha_{\mathrm{s}}^2)$. The power correction term for $1 - T$, ρ_{H}, C and D is given by

$$\langle f_{\mathrm{pow}} \rangle = c_f \mathscr{P} ,$$

where the factor c_f depends on the shape variable f and $\mathscr{P}$ is supposed to have a universal form:

$$\mathscr{P} = \frac{4C_{\mathrm{F}}}{\pi^2} \mathscr{M} \frac{\mu_{\mathrm{I}}}{\sqrt{s}} \left[\alpha_0(\mu_{\mathrm{I}}) - \alpha_{\mathrm{s}}(\sqrt{s}) - \beta_0 \frac{\alpha_{\mathrm{s}}^2(\sqrt{s})}{2\pi} \left(\ln \frac{\sqrt{s}}{\mu_{\mathrm{I}}} + \frac{K}{\beta_0} + 1 \right) \right]$$

for a renormalisation scale fixed at $\sqrt{s}$. The parameter α_0 is the value of α_{s} in the non-perturbative region below an infrared matching scale $\mu_{\mathrm{I}} (= 2$ GeV); β_0 is $(11N_{\mathrm{C}} - 2N_{\mathrm{F}})/3$, where N_{C} is the number of colours and N_{F} is the number of active flavours. $K = (67/18 - \pi^2/6)C_{\mathrm{A}} - 5N_{\mathrm{F}}/9$ and C_{F}, C_{A} are the usual colour factors. The Milan factor $\mathscr{M}$ is 1.49 for $N_{\mathrm{F}} = 3$. For the jet broadening variables, the power correction term takes the form

$$\langle f_{\mathrm{pow}} \rangle = c_f F \mathscr{P} ,$$

where

$$F = \left(\frac{\pi}{2\sqrt{a C_{\mathrm{F}} \alpha_{\mathrm{CMW}}}} + \frac{3}{4} - \frac{\beta_0}{6a\, C_{\mathrm{F}}} - 0.6137 + \mathscr{O}(\sqrt{\alpha_{\mathrm{s}}}) \right)$$

and a takes a value 1 for B_{T} and 2 for B_{W}. α_{CMW} is related to α_{s}.

DELPHI and L3 have analysed the moments in terms of the two variables α_{s} and α_0. They obtain good fits (see figure 4) and the results are summarised in table 2.

7.2 *Analysis of shape distributions*

The QCD predictions in fixed order perturbation theory cannot take into account the effect of multiple gluon emission. In second order calculations two gluons can be emitted at most. For variables like thrust, heavy jet mass, etc. this leads to a singular behaviour of the

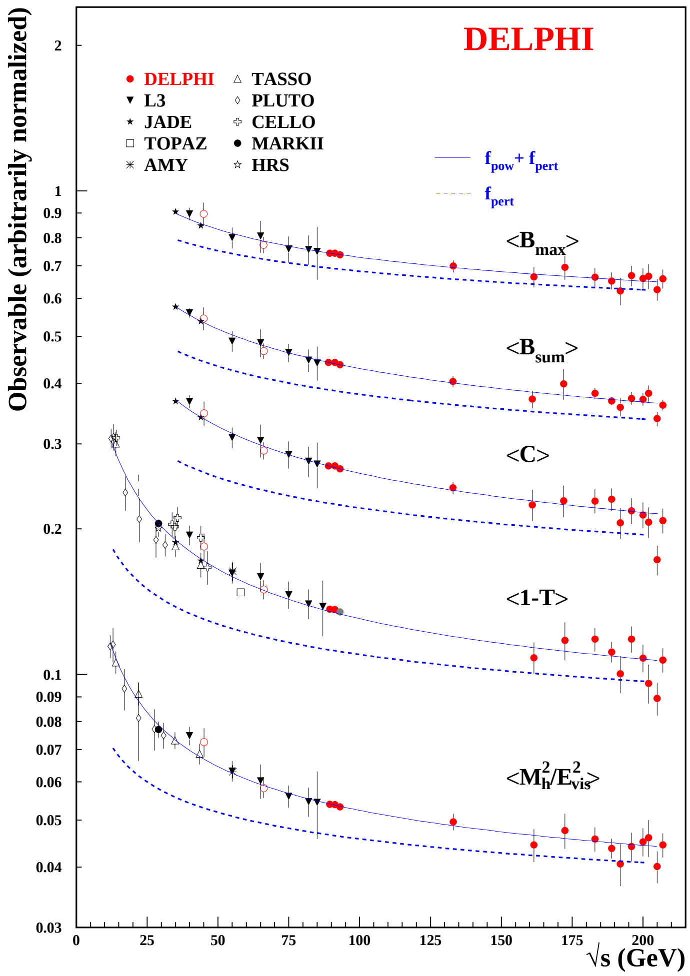

Figure 4. Moments of event shape variables as a function of centre-of-mass energies fitted to a combination of perturbative and power law terms.

Table 2. α_s determined from the energy dependence of moments of event shape variables.

	$\alpha_s(m_Z)$ from DELPHI	$\alpha_s(m_Z)$ from L3
$\langle 1-T \rangle$	0.1241 ± 0.0034	0.1162 ± 0.0049
$\langle \rho_H \rangle$	0.1177 ± 0.0036	0.1068 ± 0.0036
$\langle B_T \rangle$	0.1174 ± 0.0029	0.1163 ± 0.0034
$\langle B_W \rangle$	0.1167 ± 0.0019	0.1172 ± 0.0034
$\langle C \rangle$	0.1222 ± 0.0036	0.1161 ± 0.0030
$\langle D \rangle$		0.1371 ± 0.0092
Combined	0.1217 ± 0.0046 (expt) ± 0.0030 (theory)	0.1183 ± 0.0046 (expt) ± 0.0044 (theory)

Table 3. Schematic representation of the fixed order expansion vs. the logarithmic expansion of theoretical predictions to the event shape variables.

	Leading log	Next-to-leading log	Subleading			
First order	$\overline{\alpha}_s L^2$	$\overline{\alpha}_s L$	α_s	$\overline{\alpha}_s \mathcal{O}(1/L)$		
Second order	$\overline{\alpha}_s^2 L^3$	$\overline{\alpha}_s^2 L^2$	$\overline{\alpha}_s^2 L$	$\overline{\alpha}_s^2$	$\overline{\alpha}_s^2 \mathcal{O}(1/L)$	
Third order	$\overline{\alpha}_s^3 L^4$	$\overline{\alpha}_s^3 L^3$	$\overline{\alpha}_s^3 L^2$	$\overline{\alpha}_s^3 L$	$\overline{\alpha}_s^3$	$\overline{\alpha}_s^3 \mathcal{O}(1/L)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

distributions in kinematic regions where multi-gluon emission becomes dominant. This is a direct consequence of the collinear and infrared divergence of the gluon emission cross-section. It is possible to isolate the singular terms in every order of perturbation theory and to sum them up in the form of an exponential series. These calculations have been carried out for a few shape variables [14] to next-to-leading log terms.

One can write down the cumulative cross-section in the form

$$R(y,\alpha_s) \equiv \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy} = C(\alpha_s)\Sigma(y,\alpha_s) + D(\alpha_s, y)$$

with

$$\begin{aligned}
C(\alpha_s) &= 1 + \sum_{n=1}^{\infty} C_n \overline{\alpha}_s^n,\\
D(\alpha_s, y) &= \sum_{n=1}^{\infty} D_n(y)\overline{\alpha}_s^n,\\
\Sigma(y,\alpha_s) &= \exp\left[\sum_{n=1}^{\infty}\sum_{m=1}^{n+1} G_{nm}\overline{\alpha}_s^n L^m\right]\\
&\equiv \exp\left[L\, g_1(\overline{\alpha}_s L) + g_2(\overline{\alpha}_s L) + \alpha_s\, g_3(\overline{\alpha}_s L) + \cdots\right],\\
\overline{\alpha}_s &\equiv \frac{\alpha_s}{2\pi},\\
L &\equiv \ln\left(\frac{1}{y}\right),
\end{aligned}$$

where y is the event shape variable. In the two-jet region, y is small. Therefore, L and the corrections due to large powers of L are large.

In the fixed order calculations [15], one can write down

$$R(y,\alpha_s) = \alpha_s A(y) \;+\; \alpha_s^2 B(y) \;+\; \mathcal{O}(\alpha_s^3).$$

The two approaches are summarised in table 3. The first two rows have been completely computed in the fixed order calculations and the first two columns are known to all orders in the recent resummed calculations. In order to describe the data over a wide kinematic region, it is desirable to combine the two sets of calculations taking care of the common parts. This leads to a number of matching schemes.

The simplest matching scheme is to match the two calculations at a given value of y and use a suitable damping function so that the resummed calculations contribute to the two-jet region and the fixed order calculations dominate in the multi-jet region. A more preferable approach would be to combine the two calculations and subtract the common terms of the two calculations. This is done by taking the log of the fixed order calculations and expanding it as a power series. Then one can match in $\ln R(y)$ (called the 'ln R matching' scheme). Alternatively one can carry out a similar procedure in the function $R(y)$ rather than in $\ln R(y)$. This procedure is called the R-matching scheme. In a variation of the 'R matching' scheme, the term $G_{21}\overline{\alpha}_s^2 L$ is included in the term of the exponential and subtracted after exponentiation. This method is termed as the 'modified R matching' scheme.

One has to take care of the additional constraint coming from kinematics, namely the cross-sections vanish beyond the kinematic limit

$$R(y = y_{\max}) = 1,$$
$$\frac{\mathrm{d}R}{\mathrm{d}y}(y = y_{\max}) = 0 \; .$$

These constraints are strictly obeyed in the fixed order calculations but they are not valid for the resummed expansion. The first constraint can be taken care of by replacing $R(y)$ with $R(y) - R(y_{\max})$ for the resummed calculations. Alternatively, one can replace L in the resummed term by $L' = \ln(y^{-p} - y_{\max}^{-p} + 1)$ in the 'ln R matching' scheme to fulfil both of them. p is termed as modification degree and this scheme is referred to as 'modified ln R matching'.

An important improvement of the new QCD calculations with respect to the second order formulas is their ability to describe also the low y region. One should note that the sub-leading terms are not included beyond second order.

The calculations for the distributions of the five variables are given in the form of analytical functions

$$f^{\text{pert}}(y; s, \alpha_s(\mu), \mu) \; .$$

These calculations are carried out for massless partons. To compare the analytical calculations with the experimental distributions, one has to include the effect of hadronisation and decays using Monte Carlo programs. These have been taken care of by using parton shower programs with string or cluster fragmentation. The fragmentation parameters are determined from a comparison of predicted and measured distributions for several event shape variables. All these generators describe the experimental measurements well. The perturbative calculations for a variable y have been folded with the probability $p^{\text{non-pert}}$ (y', y) to find a value y after fragmentation and decays for a parton level value y':

$$f(y) = \int f^{\text{pert}}(y') \cdot p^{\text{non-pert}}(y', y)\, \mathrm{d}y' \; .$$

The resulting differential cross-section $f(y)$ is compared with the measurements. The correction for hadronisation and decays changes the perturbative prediction by less than 5% for the event shape variables over a large kinematic range. The corrections increase in the extreme two-jet region.

To determine α_s at each energy point, the measured distributions are fitted to the analytical predictions, using the modified-log(R) matching scheme after corrections for hadronisation effects. Figure 5 shows one such fit to the $L3$ data at $\sqrt{s} = 206$ GeV.

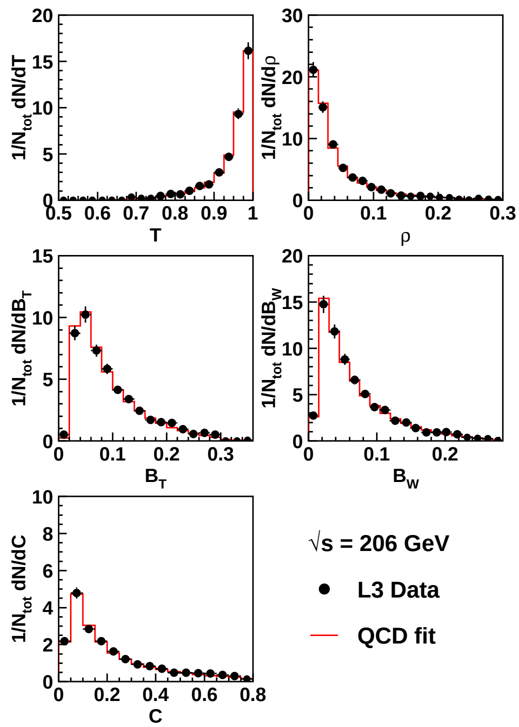

Figure 5. Measured distributions of thrust, T, scaled heavy jet mass, $\rho_{\rm H}$, total, $B_{\rm T}$, and wide, $B_{\rm W}$, jet broadenings, and C-parameter in comparison with QCD predictions at $\sqrt{s} = 206$ GeV.

The four LEP experiments use different set of variables (y), differ in fitting range, in matching scheme and also in estimation (method) of systematic errors. To combine these measurements, the LEPQCD working group chooses two matching schemes for all the four experiments (modified ln R and R schemes) and classifies the uncertainty of $\alpha_{\rm s}$ due to four sources: (1) statistical; (2) experimental systematic (dominated by backgrounds); (3) hadronisation (dominated by model differences); (4) uncalculated higher orders.

The LEPQCD has decided to estimate hadronisation uncertainty from the difference between various models. In order to determine theoretical uncertainty due to uncalculated higher orders, LEPQCD has developed a new prescription: 'uncertainty band method'. Here one obtains the uncertainty band for a fixed $\alpha_{\rm s}$ by varying (1) renormalisation scale (between $\sqrt{s}/2$ and $2\sqrt{s}$); (2) rescaling factor $x_{\rm L}$ ($L' = \ln(1/x_{\rm L}y)$); (3) kinematic constraint ($y_{\rm max}$); (4) matching scheme and (5) modification degree (p in L'). For a fixed reference prediction (ln R), one then finds $\alpha_{\rm s}$ variation which covers the band within the fit range. The uncertainty is typically 5% at LEP1 and it goes down to 3.5% at the highest LEP energy.

For combining different $\alpha_{\rm s}$ values (between experiments and between energies of a given experiment), the working group has defined a methodology of treating the correlated errors. With this prescription the $\alpha_{\rm s}$ values at different energies have been combined and they are summarised in table 4.

Table 4. α_s values measured at LEP from a fit to the event shape distributions to the matched fixed order and resummed calculation.

$\sqrt{s}$ (GeV)	α_s	$\Delta\alpha_s^{\text{Stat.}}$	$\Delta\alpha_s^{\text{Expt}}$	$\Delta\alpha_s^{\text{Had}}$	$\Delta\alpha_s^{\text{Scale}}$
41.4	0.1415	0.0024	0.0027	0.0018	0.0077
55.3	0.1260	0.0023	0.0049	0.0045	0.0067
65.4	0.1332	0.0015	0.0031	0.0041	0.0061
75.7	0.1190	0.0012	0.0051	0.0045	0.0056
82.3	0.1174	0.0013	0.0037	0.0051	0.0055
85.1	0.1140	0.0018	0.0041	0.0051	0.0056
91.2	0.1197	0.0002	0.0008	0.0010	0.0048
133.0	0.1149	0.0016	0.0012	0.0010	0.0045
161.0	0.1080	0.0025	0.0014	0.0003	0.0043
172.0	0.1046	0.0029	0.0017	0.0006	0.0040
183.0	0.1076	0.0013	0.0008	0.0007	0.0038
189.0	0.1089	0.0008	0.0009	0.0006	0.0037
200.0	0.1074	0.0009	0.0010	0.0006	0.0036
206.0	0.1073	0.0009	0.0008	0.0005	0.0034

The measured values can be fitted to the QCD evolution equation in NNLO with a χ^2 of 14.6 for 13 degrees of freedom. The fit corresponds to:

$$\alpha_s(m_Z) = 0.1198 \pm 0.0009\,(\text{expt}) \pm 0.0046\,(\text{theory}).$$

ALEPH [16] has analysed 4-jet events in e^+e^- interactions at $\sqrt{s} \simeq m_Z$. From the measured energies of the 4-jets with $k_\perp$ algorithm at $y_{\text{cut}} = 0.008$, the different angular correlations have been measured. These distributions lead to simultaneous measurement of α_s and QCD colour factors:

$$\alpha_s(m_Z) = 0.119 \pm 0.006\,(\text{stat.}) \pm 0.026\,(\text{syst.}).$$

8. Summary

A large number of measurements exist on α_s from a variety of experiments. PDG averages 11 measurements with a χ^2 of 9.

$$\alpha_s(m_Z) = 0.1171 \pm 0.0014.$$

The measurements described above give a weighted average

$$\alpha_s(m_Z) = 0.1183 \pm 0.0009.$$

The errors are however correlated and treatment of combining these correlated errors has been discussed in detail in [17]. A similar treatment provides a more realistic estimate of the error to be ± 0.003.

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