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Pseudo information entropy of a three-level atom interaction with two-laser fields in Λ-configuration

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Abstract. In this report we investigate some aspects of the pseudo entropy of multi-level system in the language of quantum information theory. The influence of the non-linear interaction and detuning parameter on the properties of the pseudo information entropy is examined.

Keywords. Information entropy; multilevel atoms; entanglement.

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Information entropy has occupied a central place in modern research because of its promise of enormous utility in quantum computing, cryptography etc. A major thrust of current research is to find a quantitative measure of entanglement for general states. Approaches to this question based on the eigenvalues and eigenfunctions of the system density matrices such as entropy methods, have given necessary but not sufficient conditions for particular states. The central object of information theory, the entropy, has been introduced in quantum mechanics by von Neumann [1]. An entropy-like quantity (pseudo entropy) in a fixed basis has been defined in ref. [2]. The most commonly encountered form of the pseudo entropy uses only diagonal part of the density matrix, therefore the pseudo entropy does not have contribution from the interference terms [3]. For a two-level system we may define the corresponding information entropy S_{I} on the basis of the eigenfunctions of the unperturbed Hamiltonian as

$$S_{\mathrm{I}} = -k[\rho_{ee} \ln \rho_{ee} + (1-\rho_{ee}) \ln(1-\rho_{ee}). \tag{1}$$

When $\rho_{ee} \to 0$ or $(1-\rho_{ee}) \to 0$, $S_{\mathrm{I}} = 0$. For $\rho_{ee} \neq 0$, the density matrix is seen to possess a certain amount of information entropy which obviously arises from mixing of unperturbed states $|e\rangle$ and $|g\rangle$ under the perturbation H_{in}. If the perturbation causes a transition to the state $|g\rangle$ at $t = t_1$, we have $\rho_{ee} = 0$ and $S_{\mathrm{I}} = 0$ again. Maximizing S_{I} with respect to ρ_{ee}, we find that $S_{\mathrm{I}}(t)$ is maximized at $\rho_{ee} = 0.5$. The transition from the state $|e\rangle$ at $t = 0$ and $\rho_{ee} = 1$ to the state $|g\rangle$, at $t = t_1$, must therefore pass through a state at some intermediate time where $S_{\mathrm{I}}(t)$ is maximized. The definition $S_{\mathrm{I}}(t)$ can be generalized to a multi-level system as [4]

$$S_{\mathrm{I}}(t) = -k\sum_{i=1}^{n}\rho_{ii}\ln\rho_{ii}. \tag{2}$$

In this report we shed some light on pseudo entropy as a purely basis mixing entropy with reference to a two-level system and extend this analysis to a multi-level system. In addition we use this argument to study the dynamical behavior of the pseudo entropy of a two-electron Rydberg atom, whose core is driven by two laser fields in a lambda configuration. The two laser pulses are near-resonant to the corresponding transitions but are slightly and equally detuned from the upper level.

We shall consider the Hamiltonian model which consists of a two-electron atom interacting with two laser pulses. First, a radial Rydberg wavepacket is excited by a short laser pulse. After a certain time delay, the core, which is assumed to represent a three-level system in the lambda configuration, is driven by two laser fields. This system can be realized in a Ca Rydberg atom, whose core has approximately the same level structure as the Ca^{+} ion. Since Ca is also convenient for an experimental investigation of the effect [5], we focus on this special element. We consider the Ca core without any decay, interacting with the core-driving laser pulses. We assume, that the two laser fields which couple the upper 4p-state of the Ca core to the 4s- and the 3d-states, respectively, are not necessarily in resonance with the transitions but are slightly detuned with the energy-matching condition $E_{4\mathrm{s}} + \hbar\Omega_1 = E_{3\mathrm{d}} + \hbar\Omega_2$ fulfilled. To explain the effect of observable in this system, we need to summarize the basic features of a lambda system [6] driven by two equally detuned fields. The optical Bloch equations corresponding to the density matrix elements are given by

$$\begin{aligned}
\dot{\rho}_{11} &= i\lambda_1(\rho_{12}-\rho_{21})/\hbar,\\
\dot{\rho}_{22} &= i\lambda_2(\rho_{23}-\rho_{32})/\hbar,\\
\dot{\rho}_{12} &= -i\Delta\rho_{12} + i\lambda_2\rho_{13}/\hbar + i\lambda_1(\rho_{11}-\rho_{22})/\hbar,\\
\dot{\rho}_{13} &= i\Delta(\rho_{13}-\rho_{11}) + i\lambda_1(\rho_{12}-\rho_{23})/\hbar,\\
\dot{\rho}_{23} &= i\lambda_1(\rho_{21}-\rho_{13})/\hbar + i\lambda_2(\rho_{23}-\rho_{33})/\hbar,
\end{aligned} \tag{3}$$

where $\Delta = (E_2-E_1)/\hbar - \Omega_1 = (E_2-E_3)/\hbar - \Omega_2$ is the detuning and Ω_1, Ω_2 are the field frequencies. We denote by λ_1 the coupling constant for the ($|1\rangle \to |2\rangle$ transition) and λ_2 for the ($|2\rangle \to |3\rangle$ transition). To obtain the analytical expression for the time development of the density matrix elements, we assume the initial condition as $\rho_{11}(0) = 0$, $\rho_{22}(0) = 0$, $\rho_{33}(0) = 1$. After some straightforward algebraic calculations we find the solution

$$\begin{aligned}
\rho_{11}(t) &= \frac{\lambda_2^2+\hbar^2\delta^2}{\lambda_1^2+\lambda_2^2+\hbar^2\delta^2}\sin^2\left(\frac{(\mu_1+\mu_2)t}{2}\right) - 4C_1^2 - C_2,\\
\rho_{22}(t) &= -4C_1 - C_2,\\
\rho_{33}(t) &= 1 - \rho_{11}(t) - \rho_{22}(t),
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
C_1 &= \left(\frac{\lambda_1\lambda_2}{\lambda_1^2+\lambda_2^2}\right)^2\left[\frac{\mu_2}{\mu_1+\mu_2}S_1^2 + \frac{\mu_1}{\mu_1+\mu_2}S_2^2\right],\\
C_2 &= \frac{\lambda_2^2\lambda_1^2}{(\lambda_1^2+\lambda_2^2)(\lambda_1^2+\lambda_2^2+\hbar^2\delta^2)}\sin^2\left(\frac{(\mu_1+\mu_2)t}{2}\right),
\end{aligned}$$

$$S_i^2 = \sin^2\left(\frac{\mu_i t}{2}\right), \quad \mu_{1,2} = \sqrt{\lambda_1^2 + \lambda_2^2 + \delta^2} \pm \delta,$$

where $\delta = \Delta/2$. Employing the results obtained above, we shall discuss below the pseudo-entropy which has received little attention in the literature.

Experimentally it is well-known that the quantity which is often measured; is the probability of the atom staying in its initial state such as the system is detuned from exact resonance. In figure 1, we have plotted the $S_{\rm I}(t)$ for the present system as a function of the scaled time λt, $(\lambda_1 = \lambda_2 = \lambda)$, and for different values of detuning. It is remarkable to point out that, the first maximum of the $S_{\rm I}(t)$ at $t > 0$ is achieved at $\lambda t \approx \pi$. Also it is noticed that with increasing detuning parameter a gradual decrease in the amplitudes of the Rabi oscillations is seen (see figure 1). With the help of the above equations, it can be easily shown that the 4p-state is hardly populated, at large detuning, and that there is a practically complete periodic population transfer between the 4s- and the 3d-states. The frequency of this Rabi-like oscillation is given by $\Omega_{\rm R} = 2\lambda^2/\Delta$. The existence of such a periodic population transfer with a well-defined frequency enables us to synchronize this Rabi oscillation with the Kepler oscillation of the radial Rydberg wavepacket in analogy to the original arrangement.

The motivation in considering the present system stems from the possibility of the lambda system having dark states, which are insensitive to the decay of the upper level [6]. It is this spontaneous decay of the upper level that may affect the wavepacket dynamics of the original system in an unfavorable way, when the principal quantum numbers involved are sufficiently large (see figure 2). Here we may mention that if the detuning parameter is stronger than the atom-field coupling, one can see that the system starts to dominate the dynamics (there is nearly decoupling of the atom and field) and there is a repetition for some kind of regularity in the evolution of the system. This is apparent from the regular oscillations present in figure 2. Also we can see that the amplitude of the oscillation becomes smaller. More examples are, of course, needed to validate the proposition as a general one. It can be shown that when $S_{\rm I}(t)$ becomes large its value becomes asymptotically independent of the choice of the basis [4], provided the transformation from one basis to another is effected by a band matrix.

In general quantum mechanical density matrices have off-diagonal terms, which, for pure states, reflect the relative quantum phase in superpositions. In other words, the pseudo entropy gives the same results of the von Neumann entropy if we ignore contribution from the interference terms and therefore [7], it is expected to use pseudo entropy as a measure of entanglement for muliparticle system.

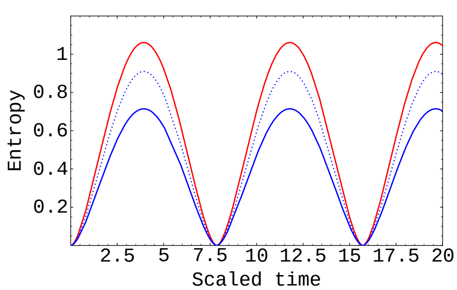

Figure 1. $S_{\rm I}(t)$ as a function of the scaled time λt, $\Delta/\lambda = 1, 2, 10$, $\bar{n} = 10$.

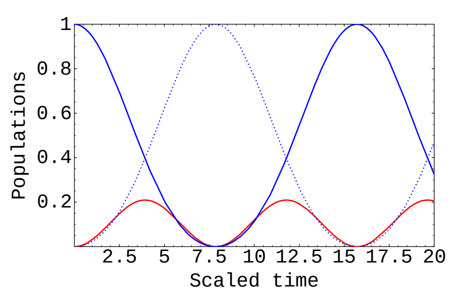

Figure 2. Populations of the states, $\Delta/\lambda = 1$.

In conclusion, we have put considerable effort in exploring the behavior of the pseudo information entropy of a two-electron atom interaction with two-laser fields in Λ-configuration. The pseudo entropy which has its origin in basis-mixing under perturbation can profitably be utilized for characterizing the dynamical features of the evolution of a pure state. The detuning effect on the pseudo entropy and atomic level occupation probabilities has been investigated.

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