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Non-linear propagation of laser beam and focusing due to self-action in optical fiber: Non-paraxial approach

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Abstract. A somewhat more general analysis for solving spatial propagation characteristics of intense Gaussian beam is presented and applied to the laser beam propagation in step-index profile as well as parabolic profile dielectric fibers with Kerr non-linearity. Considering self-action due to saturating and non-saturating non-linearity in the refractive index, a general theory has been developed without any kind of power series expansion for the dielectric constant as is usually done in other theories that make use of paraxial approximation. Result of the steady state self-focusing analysis indicates that the Kerr non-linearity acts as a perturbation on the radial inhomogeneity due to fiber geometry. Analysis indicates that the paraxial rays and peripheral rays focus at different points, indicating aberration effect. Calculated critical power matches with the experimentally reported result.

Keywords. Laser–matter interaction; Kerr non-linearity; self-focusing.

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1. Introduction

Wave propagation in optical fiber has been a subject of intensive theoritical and experimental research [1–10]. Optical fiber technology changed the communication scenario and efforts are on, in achieving the goal of dispersionless transmission over a long distance [11–14]. Focusing due to self-action has been employed for dispersionless propagation of laser beam in non-linear media [15–17]. Trapping of beam in dielectric cell had been demonstrated experimentally by Chiao *et al* [18] and Gramire *et al* [19]. Propagation of intense laser beam and its trapping in optical fibers due to self-action is of current interest because of its potential applications in optical switching, Kerr lens mode locking, optical power filter, fiber amplifiers etc. [20].

An intense laser beam with radial distribution of intensity causes a radial gradient of dielectric constant due to non-linear mechanisms such as Kerr effect, electrostriction, thermal effects etc. This leads to focusing or defocusing of beam depending on the nature of the non-linear medium in which the beam is propagating. Theories of self-focusing (focusing of beam due to self-action) have relied on the paraxial ray approximation [21–24]. These approximations are known to give large error in the critical power for self-focusing [19]. Realising that paraxial approximation may also be quantitatively in error in the saturation region, some alternative methods for self-focusing and self-trapping of beam in plasma had been suggested and used by various workers [25–30].

The self-focusing problem of non-linear interaction of intense laser beam has been analysed considering the entire spatial characteristics of the laser beam, without any paraxial ray approximation and Taylor series expansion of dielectric constant. The effect of the Kerr non-linearity on fiber parameter and propagation characteristics of laser beam have been discussed. Results of the present analysis are compared with popular paraxial ray theories [24].

The paper is organized as follows: After the introduction in §1, the mathematical expression for effective dielectric constant has been presented in §2. Section 3 is devoted to the self-focusing equations considering non-saturating Kerr non-linearity. Second-order differential equations related to step-index as well as parabolic profile fiber have been obtained and presented in this section. In §4 the condition for uniform waveguide propagation of laser beam in optical fiber is established and expression for critical power is also obtained. Numerical results of self-action effects are presented in §5. Details of results and related conclusions are presented in §6.

2. Effective dielectric constant of optical fiber

Consider a fiber of core size a_0 with dielectric constant $\varepsilon_{\rm CO}$ and cladding dielectric constant $\varepsilon_{\rm CL}$. The dielectric constant distribution within the fiber in general can be expressed as [31]

$$\begin{aligned} \varepsilon &= \varepsilon_{\rm CO}[1-2\Delta g(r)], \quad r \le a_0, \\ \varepsilon &= \varepsilon_{\rm CO}[1-2\Delta], \quad r > a_0. \end{aligned} \tag{1}$$

Here $\Delta = (\varepsilon_{\rm CO} - \varepsilon_{\rm CL})/2\varepsilon_{\rm CO}$ is the relative dielectric constant difference and it is simply called 'delta parameter'. Radial profile function $g(r)$ defines the variation of dielectric constant inside the core.

Due to practical importance, in the present analysis, we have considered step-index profile fibers for which $g(r) = 0$ and parabolic index profile fibers for which $g(r) = (r/a_0)^2$. The effective dielectric constant of the optical fiber in the presence of laser beam [15] is written as

$$\varepsilon_{\rm eff} = \varepsilon_{\rm L} + \varepsilon_{\rm NL}(\langle EE^* \rangle) \tag{2}$$

where $\varepsilon_{\rm L}$ is the linear part of the dielectric constant and $\varepsilon_{\rm NL}$ the intensity dependent non-linear part of the dielectric constant. Hence effective dielectric constant for step-index profile fiber [32] according to eqs (1) and (2) is written as

$$\begin{aligned} \varepsilon_{\rm eff} &= \varepsilon_{\rm CO} + \varepsilon_{\rm NL}\langle EE^* \rangle, \quad r \le a_0, \\ \varepsilon_{\rm eff} &= \varepsilon_{\rm CO}[1-2\Delta] + \varepsilon_{\rm NL}\langle EE^* \rangle, \quad r > a_0, \end{aligned} \tag{3}$$

and for the parabolic index profile fiber

$$\begin{aligned} \varepsilon_{\rm eff} &= \varepsilon_{\rm CO}[1-2\Delta(r/a_0)^2] + \varepsilon_{\rm NL}\langle EE^* \rangle, \quad r \le a_0, \\ \varepsilon_{\rm eff} &= \varepsilon_{\rm CO}[1-2\Delta] + \varepsilon_{\rm NL}\langle EE^* \rangle, \quad r > a_0. \end{aligned} \tag{4}$$

3. Self-focusing equation

It is well-known that up to second order in dispersion, the slowly varying electric field amplitude for a beam propagating in $+z$ direction in the given fiber is given as [15]

$$E(r,z,t) = A(r,z)\exp[i(kz-\omega t)], \tag{5}$$

where $A(r,z)$ is the amplitude of electric field of the incident beam which is supposed to be cylindrically symmetrical. It satisfies the following differential equation:

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + 2ik\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial z^2} = k^2 A - \frac{\varepsilon_{\mathrm{eff}}\omega^2}{c^2}A. \tag{6}$$

This is a parabolic equation and is extensively used by various workers for propagation and radiation problems [24].

Let

$$A(r,z) = A_0(r,z)\exp[-ikS(r,z)], \tag{7}$$

where S is eikonal and A_0 is the amplitude of the propagating beam in the non-linear medium. The mode propagating in a single mode fiber is the LP_{01} mode and the characteristics of the fiber can be derived from the model field distribution. The radial dependence of the model field distribution except for a few fibers of specific profiles, cannot be obtained by the analytical solution of wave equation [33–55]. Observations indicate that for all practical fibers the model field is maximum on the axis and decreases monotonically away from it. The variation of the model field is very nearly Gaussian [31]. Hence the amplitude of the propagating beam in the non-linear medium confined in the fiber is considered to be Gaussian and thus the intensity distribution is written as [30]:

$$A_0^2(r,z) = \frac{E_0^2}{f^2(z)}\exp\left(\frac{-r^2}{r_0^2 f^2(z)}\right). \tag{8}$$

Substituting eqs (7) and (8) into eq. (6), one gets

$$\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} = \frac{2}{k^2 r_0^2 r^2 f} - \frac{1}{k^2 r_0^4 f^3} - \frac{f\varepsilon_{\mathrm{NL}}\langle EE^*\rangle}{r^2\varepsilon_{\mathrm{L}}}, \tag{9}$$

where $f(z)$ is the dimensionless beam width parameter of the beam in medium at axial distance z. It is a second-order differential equation of beam width parameter (f) and can be used to obtain entire spatial characteristics of the beam.

3.1 *Self-focusing equation for Kerr non-linearity*

The non-linearity is an important quantity which plays a major role in self-action phenomena. In the present analysis, only Kerr non-linearity has been considered and thus non-linear part of the dielectric constant can be written as [24]

$$\varepsilon_{\mathrm{NL}}\langle EE^*\rangle = \varepsilon_{\mathrm{S}}\left[1-\exp\left(-\frac{\varepsilon_2}{\varepsilon_{\mathrm{S}}}\right)|E|^2\right], \tag{10}$$

where ε_2 is the Kerr non-linear constant and ε_S the saturated value of the non-linear part of the dielectric constant of a given medium. The self-focusing equation for Kerr non-linearity in optical fiber can be obtained by substituting the value of non-linear part of dielectric constant due to Kerr non-linearity (10) into the general self-focusing eq. (9). One obtains

$$\frac{d^2 f}{dz^2} = \frac{(2r_0^2 f^2 - r^2)}{k^2 r_0^4 r^2 f^3} - \frac{f\varepsilon_S}{r^2 \varepsilon_{CO}} \left[1 - \exp\left(-\frac{\varepsilon_2}{\varepsilon_S}|E|^2\right)\right]. \tag{11}$$

3.2 *Self-focusing equation for non-saturating non-linearity*

For non-saturating type non-linearity, the following condition must be fulfilled:

$$\frac{\varepsilon_2}{\varepsilon_S}|E|^2 \ll 1,$$

then eq. (10) is reduced to

$$\varepsilon_{NL}\langle EE^*\rangle = \varepsilon_2\langle EE^*\rangle. \tag{12}$$

Under such a situation, the self-focusing equation for step-index profile fiber is obtained by substituting eqs (3) and (12) into eq. (11) as

$$\frac{d^2 f}{dz^2} = \frac{(2r_0^2 f^2 - r^2)}{k^2 r_0^4 r^2 f^3} - \frac{\varepsilon_2 E_0^2}{2r^2 \varepsilon_{CO} f} \exp\left(\frac{-r^2}{r_0^2 f^2}\right). \tag{13}$$

Similarly, in the case of parabolic index profile fiber for non-saturating non-linearity the self-focusing equation can be obtained by substituting eqs (4) and (12) into eq. (11) as

$$\frac{d^2 f}{dz^2} = \frac{(2r_0^2 f^2 - r^2)}{k^2 r_0^4 r^2 f^3} - \frac{\varepsilon_2}{\varepsilon_{CO}} \left[\frac{E_0^2}{2fr^2} \exp\left(-\frac{r^2}{r_0^2 f^2}\right) - \frac{2\Delta f \varepsilon_{CO}}{\varepsilon_2 a_0^2}\right]. \tag{14}$$

3.3 *Self-focusing equation for saturating non-linearity*

For high intensity beam,

$$\frac{\varepsilon_2}{\varepsilon_S}|E|^2 \gg 1$$

and hence non-linear part of the dielectric as per eq. (10) is reduced to

$$\varepsilon_{NL}\langle EE^*\rangle = \varepsilon_S. \tag{15}$$

Here, non-linear dielectric constant term attains a saturating value which is independent of the intensity of laser beam.

For step-index profile fiber, self-focusing equation for saturating non-linearity is obtained by substituting eqs (3) and (15) into eq. (11) as

$$\frac{d^2 f}{dz^2} = \frac{(2r_0^2 f^2 - r^2)}{k^2 r_0^4 r^2 f^3} - \frac{\varepsilon_S f}{\varepsilon_{CO} r^2}. \tag{16}$$

Similarly, for parabolic profile fiber, self-focusing equation for saturating non-linearity is obtained by substituting eqs (4) and (15) into eq. (11) as

$$\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} = \frac{(2r_0^2 f^2 - r^2)}{k^2 r_0^4 r^2 f^3} - \frac{\varepsilon_\mathrm{S}}{\varepsilon_\mathrm{CO}}\left[\frac{f}{r^2} - \frac{2\Delta f \varepsilon_\mathrm{CO}}{\varepsilon_\mathrm{S} a_0^2}\right]. \tag{17}$$

4. Uniform waveguide propagation

The condition under which the beam can produce its own waveguide and propagate without spreading, is known as uniform waveguide propagation mode [18]. Such self-trapping in fiber (waveguide mode) is possible, when refraction terms are exactly balanced with diffraction terms in the self-focusing equation. To obtain the self-trapping condition one can use the general expression (11) for non-linearity [21]. Under this condition two terms on the RHS of eq. (11) cancel and thus $\mathrm{d}^2 f/\mathrm{d}z^2 = 0$ and $\mathrm{d}f/\mathrm{d}z$ will remain zero for all values of z. If at $z = 0$, $\mathrm{d}f/\mathrm{d}z = 0$, i.e., incident beam at $z = 0$ have a plane wavefront, then the above condition imply that $f = 1$ for all the values of z. This leads to uniform waveguide motion. Substituting $r = \rho f$, for different transverse distance (ρ) in eq. (11), we get the following relation for uniform waveguide propagation:

$$\frac{(2r_0^2 - \rho^2)}{k^2 r_0^4 \rho^2} - \left[\frac{\varepsilon_\mathrm{S}}{\varepsilon_\mathrm{CO}}\left\{\frac{1}{\rho^2}\left\{1 - \exp\left(-\alpha E_0^2 \exp\left(\frac{-\rho^2}{r^2}\right)\right)\right\}\right\}\right] = 0, \tag{18}$$

$$\frac{(2r_0^2 - \rho^2)}{k^2 r_0^4 \rho^2} = \frac{\varepsilon_\mathrm{S}}{\varepsilon_\mathrm{CO}\rho^2}\left\{1 - \exp\left(-\alpha E_0^2 \exp\left(\frac{-\rho^2}{r_0^2}\right)\right)\right\}. \tag{19}$$

But

$$k = \frac{\omega\sqrt{\varepsilon_\mathrm{CO}}}{c}.$$

Hence the normalised self-trapped radius is written as

$$\left(\frac{\omega r_0}{c}\right) = \left[\frac{\varepsilon_\mathrm{S} r_0^2\left\{1 - \exp\left(-\alpha E_0^2 \exp\left(\frac{-\rho^2}{r_0^2}\right)\right)\right\}}{(2r_0^2 - \rho^2)}\right]^{-1/2}. \tag{20}$$

4.1 *Critical power*

The critical power is the power of incident beam for which the beam propagates uniformly without convergence or divergence [19]. The critical angle for total internal reflection θ_c according to Snell's law, is written as [24]

$$\cos\theta_\mathrm{c} = \frac{n_0}{n_0 + n_2 E^2}. \tag{21}$$

Here, n_0 and n_2 are linear and non-linear part of the refractive index of the core of optical fiber, i.e., of the medium.

$$\theta_{\mathrm{c}}^2 = \frac{n_2 E^2}{n_0}. \tag{22}$$

As per the Fraunhoffer diffraction, rays carrying large fraction of power is given by the angle [24]

$$\theta_{\mathrm{d}} = \frac{1.22\lambda}{4 r_0 n_0}. \tag{23}$$

For the critical condition, i.e., without convergence and divergence of the beam one has

$$\theta_{\mathrm{c}} = \theta_{\mathrm{d}} \tag{24}$$

and using eqs (21) and (23) one can write

$$\frac{n_2 E^2}{n_0} = \left(\frac{1.22\lambda}{4 r_0 n_0}\right)^2. \tag{25}$$

But intensity distribution at $z = 0$ is in Gaussian form for which the value of E^2 is given as

$$E^2 = E_0^2 \exp(-r^2/r_0^2).$$

The critical value of the electric field is written as

$$E_{0\mathrm{CR}}^2 = \frac{c(1.22)^2\lambda^2}{16 r_0^2\, n_2 n_0} \exp\left(\frac{r^2}{r_0^2}\right). \tag{26}$$

Thus the critical power of the beam for the electric field $E_{0\mathrm{CR}}$ is given as

$$P_{\mathrm{CR}} = \frac{c}{128} \frac{(1.22)^2\lambda^2}{n_2} \exp\left(\frac{r^2}{r_0^2}\right). \tag{27}$$

5. Numerical result

The present non-paraxial analysis of the laser beam propagation through the optical fiber provides many important results which are presented in this section. The value of the effective dielectric constant of the fiber ($\varepsilon_{\mathrm{eff}}$) for different values of the incident beam intensity parameter (αE_0^2) has been calculated using eq. (10) and plotted in figure 1. For comparison, effective dielectric constant of the medium has also been computed for paraxial ray approximation using

$$\varepsilon_{\mathrm{eff}} = \varepsilon_{\mathrm{CO}} + \varepsilon_{\mathrm{S}}[1 - \exp(-\alpha E_0^2)] \tag{28}$$

and plotted in figure 1. The curves show that the effective dielectric constant increases sharply with incident beam intensity in low intensity region and attains a saturated value for higher intensities. Beam intensity for the saturation of dielectric constant is found to be higher than paraxial case.

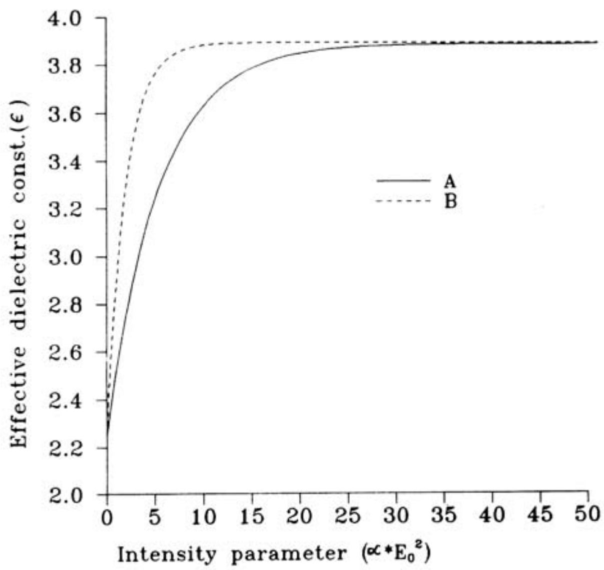

Figure 1. Variation of the effective dielectric constant (ε) with the electric field intensity parameter (αE_0^2) for Kerr non-linearity. Curve A denotes the present analysis and Curve B the paraxial ray approximation method. Here $\varepsilon_{\text{CO}} = 2.25$, $\varepsilon_{\text{S}} = 0.73$, $r_0 = 25\ \mu$m and $a_0 = 25\ \mu$m.

It is difficult to solve the non-linear equations ((13)–(17)) analytically. Runge–Kutta method and numerical techniques are used to solve these equations. For different parameters the following values have been used for the analysis: $\varepsilon_{\text{CO}} = 2.25$, $\varepsilon_2 = 9 * 10^{-22}$ CGS units, $\varepsilon_{\text{S}} = 0.01125$, $\Delta = 0.001$, $a_0 = 25\ \mu$m and $r_0 = 25\ \mu$m.

For non-saturating type of non-linearity, the variation of self-focusing beamwidth parameter (f) in the case of step-index profile fiber has been plotted with axial distance (z) for different transverse coordinates (ρ), in figure 2. The curves demonstrate that focusing parameter (f) first decreases and then increases sharply for different radial coordinates (ρ).

In figure 3, the self-focusing parameter (f) obtained from the numerical solution of eq. (14) is plotted for parabolic profile fiber considering non-saturating non-linearity. It appears that the peripheral rays focused at a finite value of axial distance (z) but paraxial rays with $\rho \leq 10\ \mu$m (small value of ρ) diverge, before a sharp focus is achieved.

Many a time, the non-linear part of the dielectric constant shows saturating tendency for intense laser beam. In the present analysis such type of cases has also been considered. The variation of focusing parameter (f) with z in step-index as well as parabolic profile fiber had been calculated for saturating type non-linearity using eqs (16) and (17) and results are plotted in figure 4. For both type of fibers, these curves demonstrate that the beamwidth parameter (f) oscillates between well-defined values as it propagates. Results indicate that emerging beam from different spatial positions (ρ) focuses at different points on the axis, i.e., focal length is found to be different for different values of ρ. For comparison, the focal

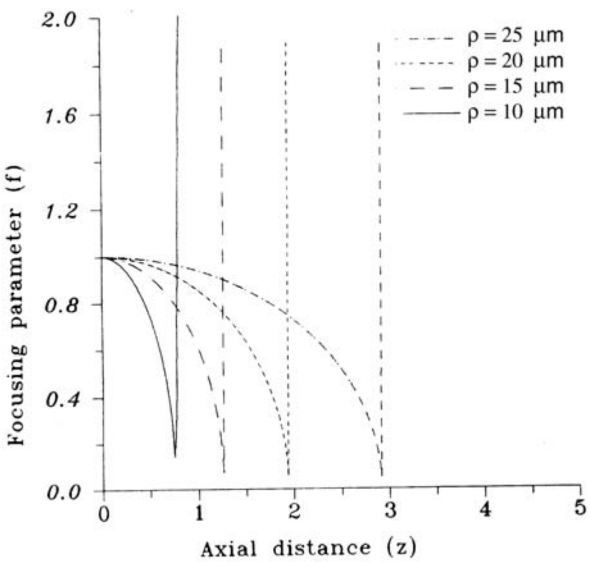

Figure 2. Axial dependence of the beam width focusing parameter (f) on axial distance (z) for different values of initial transverse distance (ρ), for step-index fiber in the case of non-saturating non-linearity. Here $\varepsilon_{\mathrm{CO}} = 2.25$, $\varepsilon_{\mathrm{NL}} = 9 * 10^{-22}$, $E_0 = 1 * 10^{10}$, $\varepsilon_{\mathrm{S}} = 0.01125$, $\lambda = 1.55\ \mu$m, $\Delta = 0.001$, $r_0 = 25\ \mu$m and $a_0 = 25\ \mu$m.

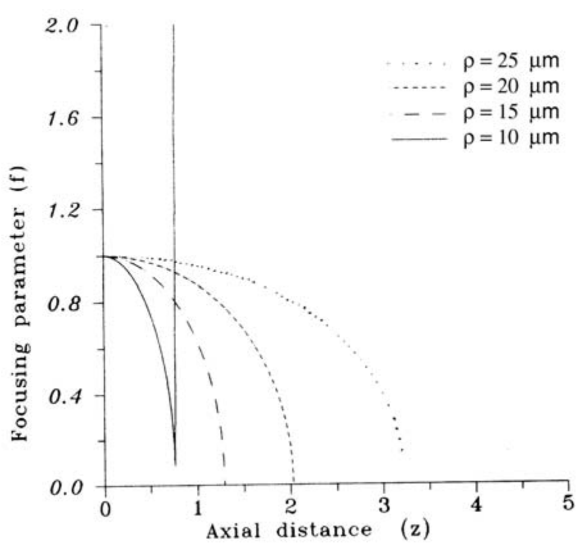

Figure 3. Variation of beam width parameter (f) with axial distance (z) for different values of transverse distance (ρ) for parabolic profile fiber corresponding to non-saturating non-linearity. Here $\varepsilon_{\mathrm{CO}} = 2.25$, $\varepsilon_{\mathrm{S}} = 0.01125$, $\varepsilon_{\mathrm{NL}} = 9 * 10^{-22}$, $\lambda = 1.55\ \mu$m, $E_0 = 1 * 10^{10}$, $\Delta = 0.001$, $r_0 = 25\ \mu$m and $a_0 = 25\ \mu$m.

Table 1. Minimum value of beam width parameter (f_{min}) and corresponding axial distance (z_{min}) for the step-index and parabolic profile fibers for saturating Kerr non-linearity. Here for both the fibers $\varepsilon_{\text{CO}} = 2.25$, $\varepsilon_2 = 9 * 10^{-22}$, $\varepsilon_{\text{S}} = 0.0112$ and $r_0 = 25\ \mu$m.

Transverse distance (ρ) (μm)	Step-index fiber		Parabolic fiber	
	$f_{\text{min}} \times 10^{-2}$	z_{min}	$f_{\text{min}} \times 10^{-2}$	z_{min}
25	3.61	4.47	3.75	5.36
20	4.34	3.58	4.44	3.99
15	4.84	2.69	4.91	2.85
10	5.21	1.79	5.22	1.84

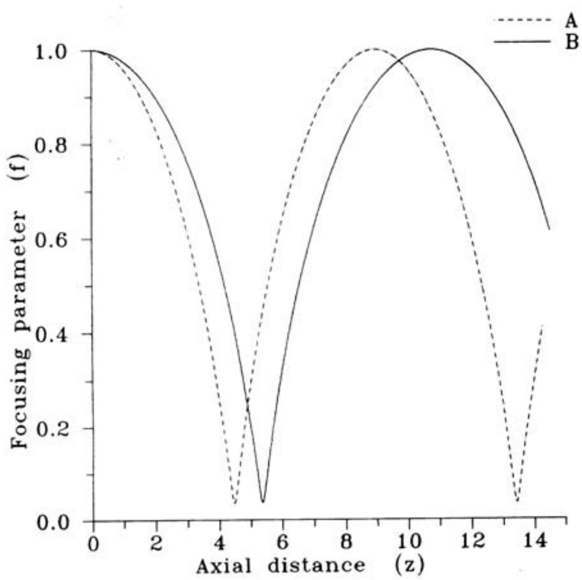

Figure 4. Oscillatory behaviour of the focusing parameter (f) with the axial distance (z) for saturating Kerr non-linearity. Curve A denotes the step-index profile fiber and Curve B the parabolic profile fiber. Here $\varepsilon_{\text{CO}} = 2.25$, $\varepsilon_{\text{S}} = 0.01125$, $\varepsilon_{\text{NL}} = 9 * 10^{-22}$, $\lambda = 1.55\ \mu$m, $E_0 = 1 * 10^{10}$ and $a_0 = 25\ \mu$m.

length and minimum value of f (i.e. f_{min}) at the focus for step-index and parabolic profile fibers are tabulated in table 1. The results in figures 5 and 6 as well as in table 1 show that for a given initial transverse coordinate of the beam, the focal length as well as minimum value of focusing parameter (f_{min}) have higher values for parabolic fiber compared to the step-index profile fiber.

The self-trapping of the beam is an important phenomenon of the self-action. This process leads to a uniform waveguide propagation of the beam in the fiber. For non-paraxial analysis, normalised self-trapped radius for different intensities have been obtained using eq. (20). The value of self-trapped radius has also been calculated for paraxial ray approximation [24], using the following equation:

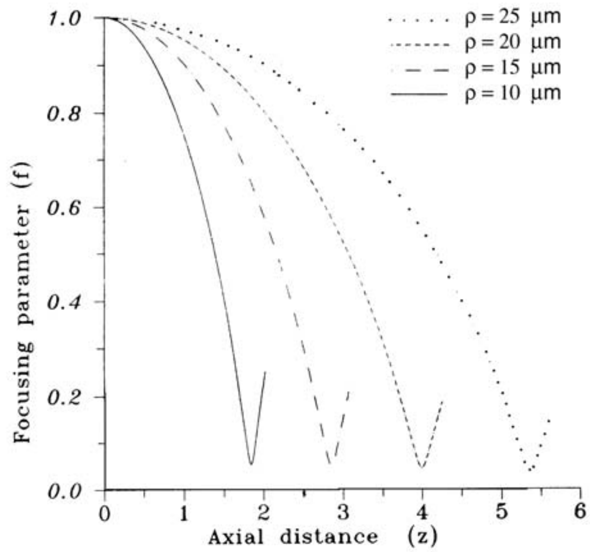

Figure 5. Variation of beam width parameter (f) with the axial distance (z) for the saturating Kerr non-linearity in the case of step-index profile propagation. Here $\varepsilon_{\mathrm{CO}} = 2.25$, $\varepsilon_{\mathrm{S}} = 0.01125$, $\varepsilon_{\mathrm{NL}} = 9 * 10^{-22}$, $E_0 = 1 * 10^{10}$, $\lambda = 1.55\ \mu$m, $\Delta = 0.001$, $r_0 = 25\ \mu$m and $a_0 = 25\ \mu$m.

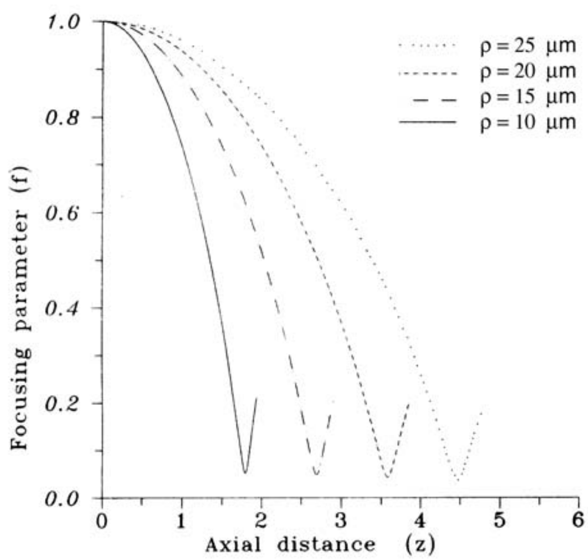

Figure 6. Plot of beam width parameter (f) with axial distance (z) for the saturating Kerr non-linearity in the case of parabolic fiber for different values of transverse distance (ρ) from the axis of fiber. $\varepsilon_{\mathrm{S}} = 0.01125$, $E_0 = 1 * 10^{10}$, $\lambda = 1.55\ \mu$m, $\Delta = 0.001$, $r_0 = 25\ \mu$m and $a_0 = 25\ \mu$m.

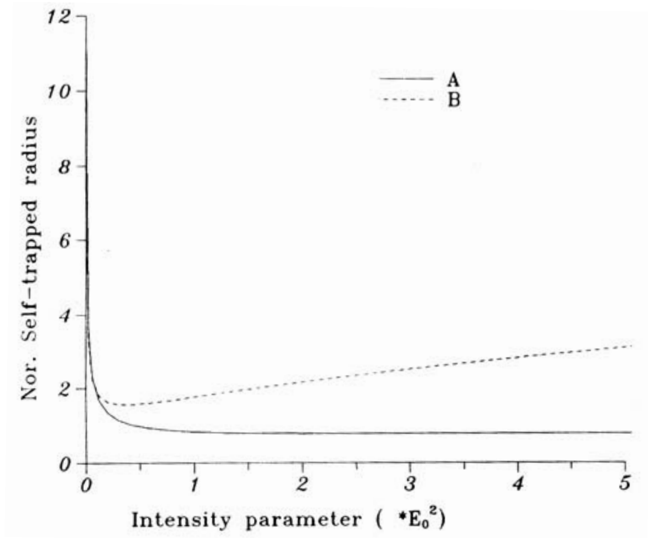

Figure 7. Normalised self-trapped radius of beam $(\omega r_0/C)$ dependence on beam intensity parameter (E_0^2). Curve A for the present non-paraxial analysis and curve B for the paraxial ray approximation. Here $\varepsilon_{CO} = 2.25$, $\varepsilon_{NL} = 9 * 10^{-22}$, $\lambda = 1.55$ μm and $r_0 = 25$ μm.

$$\left(\frac{\omega r_0}{c}\right) = \left[\frac{\varepsilon_2 E_0^2}{2(1+(\varepsilon_2 E_0^2/2\varepsilon_S))}\right]^{-1/2}. \tag{29}$$

Numerical results are plotted in figure 7. Considering the entire spatial part of the beam, the critical power of the beam has been calculated by eq. (27).

For paraxial approximation using Sodha *et al* [24] equations, again critical power is calculated. For calculating the critical power, same values of parameters used by Chiao *et al* [18] in their experiments are used for both type of calculations.

The value of critical power calculated by the present non-paraxial analysis as well as paraxial results are given in table 2.

6. Discussion

In the present analysis, the exact numerical solution of the second-order differential equation for beamwidth parameter has been obtained without approximations which are usually adopted in the popular paraxial ray theories [21–24]. For these reasons, present analysis is expected to provide better results for the beam as well as medium characteristics.

Optical fiber filled with dielectric medium which follows Kerr non-linear behaviour has been considered in the present study. At high intensity region, strong electric field associated with propagating beam tends to orient the anisotropic molecules of Kerr non-linear medium, owing to interaction with induced dipoles. The non-linear part of the dielectric constant in the case of some dielectric materials follows saturating behaviour as suggested

Table 2. Calculated value of critical power (P_{CR}) for step-index type optical fiber filled with Kerr medium at different values of radial distance (ρ) from the present non-paraxial analysis and its ratio to the value obtained by paraxial ray approximation. Here $P^*_{\text{CR (paraxial)}} = 9.34$ kW.

Radial distance (ρ) (μm)	P_{CR} (kW) (non-paraxial)	Critical power ratio $P_{\text{CR}}/P_{\text{CR (paraxial)}}$
0.01	9.34	1
5	9.72	1.04
10	10.96	1.17
15	13.39	1.43
20	17.71	1.89
25	25.39	2.71

*Using ref. [24].

by Konar *et al* [36]. Results of the present analysis indicate that the effective dielectric constant attains saturation for higher values of the incident beam intensity, i.e., about four times higher (see figure 1) as compared to paraxial theory result. It is because the entire cross-section is considered in the present case in place of only near-axis part as used in the paraxial ray approximation.

As shown in figure 2, for step-index fiber, beamwidth focusing parameter (f) shows non-oscillatory behaviour with axial distance (z) in the case of non-saturating-type Kerr non-linearity. For incident beam power higher than the critical value, initially the beam start converging but after an appreciable propagation in the fiber, the influence of the diffraction divergence dominate the converging effect due to non-linearity and beam start diverging much before the sharp focus is reached. In the case of parabolic fiber, the results are quite different from that for step-index fiber. This difference can easily be attributed to the geometry effect of the fiber. Here effect due to geometry of the fiber also superimpose on the non-linear effect which contribute to self-action focusing. In the case of non-saturating non-linearity (results in figure 3) for larger part of incident beam (i.e. $\rho \geq 15\ \mu$m) indicates, that converging effects are stronger as compared to step-index fiber, as reflected in the lower value of $f_{\min}$. For paraxial region, i.e., $\rho \leq 10\ \mu$m beam aperture after attaining certain minimum value, sharply diverges due to the domination of diffraction effect over the focusing non-linear as well as geometric effect (see figure 3).

The observed oscillatory behaviour for the focusing parameter (f) with propagation distance (z) (see figure 4) in the case of saturating-type Kerr non-linearity may be correlated to the beam intensity independent value attained by non-linearity. As beam propagates in the medium it converges and due to this beam intensity increases. Non-linear effect continuously increases till beam intensity attains the value which causes saturation in non-linearity. After this converging effect related to non-linearity ceased to increase, diffraction term starts dominating. The competition between the diffraction and refraction effects yields oscillatory behaviour. This type of oscillatory behaviour is observed both for step index and parabolic profile fibers with saturating non-linearity (figure 4) and such a type of medium acts as an oscillatory waveguide. An analysis of self-focusing of elliptic Gaussian laser beam in saturable non-linear medium by Konar and Sengupta [36] also indicates oscillatory behaviour but pattern is somewhat different from that predicted by the present study. This difference may be because the present analysis is for cylindrically Gaussian beam while they deal with elliptical Gaussian form in optical fiber.

Strong aberration effects are observed for both type of fibers under study. For the near-axis parts of the beam, paraxial analysis indicates only one focal point [24]. But the result of the present analysis which consider the entire spatial part of the beam shows that far-axis beam focuses at a later point (i.e. large focal length) as compared to near-axis beam. In the case of parabolic fiber (figure 6), aberration effects are more prominent as compared to step-index fiber (figure 5). As shown in table 1, first focal length of the beam in step-index fiber is smaller than in parabolic fiber. Apart from that, focusing effect is strong in the case of step index as compared to parabolic fiber, reflected by the value of $f_{\min}$ (see table 1). All these differences in the step-index and parabolic fiber can easily be correlated with the geometric and associated refraction effect.

For step-index fiber the normalised self-trapped beam radius $(\omega r_0/C)$ has been plotted for different values of intensities and shown in figure 7. Results demonstrate that the present analysis gives much flatter curve in the saturation region as compared to paraxial method. Such type of results had also been observed for detailed non-paraxial analysis by various authors in the case of plasma [25,26]. Observed self-trapping behaviour of the beam in fiber medium can easily be related to the saturating behaviour of orientation of anisotropic molecules of Kerr non-linear medium at high intensities and associated Kerr non-linearity saturation effect. The critical power is an important parameter in self-action problem. Results of analysis presented in table 2 show very interesting behaviour for critical power. For near-axis region where spatial distance $\rho \leq 5\ \mu$m, the calculated value of critical power is 9.34 kW and it compares well with the value obtained by Chiao *et al* [18] considering paraxial ray method. Present analysis indicates that the value of the calculated critical power increases as ρ increases. For the entire beam, value of the critical power comes out to be 25.39 kW, much higher than the paraxial results. It resolves the controversy and compare well with the observed experimental value of critical power $P_{\mathrm{CR}} = 25$ kW by Gramire *et al* [19]. This indicates that this analysis leads to better results for critical power as compared to paraxial ray theories. For laser beam in transverse direction, non-linear dielectric inhomogeneity perceived by the wave cannot be approximated using simple Taylor series expansion in the transverse cylindrical coordinates. This may be the reason why the paraxial ray theories fail in predicting the correct value of critical power and other important parameters related to self-focusing and self-trapping. Present analysis based on non-paraxial approach considering the entire spatial part of the incident beam predicts much flatter curve for normalised self-trapped beam radius and results compare well with the other complex theories such as variation method [26] and moment method [27], considering Laguerre–Gauss function [28] and angular spectrum theory of Subbarao and Sodha [29]. Present theory indicates that the paraxial rays and peripheral rays focus at different points predicting strong aberration effects. These effects are found to depend on the type of fibers. The value of the calculated critical power matches well with the only available experimental value.

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