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Anisotropic cosmological models and generalized scalar tensor theory

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Abstract. In this paper generalized scalar tensor theory has been considered in the background of anisotropic cosmological models, namely, axially symmetric Bianchi-I, Bianchi-III and Kortowski–Sachs space-time. For bulk viscous fluid, both exponential and power-law solutions have been studied and some assumptions among the physical parameters and solutions have been discussed.

Keywords. Anisotropic cosmological models; general scalar tensor theory; inflation.

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1. Introduction

Brans–Dicke theory [1] (BD theory) is so far the best-known alternative to Einstein theory of gravity. Hence it is the most studied and the best-known theory in place of Einstein theory of gravity. BD theory can be thought of as a minimal extension of general relativity and is nicely matched to Mach's principle [2] and Dirac's large number hypothesis [3]. According to this theory, the gravitational effects are partly due to geometrical and partly due to scalar interactions. Here the gravitational constant G is a variable scalar and is related to the scalar field $\varphi \sim G^{-1}$. Thus the action for the BD theory may be written in the form

$$A = \frac{C^3}{16\pi}\int (\varphi R + \omega\varphi^{-1}\varphi^{\mu}\varphi_{\mu})(-g)^{1/2}\mathrm{d}^4x. \tag{1}$$

Power-law inflation is possible for BD theory with the constant vacuum energy density and also for this type of extended inflation [4,5]. It is possible to 'slow roll over' for the Universe during phase transitions. Hence the BD theory generated a lot of interest as it solves some problems of the traditional exponential inflationary scenario. Further, for large values of ω (the coupling parameter) this theory is indistinghishable from Einstein gravity and experimental evidence [6] indicates very high value of ω (≥ 500). However, in scalar tensor theory [7–10] (where ω is not a constant but a function of the BD scalar field φ) it is possible to assume the coupling parameter to have such large values at present but it might have been much smaller during the early stages of the evolution of the Universe. As a consequence, the results predicted by scalar tensor theory may differ significantly from the prediction of Einstein gravity.

In this paper, we have considered axially symmetric Bianchi-I, Bianchi-III and Kantowski–Sachs space-time model and have studied both power law and inflationary solutions in the generalized scalar tensor theory along with a bulk viscous fluid. Due to the complicated form of the original BD action (see eq. (1)), we have used Dicke's revised units [11] namely

$$\tilde{g}_{\mu\nu} = \varphi \cdot g_{\mu\nu}.$$

As a consequence, G is a constant but the rest mass of the test particle is a variable. So the Lagrangian corresponding to the action (1) in this revised unit takes the form

$$L = \tilde{R} - \frac{\{2\omega(\varphi)+3\}}{2} \cdot \frac{(\varphi^{\mu} \cdot \varphi_{\mu}}{\varphi^2} + \frac{16\pi G}{C^4} L_{\mathrm{m}}, \tag{1a}$$

where $\tilde{R}$ is the Ricci scalar corresponding to the metric $\tilde{g}_{\mu\nu}$ and L_{m} is the Lagrangian corresponding to the matter distribution. Thus the field equation is (by variational principle)

$$\tilde{G}_{\mu\nu} = -T_{\mu\nu} - \frac{(2\omega+3)}{\varphi^2}\left[\varphi_{,\mu} \cdot \varphi_{,\nu} - \frac{1}{2}\tilde{g}_{\mu\nu} \cdot \varphi_{,\alpha}\varphi'^{\alpha}\right] \tag{2}$$

and the wave equations

$$\Box(\ln\varphi) = \frac{1}{(2\omega+3)}\left[T - \frac{1}{\varphi}(\varphi_{,\mu} \cdot \varphi'^{\mu})\frac{\mathrm{d}\omega}{\mathrm{d}\varphi}\right]. \tag{3}$$

Here $T_{\mu\nu}$ is the energy–momentum tensor corresponding to the matter distribution, T is the trace of the energy–momentum tensor and we have chosen units such that $8\pi G/C^4 = 1$.

2. Basic equations for the anisotropic models

We consider the anisotropic space-time model described by the line-element

$$\mathrm{d}S^2 = -\mathrm{d}t^2 + \mathrm{e}^{2\alpha}\mathrm{d}x^2 + \mathrm{e}^{2\beta}\mathrm{d}\Omega_K^2, \tag{4}$$

where α and β are functions of time alone and

$$\mathrm{d}\Omega_K^2 = \begin{cases} \mathrm{d}y^2 + \mathrm{d}z^2, & \text{when } K=0 \text{ (axially symmetric Bianchi-I model)},\\ \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2, & \text{when } K=+1 \text{ (Kantowski–Sachs model)},\\ \mathrm{d}\theta^2 + \sin h^2\theta\,\mathrm{d}\theta^2, & \text{when } K=-1 \text{ (Bianchi-III model)}.\end{cases}$$

If we consider energy–momentum tensor for a viscous flow with bulk viscosity then the above field equation (eq. (2)) for the metric (4) becomes

$$\dot{\beta}^2 + 2\dot{\alpha}\dot{\beta} + K\mathrm{e}^{-2\beta} = \rho + \frac{(2\omega+3)}{4}\dot{\psi}^2, \tag{5}$$

$$2\ddot{\beta} + 3\dot{\beta}^2 + K\mathrm{e}^{-2\beta} = -p - \frac{(2\omega+3)}{4}\dot{\psi}^2 + \eta(\dot{\alpha}+2\dot{\beta}), \tag{6}$$

$$\ddot{\alpha} + \ddot{\beta} + \dot{\alpha}^2 + \dot{\beta}^2 + \dot{\alpha}\dot{\beta} = -p - \frac{(2\omega+3)}{4}\dot{\psi}^2 + \eta(\dot{\alpha}+2\dot{\beta}), \tag{7}$$

and the wave equation (4) becomes

$$\ddot{\psi} + \dot{\psi}(\dot{\alpha}+2\dot{\beta}) = \frac{1}{(2\omega+3)}((\rho-3p)+3\eta(\dot{\alpha}+2\dot{\beta}) - \dot{\omega}\dot{\psi}). \tag{8}$$

Here, as usual, ρ, p and η are the energy density, pressure and coefficient of bulk viscosity respectively and $\psi = \ln\varphi$. Thus we have a set of four field equations containing seven unknown (physical or geometrical) parameters namely $\rho, p, \eta, \omega, \psi, \alpha$ and β. Hence for a unique solution one has to assume three more functional relations among these variables. A reasonable set of three assumptions are:

(i) Barotropic equation of state:

$$p = \varepsilon\rho, \quad 0 \le \varepsilon \le 1. \tag{9}$$

(ii) A particular choice of ω [12,13]

$$2\omega + 3 = \omega_0 \mathrm{e}^{-\psi}. \tag{10}$$

(iii) Scale factors in exponential form

$$\alpha = xt, \quad \beta = yt. \tag{11}$$

Scale factors in power-law form

$$\alpha = \ln\alpha_0 + x\ln t, \quad \beta = \ln\beta_0 + y\ln t, \tag{12}$$

where (x,y), (α_0,β_0), and ω_0 are constants.

The explicit evolution of the unknown physical parameters are as follows:

Case I: *Expansion of the Universe in exponential form*

By choosing the scale factors from eq. (11) we can eliminate p,η and ψ from the field equations and we have the following restrictions (only for $K=0$): (i) $x = y$, or (ii) $x + 2y = 0$.

The choice of the first restriction reduces the space-time metric for flat FRW model for which solutions have been given by Banerjee and Beesham [14]. The other restriction gives a static Universe (neither expansion nor contraction) where viscous flow with bulk viscosity is not possible as claimed by Chakraborty and Ghosh. Hence the result is not of much physical interest [15].

Case II: *Scale factors in power-law form*

For the power-law expansion of the Universe we shall consider $K = 0$ and $K \neq 0$ cases separately. As before, for $K = 0$, let the restriction on the parameters be (i) $x = y$ or (ii) $x + 2y = 1$.

The first case corresponds to flat FRW as before and so we omit it. For the second case (i.e. $x + 2y = 1$) the explicit expressions for the unknown parameters are

$$P = \frac{2y - 3y^2}{t^2} - \frac{\omega_0}{4} x \frac{(K_0 \cdot (\log t/t) + C_0)^2}{(C_0 t + C_1 - (K_0/2)(\log t)^2)},$$

$$p = \varepsilon\rho,$$

$$\eta = \frac{(1-\varepsilon)}{T}\left\{\frac{\omega_0}{4} + (3y^2 - 2y)\right\} + \frac{(K_0(\log t/t) + C_0)^2}{(C_0 \cdot t + C_1 - (K_0/2)(\log t)^2)},$$

$$\phi = \mathrm{e}^{\psi} = \left(C_0 t + C_1 - \frac{K_0}{2}(\log t)^2\right)^{-1},$$

where $K_0 = 10y^2 + 2xy - 6y$ and C_0, C_1 are integration constants.

For $K \neq 0$, i.e. for $K = \pm 1$, the restrictions on the parameters are

$$y = 2, \quad \text{and} \quad (x-2)(x+3) = K\beta_0^2.$$

In this case also the unknown physical parameters have the following expressions:

$$\rho = \frac{4(x+1)}{t^2} + \frac{(K/\beta_0^2)}{t^4} - \left(\frac{\omega_0}{4}\right) \cdot \frac{\left(C_2(x+3) \cdot t^{-(x+4)} + \frac{4(x+1)^2}{\omega_0(x+3)} \cdot \frac{1}{t}\right)^2}{\left(C_1 + C_2 t^{-(x+3)} - \frac{4(x+1)^2}{\omega_0(x+3)} \log t\right)},$$

$$p = \varepsilon\rho,$$

$$\eta = \frac{1}{(x+4)}\left(\frac{4(x+3)}{t} + \frac{(2K/\beta_0^2)}{t^3} - \rho \cdot t(1-\varepsilon)\right),$$

$$\phi = \mathrm{e}^{\psi} = \left(C_1 + C_2 \cdot t^{-(x+3)} - \frac{4(x+1)^2}{\omega_0(x+3)} \log t\right)^{-1},$$

with C_1, C_2 as integration constants.

3. Discussion

General scalar tensor theory has been studied in this paper for a class of anisotropic models, considering a viscous flow with bulk viscosity. We have examined both exponential expansion and power-law form of the scale factors for these models. The exponential form of the scale factors is possible only for axially symmetric Bianchi-I ($K = 0$) model where either it is reduced to flat FRW model (which was discussed earlier) or volume becomes a constant (static model).

The power-law expansion is possible for all the three space-time models ($K = 0, \pm 1$), with some restrictions on the parameters involved, and unknown physical parameters have been explicitly evaluated.

Finally, using Dicke's revised units, i.e., $\bar{g}_{\mu\nu} = \varphi g_{\mu\nu}$, the actual form of the metric coefficients in atomic units are

$$e^{2\alpha} = e^{2xt} \cdot \left(C_0 + C_1 t - \frac{K_0}{2} (\log t)^2 \right),$$
$$e^{2\beta} = e^{2yt} \cdot \left(C_0 + C_1 t - \frac{K_0}{2} (\log t)^2 \right),$$

when in exponential form and

$$e^{2\alpha} = \alpha_0^2 t^{2x} \cdot \left(C_1 + C_2 \cdot t^{-(x+3)} - \frac{4(x+1)^2}{\omega_0 (x+3)} \log t \right),$$
$$e^{2\beta} = \beta_0^2 t^{2y} \cdot \left(C_1 + C_2 \cdot t^{-(x+3)} - \frac{4(x+1)^2}{\omega_0 (x+3)} \log t \right),$$

for power-law form of expansion.

Thus the actual scale factors are not simple exponential functions or power functions of time but rather a combination of polynomial and logarithmic functions (for power form). Hence it is possible to have 'graceful exit' problem for generalized scalar tensor theory with bulk viscosity by a proper choice of $\omega(\varphi)$.

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