

Radial oscillations of magnetized proto strange stars in temperature- and density-dependent quark mass model

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Abstract. We report on the study of the mass–radius (M – R) relation and the radial oscillations of magnetized proto strange stars. For the quark matter we have employed the very recent modification, the temperature- and density-dependent quark mass model of the well-known density-dependent quark mass model. We find that the effect of magnetic field, both on the maximum mass and radial frequencies, is rather small. Also a proto strange star, whether magnetized or otherwise, is more likely to evolve into a strange star rather than transform into a black hole.

Keywords. Proto strange stars; magnetic field; radial oscillations.

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1. Introduction

With the discovery of magnetars there has been a lot of interest in the study of the effects of strong magnetic fields on various astrophysical phenomena. The observed soft gamma ray repeaters discovered by Kuoveliotou *et al* [1] (see also [2]) exhibit the presence of strong magnetic fields of up to 10^{15} G. Since in most models, the core magnetic fields may be higher by a factor of 10^3 – 10^5 than the surface fields, the magnetic field in the core could be as high as 10^{18} – 10^{19} G. Such intense magnetic fields may not be ruled out by the virial theorem along with the general relativistic corrections [3]. However for the quark matter (QM) another problem crops up at extremely high magnetic fields. Agasian *et al* [4] have studied the effect of a strong magnetic field on QM and reached a tentative conclusion that the gluon condensate could break down at fields above $(0.5$ – $1) \times 10^{19}$ G leading to dramatic consequences for an effective QCD model like the density-dependent quark mass (DDQM) model that we have employed in the present study or the Bag model for that matter. Keeping this in mind the maximum magnetic field that we have employed in the present study is $\sim 10^{19}$ G (7.5×10^4 MeV²; 1 MeV² $\sim 1.6 \times 10^{14}$ G).

At present, it is believed that at high densities ($\rho \geq 3\rho_0$ where ρ_0 is the saturation nuclear density) hadronic matter undergoes a phase transition to the unconfined state of quarks and gluons which is composed of roughly equal number of u , d and s quarks together with a

gas of electrons and muons necessary to maintain charge neutrality. This strange quark matter (SQM) may in fact be the true ground state of matter [5–7]. It has been conjectured by Glendenning [6] that as a result of nucleation of SQM bubbles, most of the currently believed neutron stars may be strange stars (SS).

The formation of proto neutron stars (PNS) is well-understood [8,9]; however how a PNS converts to a proto strange star (PSS) during the supernova explosion is not very clear at the moment due to complex nuclear burning of nuclear matter (NM) to SQM. Some studies have been undertaken for the phase transition in a two-step process, viz., $NM \rightarrow ud$ matter $\rightarrow uds$ matter [10–14] where the first conversion is a strong process while the second conversion is a weak process. Recently, Gupta *et al* [14] have studied the first step in this process, i.e., phase transition from NM to ud matter in a magnetized PNS and found that the presence of strong magnetic field aids this conversion. Once the two-flavor quark matter is formed it converts into strange matter via the well-understood weak processes. In view of this one can expect a fairly significant fraction of proto stars to be actually magnetized PSS.

Once a PSS is formed it loses its excess neutrinos in a few tens of seconds and simultaneously cools to a temperature of about 1 MeV. After that it keeps on losing energy by thermal radiation to become a cold SS. Alternatively the PSS may transform itself into a black hole with certain observable consequences. What course a PSS will follow depends upon its structure and the M – R relationship. Recently, we have studied some aspects of the PSS with a view to study its stability. Since a fair proportion of such dense proto stars are likely to be magnetized PSS, it would be interesting to study the effect of a strong magnetic field on their properties. Motivated by these considerations, in this paper we study the M – R relation and radial oscillations of a magnetized PSS. For this purpose we have employed the temperature- and density-dependent quark mass (TDDQM) model which we had employed in our previous study of the PSS as well. The TDDQM model is an improvement over the usual and much-investigated DDQM model. Such a model of quark confinement through the density dependence of quark masses was initially proposed by Pati and Salam [15] who picturized confinement as a quark having a small mass inside and a very large mass outside the confined region. Fowler *et al* [16] and Plumber *et al* [17] introduced a slightly different description in which the confinement is treated by assuming a baryon density dependence of quark masses:

$$\begin{aligned} m_u &= m_{u0} + \frac{C}{3n_B}, \\ m_d &= m_{d0} + \frac{C}{3n_B}, \\ m_s &= m_{s0} + \frac{C}{3n_B}, \end{aligned} \tag{1}$$

where C , m_{u0} , m_{d0} and m_{s0} are free parameters and n_B is the baryon density. The u and d quark masses, m_{u0} and m_{d0} , are only a few MeV and usually taken to be zero. Dey *et al* [18] have given an alternative formulation for the density dependence of quark masses.

In this model at high quark chemical potentials (high baryon density) the masses approach their constant values only asymptotically. On the other hand the phenomenological Bag model assumes that within the Bag the quarks are free. But results from lattice calculations show that QM does not become asymptotically free immediately after phase transition [19]. In this context the DDQM model is an improvement over the usual MIT

Bag model at high chemical potentials. However, as pointed out by Fowler *et al* in their original paper [16], confinement in the DDQM model, as also in the Bag model, is permanent: this model cannot produce a correct lattice QCD phase diagram even qualitatively at non-zero temperatures, since in lattice QCD at zero baryon density there is a phase transition at a finite temperature. To rectify this situation, Zhang and Su [20] suggested that apart from the density, the quark masses should also depend upon the temperature. In TDDQM model of Zhang and Su [20] guided by Friedman–Lee soliton Bag model [21], C is taken as a function of temperature and is parameterized as

$$C(T) = C_0 \left[1 - \left(\frac{T^2}{T_c} \right) \right]; \quad 0 \leq T \leq T_c$$

$$= 0; \quad T > T_c \quad (2)$$

where T_c is the critical temperature of deconfinement. At $T = 0$, the TDDQM model goes over to the usual DDQM model.

In §2, we present the formalism and the thermodynamics used. Section 3 deals with the structure and radial oscillations of the magnetized PSS and §4 is devoted to results and discussion.

2. Formalism and thermodynamics

We consider PSS as a system composed of u , d and s quarks, electrons, muons and their neutrinos. The thermodynamical potential for the system is given by $\Omega = \sum \Omega_i$, where Ω_i for a charged particle in the presence of magnetic field B along the z -axis is given by [14]

$$\Omega_i = -\frac{g'_i e_i B T}{2\pi^2} \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int dp_z [\log(1 + \exp(-\beta[E_i - \mu_i]))$$

$$+ \log(1 + \exp(-\beta[E_i + \mu_i]))]. \quad (3)$$

Here $i = u, d, s, e, \mu$,

$$E_i = \sqrt{p_z^2 + m_i^2 + 2ne_i B} \quad (4)$$

and $n (= 0, 1, 2, \dots)$ are the Landau levels for the energy of a particle of charge e_i in the magnetic field. The degeneracy factor, g'_i , is three for the quarks and unity for the charged leptons, since spin degeneracy is taken care of by the factor $(2 - \delta_{n0})$. Since the neutrinos are neutral, their thermodynamical potential (as also for charged particles in the absence of magnetic field) is given by the usual expression

$$\Omega_i = -\frac{g_i T}{2\pi^2} \int p^2 dp [\log(1 + \exp(-\beta[E_i - \mu_i]))$$

$$+ \log(1 + \exp(-\beta[E_i + \mu_i]))] \quad (5)$$

and

$$E_i = \sqrt{p^2 + m_i^2}. \quad (6)$$

The degeneracy factor g_i includes spin degeneracy as well and is six for u , d and s quarks, two for e and μ and one for ν_e and ν_μ . The expressions for the pressure, P_i , the energy density, ρ_i , and the number density, n_i , in terms of the thermodynamical potential are well-known, and are

$$P_i = n_B \frac{\partial \Omega_i}{\partial n_B} - \Omega_i, \quad (7)$$

$$\rho_i = \Omega_i + \mu_i n_i - T \frac{\partial \Omega_i}{\partial T}, \quad (8)$$

$$n_i = - \frac{\partial \Omega_i}{\partial \mu_i}. \quad (9)$$

The contribution of gluons should also be added to the above expressions for the total pressure and energy. For free gluons this contribution is $(8/45)\pi^2 T^4$ to the pressure and $(8/15)\pi^2 T^4$ to the energy density. However at densities $\geq n_0$, where n_0 is the saturation nuclear density one has to include the corrections to the free gluonic behaviour which suppress the free gluon contribution considerably [22]. Since the contribution of gluons at densities and temperatures considered here is anyway very small compared to that of the quarks and leptons, whether the gluonic contribution is added or not is only of academic interest.

The entire system is in β equilibrium maintained by the weak interactions:

$$\begin{aligned} d, s &\leftrightarrow u + e + \bar{\nu}_e, \\ d, s &\leftrightarrow u + \mu + \bar{\nu}_\mu, \\ s + u &\leftrightarrow u + d. \end{aligned}$$

This leads to the following relations among the various chemical potentials under equilibrium conditions:

$$\begin{aligned} \mu_d &= \mu_u + \mu_e - \mu_{\nu_e}, \\ \mu_d &= \mu_s, \end{aligned} \quad (10)$$

$$\mu_\mu - \mu_{\nu_\mu} = \mu_e - \mu_{\nu_e}. \quad (11)$$

Charge neutrality of the system demands

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e - n_\mu = 0. \quad (12)$$

The total baryon density is given by

$$n_B = \frac{1}{3}(n_u + n_d + n_s). \quad (13)$$

Equations (10)–(12) can be solved self-consistently for any given baryon density, temperature and the two neutrino chemical potentials. Once the chemical potentials are known the equation of state $P(\rho)$ can be obtained.

3. Radial pulsations of magnetized PSS

The radial pulsations were first studied by Chandrasekhar [23] for non-rotating stars in general theory of relativity. The metric used is

$$ds^2 = -e^{\nu}c^2 dt^2 + e^{\lambda} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (14)$$

The structure of the star is described by the Tolman–Openheimer–Volkoff equations

$$\frac{dm}{dr} = 4\pi r^2 \rho, \quad (15)$$

$$\frac{dP}{dr} = \frac{-G(P + \rho c^2)(m + 4\pi r^3 P c^{-2})}{c^2 r^2 (1 - \frac{2GM}{c^2 r})}, \quad (16)$$

$$\frac{d\nu}{dr} = \frac{2GM(1 + 4\pi r^3 P m c^{-2})}{c^2 r (1 - \frac{2GM}{c^2 r})}. \quad (17)$$

For the given equation of state $P(\rho)$, eqs (15)–(17) can be numerically integrated outwards starting from $r = 0$ with a given central density ρ_c and the corresponding pressure $P_c(\rho_c)$ up to the point where $P = 0$; this point defines the surface of the star with radius R . If δr is a small radial perturbation, define

$$\xi = \frac{\delta r}{r}, \quad \zeta = r^2 e^{-\nu/2} \xi, \quad (18)$$

and project the time dependence as $\exp(i\sigma t)$. The equation governing radial adiabatic oscillations then takes the form [23–25]

$$F \frac{d^2 \zeta}{dr^2} + G \frac{d\zeta}{dr} + H \zeta = \sigma^2 \zeta, \quad (19)$$

where

$$F = -\frac{e^{\nu-\lambda}}{P + \rho c^2} (\Gamma P), \quad (20)$$

$$G = -\frac{e^{\nu-\lambda}}{P + \rho c^2} \left[\frac{1}{2} \Gamma P (\lambda + 3\nu) + \frac{d(\Gamma P)}{dr} - \frac{2}{r} (\Gamma P) \right], \quad (21)$$

$$H = \frac{e^{\nu-\lambda}}{P + \rho c^2} \left[\frac{4}{r} \frac{dP}{dr} + \frac{8\pi G e^{\lambda} P (P + \rho c^2)}{c^4} - \frac{1}{P + \rho c^2} \left(\frac{dP}{dr} \right)^2 \right]. \quad (22)$$

λ is related to the metric function through

$$e^{-\lambda} = \left(1 - \frac{2GM(r)}{rc^2} \right) \quad (23)$$

and Γ is the adiabatic index

$$\Gamma = \frac{P + \rho c^2}{c^2 P} \frac{dP}{d\rho}. \quad (24)$$

The boundary conditions to solve the pulsation equations (19)–(24) are

$$\zeta(r = 0) = 0, \quad (25)$$

$$\delta P(r = R) = 0. \quad (26)$$

Equation (19) with the boundary conditions (25) and (26) represents a Sturm–Liouville eigenvalue problem for σ^2 with the well-known result that the frequency spectrum is discrete. For $\sigma^2 > 0$, σ is real, and the solution is purely oscillatory, whereas for $\sigma^2 < 0$, σ is imaginary, resulting in exponentially growing unstable radial oscillations.

4. Results and discussion

In the TDDQM model there are three free parameters C_0 , m_{s0} and T_c . The choice of C_0 and m_{s0} is motivated by the usual stability argument. The formation of SS or PSS is based on the premise that the strange matter is the true ground state of matter [5,6]. For this to be so, the parameters, C_0 and m_{s0} , must be such that energy/baryon, $\rho/n_B \leq 930$ MeV for uds matter while $\rho/n_B > 940$ MeV for ud matter [6]. This gives us a stability window for C_0 and m_{s0} and the choice of these two parameters must lie within this window. In paper 1, we had chosen two sets: (i) $C_0 = 185$ MeV fm⁻³, $m_{s0} = 150$ MeV and (ii) $C_0 = 210$ MeV fm⁻³, $m_{s0} = 100$ MeV, both of which lie within the stability window. Since in the present calculation our aim is to study the effect of magnetic field on the results obtained in paper 1, we have chosen one of these sets, viz. $C_0 = 185$ MeV fm⁻³, $m_{s0} = 150$ MeV for this study. As for T_c , which is the critical temperature for transition from hadron to quark phase, in paper 1 following Zhang and Su [20] we had chosen $T_c = 170$ MeV. However, according to recent improved calculations of T_c based on the Clover-improved Wilson fermions and the improved staggered fermions, a value of $T_c \sim 150$ MeV is more appropriate for uds matter instead of $T_c \sim 170$ MeV which is a favoured value for the ud matter [26]. In the present calculation, we have considered both $T_c = 150$ MeV and $T_c = 170$ MeV. As for the neutrino chemical potentials, we have taken $\mu_{\nu_e} = \mu_{\nu_\mu} = 200$ MeV, which are quite realistic, since the PNS is initially born with a high lepton excess in the form of the electron and muon neutrinos. However, we have included one set for $\mu_{\nu_e} = \mu_{\nu_\mu} = 0$ MeV, the value at the end of ~ 20 s when the PSS converts to an ordinary cold SS.

In figure 1, we show the mass–radius relation for the case $\mu_{\nu_e} = \mu_{\nu_\mu} = 200$ MeV and $T = 60$ MeV, the value expected if the PNS converts to a PSS at the beginning of its formation. The maximum mass for the various sets shown are all in the region of 1.6–1.8 solar masses. The effect of the increase in critical temperature from 150 MeV to 170 MeV is to reduce the maximum mass by a few per cent. The effect of the magnetic field is also to reduce the maximum mass, the change again being a few per cent for a magnetic field of 75,000 MeV² ($1 \text{ MeV}^2 \sim 1.6 \times 10^{14}$ G). The effect on the radius corresponding to a given mass is also similar – a few percent decrease with increasing T_c or increasing B .

In figure 2, we show the effect of temperature on the M – R relation for fixed neutrino chemical potentials, $\mu_{\nu_e} = \mu_{\nu_\mu} = 200$ MeV, fixed magnetic field $B = 50,000$ MeV² and fixed $T_c = 150$ MeV. We find that both the maximum mass and the radius for a given mass increase with increasing temperature, the increase becoming more significant as the

Radial oscillations of magnetized proto strange stars

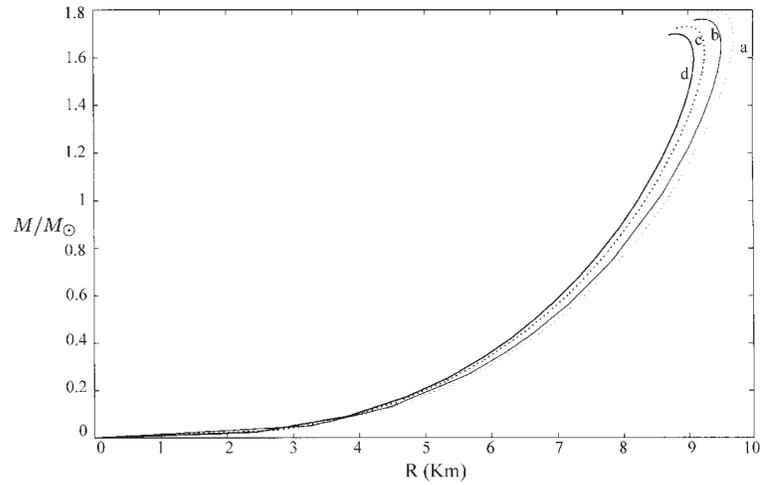


Figure 1. Radius R in km vs. mass M/M_{\odot} (where M_{\odot} is the solar mass) for $T = 60$ MeV and $\mu_{\nu_e} = \mu_{\nu_{\mu}} = 200$ MeV. The curves a, b correspond to $B = 0$ and $T_c = 150$ and 170 MeV respectively. The curves c, d correspond to $B = 75,000$ MeV² (1 MeV² $\sim 1.6 \times 10^{14}$ G) and $T_c = 150$ and 170 MeV respectively.

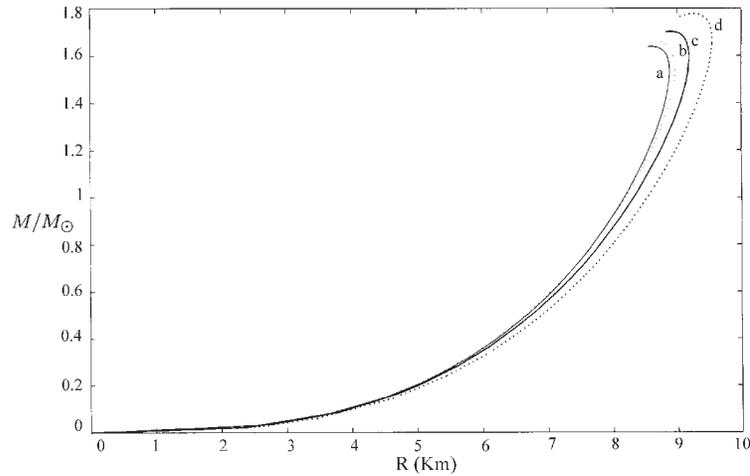


Figure 2. Radius R in km vs. mass M/M_{\odot} for $T_c = 150$ MeV, $B = 50,000$ MeV² and $\mu_{\nu_e} = \mu_{\nu_{\mu}} = 200$ MeV. The curves a, b, c and d correspond to $T = 0, 20, 40$ and 60 MeV respectively.

temperature increases. Thus the increase in maximum mass is only about one per cent as temperature increases from 0 to 20 MeV but is about four per cent as it increases from 40 to 60 MeV. It is clear that the closer one gets to T_c , the more rapid is the variation of $M-R$ relation with T . As the temperature comes closer to T_c , the quark masses decrease rapidly which leads to an increase in the maximum mass of the PSS.

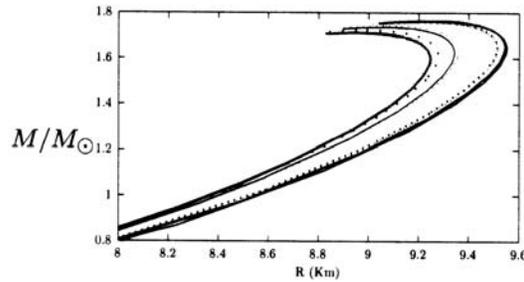


Figure 3. Radius R in km vs. mass M/M_{\odot} for $T_c = 170$ MeV and $T = 60$ MeV. Starting from top to bottom, first four curves are for $B = 0$ and next four are for $B = 50,000$ MeV^2 . The four curves in each case correspond to four sets of μ_{ν_e} and $\mu_{\nu_{\mu}}$ namely $(200, 200)$; $(200, 0)$; $(0, 200)$; $(0, 0)$ respectively.

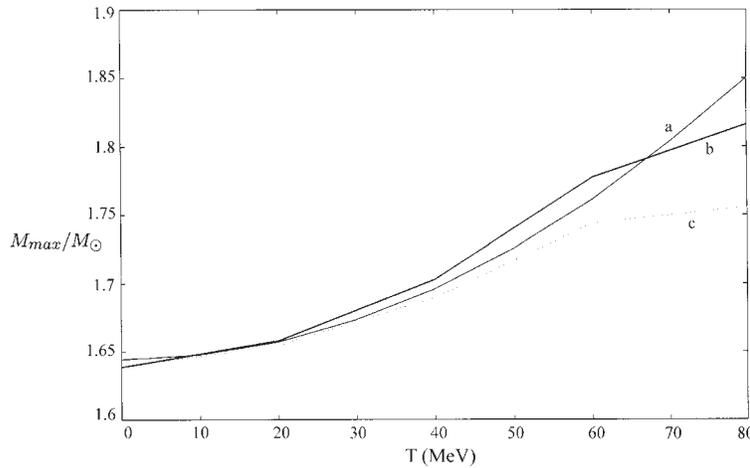


Figure 4. Temperature T vs. M_{max}/M_{\odot} for $\mu_{\nu_e} = \mu_{\nu_{\mu}} = 200$ MeV. Curve a corresponds to $B = 0$ and $T_c = 170$ MeV. Curves b, c correspond to $B = 50,000$ MeV^2 and $T_c = 150$ and 170 MeV respectively.

In figure 3 we depict the effect of neutrino chemical potentials on the M – R relation for two different magnetic fields, viz. $B = 0$ and $50,000$ MeV^2 . We find that the effect of the neutrino chemical potentials is quite small at $B = 0$ and hardly any more significant at $B = 50,000$ MeV^2 . This is consistent with our earlier calculations [14]; there too we had found that the effect of chemical potentials was equally small. Figure 4 deals with the effect of temperature T , the critical temperature T_c and magnetic field B on the maximum mass of the PSS. We find that at lower temperatures ($T < 40$ MeV) the effect of magnetic field is negligible on the maximum mass, but increases appreciably at $T > 40$ MeV.

In figure 5, we plot frequency of radial pulsations against the mass of the PSS for the fundamental ($n = 0$) and the first excited ($n = 1$) mode. We find that there is not much of a qualitative change in the frequency pattern for either the $n = 0$ or $n = 1$ modes. However for the low mass PSS the effect is slightly more than for the high mass star.

Radial oscillations of magnetized proto strange stars

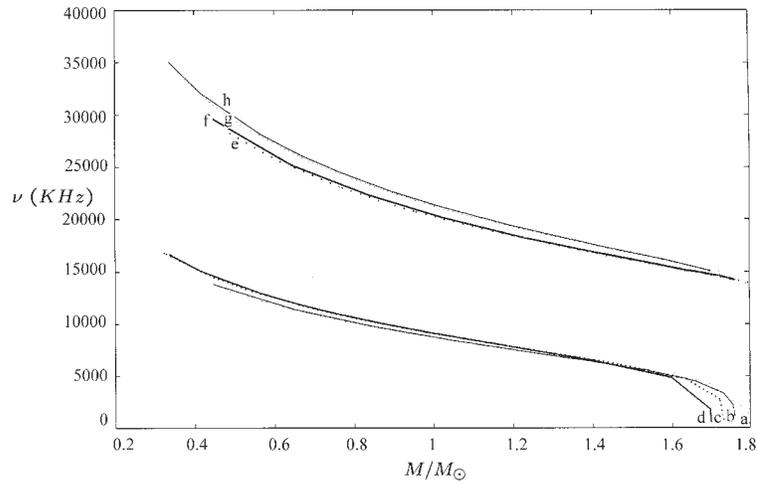


Figure 5. Mass M/M_{\odot} vs. frequency ν (KHz) for $T = 60$ MeV and $\mu_{\nu_e} = \mu_{\nu_{\mu}} = 200$ MeV. Curves a, b, c, d correspond to two sets of B and T_c namely $(0, 150)$; $(0, 170)$; $(75,000, 150)$; $(75,000, 170)$ for the fundamental mode, $n = 0$. Curves e, f, g, h correspond to the same sets respectively but at $n = 1$ mode.

To conclude, we find that the maximum mass increases with increase of temperature but decreases with increasing magnetic field. This may appear contrary to the expectation, since the effect of temperature and magnetic field is believed to be in the same direction. However, with an increase in temperature both the energy and pressure of the system increase but with magnetic field whereas the energy of the system increases, the pressure decreases. Hence it is quite understandable that the two effects are opposite in nature.

The maximum mass a star can support decreases with increase in the magnetic field both for the cold SS ($T = 0$, $\mu_{\nu_e} = \mu_{\nu_{\mu}} = 0$ case in our study) as well as for a PSS. The effect of magnetic field on the $M-R$ relationship of a PNS is not yet studied. However for a neutron star (NS) the effect was found to be just the opposite [27]; the maximum possible mass of the star configuration increases with B . Also contrary to the case of PNS, where the trapped neutrinos lead to a significant decrease in the maximum mass [8], in the present case we find that the effect is rather insignificant.

The change in the maximum mass with temperature and neutrino trapping can also have some observational consequences. Most of the measured NS masses are in the neighbourhood of $1.4M_{\odot}$. In the case of PNS the maximum mass increases as the neutrinos leak out, but the system remains stable and black hole (BH) formation is unlikely during the long-term deleptonization era. On the other hand in the case of PSS as we have seen, the maximum mass is rather insensitive to the presence of leptons but decreases due to a fall in temperature which takes place simultaneously with deleptonization. If the maximum mass comes close to the observed mass, BH formation could occur as the neutrinos leak out of the PSS, leading to a dramatic cessation of the neutrino signal [28], an event which can in principle be observed. This would imply that the equations of state that lead to a maximum mass just above the observed masses, produce a metastable PNS which transforms into a BH during leptonization instead of becoming a cold SS. In our model we find that the maximum mass is significantly above the observed NS masses, thus a PSS is likely to become

a cold SS rather than transform into a BH. Though the maximum mass in the presence of a magnetic field is considerably reduced, it still remains greater than $1.4M_{\odot}$ both for cold neutrinoless and hot neutrino-rich stars. Thus the formation of a BH is rather unlikely even in the presence of a magnetic field. If this is so, a reasonable number of observed NS could actually be SS.

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