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On Bianchi-I cosmic strings coupled with Maxwell fields in bimetric relativity

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Abstract. Axially symmetric Bianchi-I model is studied with source cosmic cloud strings coupled with electromagnetic field in Rosen's bimetric theory of relativity and observed that there is no contribution from cosmic strings and Maxwell fields in this theory.

Keywords. Bianchi-I; cosmic strings; Maxwell fields; bimetric relativity; axially symmetric; singularity; vacuum model.

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1. Introduction

Rosen [1] proposed the bimetric theory of relativity to remove some of the unsatisfactory features of the general theory of relativity, in which there exist two metric tensors at each point of space-time g_{ij}, which describes gravitation and the background metric γ_{ij}, which enters into the field equations and interacts with g_{ij} but does not interact directly with matter. One can regard γ_{ij} as describing the geometry that exists if matter were not present. Accordingly, at each space-time point one has two line-elements.

$$\mathrm{d}s^2 = g_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

and

$$\mathrm{d}\sigma^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

where ds is the interval between two neighbouring events as measured by a clock and a measuring rod. The interval dσ is an abstract or a geometrical quantity which is not directly measurable. One can regard it as describing the geometry that exist if no matter were present.

Earlier Bhattacharya and Karade [2] studied this model in the context of general theory of relativity and has shown that, in the presence of $\wedge$, the initial singularity occurs.

Here we studied Bianchi-I cosmological model in the context of bimetric-relativity with cosmic strings coupled with an electromagnetic field and observed the cosmic strings and electromagnetic field does not exist in this theory. Further we obtained an empty space-time which is free from singularity.

2. Bianchi-I model and field equations

An axially symmetric Bianchi-I metric is

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{2\alpha}\mathrm{d}x^2 - \mathrm{e}^{2\beta}(\mathrm{d}y^2 + \mathrm{d}z^2), \tag{1}$$

where α and β are functions of t only and background metric of flat space-time is

$$\mathrm{d}\sigma^2 = \mathrm{d}t^2 - (\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2). \tag{2}$$

The matter distribution consists of cosmic cloud strings coupled with a magnetic field along the x-direction and it is given by the energy-momentum tensor T_i^j as

$$T_i^j = T_{i\ \mathrm{strings}}^j + E_{i\ \mathrm{mag}}^j, \tag{3}$$

where

$$T_{i\ \mathrm{strings}}^j = \rho v_i v^j - \lambda x_i x^j \tag{4}$$

together with $v_4 v^4 = 1$ and $x_1 x^1 = -1$ (i.e. along x-axis) and

$$E_{i\ \mathrm{mag}}^j = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j, \tag{5}$$

where $E_{i\ \mathrm{mag}}^j$ is electromagnetic energy tensor, F_i^j the electromagnetic field tensor, v^i the four-velocities of the cloud of particles, x^i the four-vector representing the direction of anisotropy (i.e. x-axis) and ρ is the rest energy density for a cloud of strings with particles attached along the extension. Thus

$$\rho = \rho_\mathrm{p} + \lambda,$$

where ρ_p is the particle energy density and λ is the tension density of the strings.

In the co-moving coordinate system we have

$$T_{1\ \mathrm{strings}}^1 = \lambda, \quad T_{4\ \mathrm{strings}}^4 = \rho$$

and

$$T_{i\ \mathrm{strings}}^j = 0 \quad \text{for } i, j = 2, 3 \text{ and for } i \neq j.$$

As the electromagnetic field is along the x-direction alone, F_{23} is the only non-zero component of Maxwell tensor F_{ij}. Maxwell's equation

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \tag{6}$$

gives rise to $F_{23} = F$ (constant). Then

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = -\eta, \tag{7}$$

where

$$\eta = \frac{F^2 e^{-4\beta}}{2}. \tag{8}$$

The field equations of bimetric relativity proposed by Rosen [3] are

$$K_i^j = N_i^j - \frac{1}{2} N g_i^j = -8\pi k T_i^j, \tag{9}$$

where

$$N_i^j = \frac{1}{2} \gamma^{\alpha\beta} (g^{hj} g_{hi|\alpha})_{|\beta}, \tag{10}$$

$$K = \left(\frac{g}{\gamma}\right)^{1/2},$$

$$g = \det(g_{ij}), \quad \gamma = \det(\gamma_{ij})$$

and

$$N = N_\alpha^\alpha = N_1^1 + N_2^2 + N_3^3 + N_4^4, \tag{11}$$

where a vertical bar $(|)$ denotes the γ-differentiation with respect to $-\gamma_{ij}$.
Using eqs (1)–(11), the field equations are

$$K_1^1 = \alpha^{\cdot\cdot} - 2\beta^{\cdot\cdot} = -16\pi k(\lambda - \eta), \tag{12}$$

$$K_2^2 = K_3^3 = \alpha^{\cdot\cdot} = -16\pi k \eta, \tag{13}$$

$$K_4^4 = \alpha^{\cdot\cdot} + 2\beta^{\cdot\cdot} = 16\pi k(\rho - \eta), \tag{14}$$

when $\alpha^{\cdot} = \partial\alpha/\partial t$, $\alpha^{\cdot\cdot} = \partial^2\alpha/\partial t^2$ etc. Using eqs (12)–(14) we have

$$8\pi k(\rho - \lambda + 2\eta) = 0. \tag{15}$$

Equation (15) gives us

$$\rho = 0 = \lambda \quad \text{and} \quad \eta = 0. \tag{16}$$

Equation (16) shows that there is no contribution from cosmic cloud strings as well as from Maxwell fields to Bianchi-I axially symmetric model in bimetric relativity.
Thus using eq. (16) with eqs (12)–(14), we obtain that

$$\alpha^{\cdot\cdot} - 2\beta^{\cdot\cdot} = 0, \tag{17}$$

$$\alpha^{\cdot\cdot} = 0 \tag{18}$$

and

$$\alpha^{\cdot\cdot} + 2\beta^{\cdot\cdot} = 0. \tag{19}$$

Using eqs (17)–(19), as the number of equations are more than the variables, we can find the values of $\alpha^{\cdot\cdot}$ and $\beta^{\cdot\cdot}$. They are $\alpha^{\cdot\cdot} = 0$ and $\beta^{\cdot\cdot} = 0$.

After integration, we get

$$\alpha = c_1 t + c_3 \tag{20}$$

and

$$\beta = c_2 t + c_4, \tag{21}$$

where c_3 and c_4 are constants of integration.

Absorbing the constants c_3 and c_4 and

$$c_1 = c_2 = c = \text{ constant,}$$

in differentials the line-element (1) takes the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{2ct}(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2). \tag{22}$$

Here we observe that the vacuum model (22) is free from singularity at $t = 0$.

References

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