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Surface plasma waves over bismuth–vacuum interface

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Abstract. A surface plasma wave (SPW) over bismuth–vacuum interface has a signature of mass anisotropy of free electrons. For SPW propagation along the trigonal axis there is no birefringence. The frequency cutoff of SPW $\omega_{\text{cutoff}} = \omega_p/\sqrt{2(\varepsilon_L + \varepsilon)}$ lies in the far infrared region and can be accessed using free electron laser. The damping rate of waves at low temperatures is low. The surface plasma wave may be excited by an electron beam of current ∼100 mA propagating parallel to the interface in its close proximity.

Keywords. Surface plasma waves; bismuth; semimetals; plasma processing.

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1. Introduction

Laser ablation of materials is emerging as a powerful tool in microelectronics [1–3]. It causes heating, evaporation and plasma formation only over a small width of the target surface, leaving the rest of the target fairly unaffected. The promptness of the process, using picosecond laser pulses, makes it ideally suited for depositing superlattices. Experiments on laser ablation of metals have revealed an order of magnitude enhancement in ablation yield when the laser gets mode converted into a surface plasma wave (SPW) [4,5]. A SPW is a mode of electromagnetic wave propagation over a free space–metal interface whose amplitude falls off sharply away from the interface on both sides [6–8]. Ursu *et al* [9] have developed an elegant formalism of energy coupling from laser radiation to metallic surfaces and SPW. Zherikhin *et al* [10] have studied laser induced decomposition, evaporation and ablation processes in $1-2-3$ superconductors. Akhmanov *et al* [11] have reviewed the physical effects occurring at the surfaces due to high power radiation. Liu and Tripathi [12] have studied coupling between laser and SPW in the presence of a surface ripple. Smolyaninov *et al* [13] have reported *in situ* measurements of surface plasmon scattering by the individual defects created by a fiber tip at various sizes and shapes of the defects. Bozhevolnyi and Pudonin [14] have developed microcomponents of two-dimensional (2D) optics of surface plasma waves, viz., micromirrors, microcavities, and corner square micromirrors.

In this paper we study the nature of surface plasma modes over bismuth, a semimetal with non-spherical energy surfaces [15,16]. Bismuth has wide ranging applications in Hall effect devices, hyper frequency power sensors, thermopiles, microwave detectors, etc. [17,18]. Further, due to low free carrier density as compared to metals, the frequency cutoff of surface plasma wave may fall in the infrared region and could be accessed by free electron laser, a fast emerging device of high power coherent radiation. In §2 we derive the dispersion relation for surface plasma waves using a fluid theory. In §3 we study the SPW excitation by an electron beam. A discussion of results is given in §4.

2. SPW propagation

Consider a bismuth (Bi)–free space interface $(x = 0)$ with bismuth occupying the $x < 0$ half space, z-axis is along the trigonal axis of bismuth. A surface wave propagates along z-axis, with t, z variation as

$$\vec{E} = \vec{A}(x)\mathrm{e}^{-i(\omega t - k_z z)}. \tag{1}$$

The y-variation of fields is zero while the x-variation has to be obtained by solving the wave equation. In free space $(x > 0)$, the wave equation $\partial^2\vec{E}/\partial x^2 + \alpha_1^2\vec{E} = 0$ in compliance with the Poisson's equation, $\nabla\cdot\vec{E} = 0$ gives $\vec{A}(x) = \vec{A}_1\mathrm{e}^{-\alpha x}$ where $\vec{A}_1 = A_{1z}(\hat{z} + \hat{x}ik_z/\alpha_1)$ and $\alpha_1^2 = k^2 - \omega^2/c^2$.

In bismuth the electrons in the conduction band are equally divided into three groups. Group I electrons have an effective mass $\underline{\underline{m}}^{\mathrm{I}}$, where

$$\underline{\underline{m}}^{\mathrm{I}} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & m_4 \\ 0 & m_4 & m_3 \end{bmatrix}. \tag{2}$$

Group II, III electrons have the same mass tensor, in the frames obtained by rotating the reference frame about the trigonal axis by $\pm 120°$. The mass tensor of second and third group electrons in the first frame are $\underline{\underline{m}}^{\mathrm{II}} = \underline{\underline{O}}'\cdot\underline{\underline{m}}^{\mathrm{I}}\cdot\underline{\underline{O}}$, and $\underline{\underline{m}}^{\mathrm{III}} = \underline{\underline{O}}\cdot\underline{\underline{m}}^{\mathrm{I}}\cdot\underline{\underline{O}}'$, respectively. Here $\underline{\underline{O}}$ is the transformation matrix, with $O_{xx} = O_{yy} = \cos\theta, O_{xy} = O_{yx} = \sin\theta$, $O_{zz} = 1, O_{xz} = O_{zx} = O_{zy} = O_{yz} = 0$. $\underline{\underline{O}}'$ is transpose of $\underline{\underline{O}}$, i.e. $O'_{ij} = O_{ji}$, $\underline{\underline{O}}' = \underline{\underline{O}}^{-1}$ and $\theta = 120°$. The oscillatory velocity of group I electrons due to the laser can be obtained by solving the equation of motion,

$$\vec{v}^{\mathrm{I}} = \frac{e}{i(\omega + i\nu)}\underline{\underline{\beta}}^{\mathrm{I}}\cdot\vec{E}, \tag{3}$$

where $-e$ is the electronic charge, ν the collision frequency and $\underline{\underline{\beta}}^{\mathrm{I}} = (\underline{\underline{m}}^{\mathrm{I}})^{-1}$ the inverse mass tensor, with $\beta^{\mathrm{I}}_{xx} = (m_2m_3 - m_4^2)/M$, $\beta^{\mathrm{I}}_{yy} = m_1m_3/M$, $\beta^{\mathrm{I}}_{yz} = \beta^{\mathrm{I}}_{zy} = -m_1m_4/M$, $\beta^{\mathrm{I}}_{zz} = m_1m_2/M$, other components being zero and $M = m_1m_2m_3 - m_1m_4^2$. Similarly the oscillatory velocities of groups II and III electrons are

$$\vec{v}^{\mathrm{II,III}} = \frac{e}{i(\omega + i\nu)}\underline{\underline{\beta}}^{\mathrm{II,III}}\cdot\vec{E}, \tag{4}$$

where $\underline{\underline{\beta}}^{\mathrm{II}} = (\underline{\underline{m}}^{\mathrm{II}})^{-1}, \underline{\underline{\beta}}^{\mathrm{III}} = (\underline{\underline{m}}^{\mathrm{III}})^{-1}$.

$$\underline{\underline{\beta}}^{\text{II}} = \frac{1}{M}\begin{bmatrix} \frac{1}{4}(3m_1m_3 + m_2m_3 - m_4^2) & \frac{\sqrt{3}}{4}(m_1m_3 - m_2m_3 + m_4^2) & \frac{\sqrt{3}}{2}m_1m_4 \\ \frac{\sqrt{3}}{4}(m_1m_3 - m_2m_3 + m_4^2) & \frac{1}{4}(m_1m_3 + 3m_2m_3 - 3m_4^2) & \frac{1}{2}m_1m_4 \\ \frac{\sqrt{3}}{2}m_1m_4 & \frac{1}{2}m_1m_4 & m_1m_2 \end{bmatrix},$$

and $\underline{\underline{\beta}}^{\text{III}}$ can be deduced from $\underline{\underline{\beta}}^{\text{II}}$ by replacing $\sqrt{3}$ by $-\sqrt{3}$. The net current density in the material due to all the three groups of electrons is

$$\vec{J} = -\frac{n_0}{3}e(\vec{v}^{\text{I}} + \vec{v}^{\text{II}} + \vec{v}^{\text{III}}) = \underline{\underline{\sigma}} \cdot \vec{E}, \tag{5}$$

where $\underline{\underline{\sigma}} = (i\omega_p^2 m)/(12\pi(\omega + i\nu))[\underline{\underline{\beta}}^{\text{I}} + \underline{\underline{\beta}}^{\text{II}} + \underline{\underline{\beta}}^{\text{III}}]$ is the conductivity tensor and $\omega_p^2 = 4\pi n_0 e^2/m_1$. Using eq. (14) in the wave equation governing the propagation of electromagnetic waves, $-\nabla^2\vec{E} + \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) = (-4\pi i\omega\vec{J} - \omega^2\varepsilon_L\vec{E})/c^2$ we get,

$$-\nabla^2\vec{E} + \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) = \frac{\omega^2}{c^2}\underline{\underline{\varepsilon}} \cdot \vec{E}, \tag{6}$$

where $\underline{\underline{\varepsilon}} = \varepsilon_L \underline{\underline{I}} + i\underline{\underline{\sigma}}/\omega$ and $\underline{\underline{I}}$ is the identity matrix. We presume a well-behaved solution of this equation $\vec{E} = \vec{A}_2 e^{\alpha_2 x} e^{-i(\omega t - k_z z)}$. Then we can write $\vec{\nabla} \equiv ik_z\hat{z} + \alpha_2\hat{x}$ in (6) to obtain, $\underline{\underline{D}}\cdot\vec{E} = 0$, where $\underline{\underline{D}} = \lfloor (k_x^2 + k_z^2)\underline{\underline{I}} - \vec{k}\vec{k} - (\omega^2/c^2)\underline{\underline{\varepsilon}} \rfloor$ and $k_x = -i\alpha_2$. For non-trivial solution, we should have, $|\underline{\underline{D}}| = 0$, which gives

$$k_x = \pm\frac{1}{\sqrt{2}}\left[\left(-2k_z^2 + 2\varepsilon_L\frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2} - \frac{\omega_p^2 m_1 m_3}{c^2(m_2m_3 - m_4^2)}\right)^{1/2}\right]. \tag{7}$$

Using the Gauss's law $\vec{\nabla}\cdot\vec{D} = 0$, $(i\vec{k}\cdot\underline{\underline{\varepsilon}}\cdot\vec{E} = 0)$ we get the value of E_x in bismuth:

$$E_x = \frac{k_z}{i\alpha_2}\frac{\varepsilon_{zz}}{\varepsilon_{xx}}E_z. \tag{8}$$

Using eqs (1) and (8) in the Maxwell's equation $\vec{\nabla}\times\vec{E} = -(1/c)(\partial\vec{B}/\partial t)$, we obtain the y-component of the magnetic field in free space and bismuth as,

$$H_y = \frac{c}{i\omega}\left[-k_z\frac{k_z}{\alpha_1} + \alpha_1\right]E_z, \tag{9}$$

and

$$H_y = -\frac{c}{i\omega}\left[-k_z\frac{k_z}{\alpha_2}\frac{\varepsilon_{zz}}{\varepsilon_{xx}} + \alpha_2\right]E_z, \tag{10}$$

respectively. Applying the continuity of displacement vector, $D_{x\text{I}} = D_{x\text{II}}$ at the interface and using (1) and (8) we obtain the dispersion relation, $\alpha_2/\alpha_1 = -\varepsilon_{zz}$, which on simplification gives

$$k_z = \left[\frac{m_p - 2m_q\varepsilon_L^2 - \omega^2\varepsilon_L/\omega_p^2(1-\varepsilon_L) + (m_q)^2\varepsilon_L^2\omega_p^2/\omega^2}{c^2\{-2m_q\varepsilon_L^2/\omega^2 - (1-\varepsilon_L^2)/\omega_p^2 + (m_q)^2\varepsilon_L^2\omega_p^2/\omega^4\}}\right]^{1/2}, \tag{11}$$

where $m_p = m_1m_3/(m_2m_3 - m_4^2)$ and $m_q = m_1m_2/(m_2m_3 - m_4^2)$.

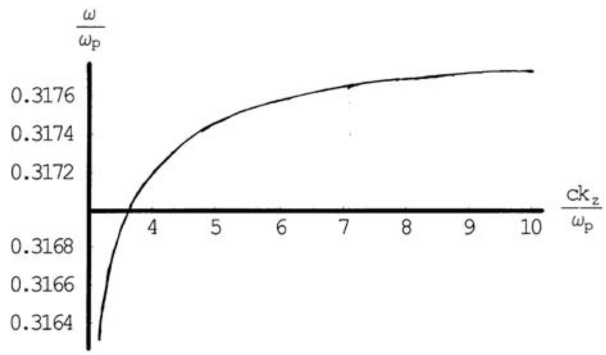

Figure 1. Variation of ω/ω_p with ck_z/ω_p for the following parameters of bismuth: $m_0 = 9.1 \times 10^{-28}$ g, $m_1 = 2.5 \times 10^{-3} m_0$, $m_2 = 2.5 m_0$, $m_3 = 0.05 m_0$, $m_4 = -0.25 m_0$, $v = 5 \times 10^{10}$/s, $c = 3 \times 10^{10}$ cm/s and $\varepsilon_L = 100$.

In figure 1 we have plotted the variation of normalized frequency ω/ω_p with normalized wave vector ck_z/ω_p for the following parameters of bismuth: $m_0 = 9.1 \times 10^{-28}$ g, $m_1 = 2.5 \times 10^{-3} m_0$, $m_2 = 2.5 m_0$, $m_3 = 0.05 m_0$, $m_4 = -0.25 m_0$, $v = 5 \times 10^{10}$/s, $c = 3 \times 10^{10}$ cm/s and $\varepsilon_L = 100$. The SPW frequency saturates at larger k_z.

3. Excitation of surface plasma waves

We consider the propagation of an electron beam with density n_{0b} and velocity $v_b\hat{z}$, slightly above the bismuth surface. The beam has a Gaussian density profile in x and has y-width as b:

$$n_{0b} = N_0 \mathrm{e}^{-(x-x_0)^2}/a^2. \tag{12}$$

The beam current is $I_b = \sqrt{\pi} N_0 abev_b$. The oscillatory velocity and density of the beam due to SPW are governed by the equations of motion and continuity as:

$$\frac{\partial}{\partial t}(\gamma\vec{v}) = -\frac{e}{m}\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) \tag{13}$$

and

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0, \tag{14}$$

respectively. Here $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic gamma factor. We perturb $\vec{v}$, n and γ as $(n, \vec{v}) = (n_{0b}, v_b\hat{z}) + (n_1, \vec{v}_1)\mathrm{e}^{-i(\omega t - k_z z)}\mathrm{e}^{-\alpha z}$, $\gamma = \gamma_0 + \gamma_0^3 v_b v_{1z}/c^2$, and solve the linearized equations [eqs (13) and (14)] to obtain the perturbed current density

$$\begin{aligned} \vec{J}_1 &= -n_{0b}e\vec{v}_1 - n_1 e v_b \hat{z}, \\ J_{1x} &= -\frac{n_{0b}e^2}{mi\gamma\omega}E_x - \frac{n_{0b}e^2\alpha_1 v v_{0b} E_z}{\omega m \gamma_0(\omega - k_z v_b)}, \end{aligned} \tag{15}$$

$$J_{1z} = -\frac{n_{0b}e^2}{mi\gamma_0^3(\omega - k_z v_b)^2}\left(1 - \frac{\alpha^2 v_b^2}{\omega^2}\gamma_0^2\right)E_z - \frac{n_{0b}e^2\alpha_1 v_b E_x}{m\gamma_0\omega(\omega - k_z v_b)}$$
$$-\frac{\partial n_{0b}}{\partial x}\frac{e^2\alpha_1 v_b^2 E_z}{mi\omega\gamma_0(\omega - k_z v_b)^2}. \tag{16}$$

For Cerenkov resonance $\omega \approx k_z v_b$, i.e., $v_b = c((1+\varepsilon)/\varepsilon)^{1/2}$, $\gamma_0 = |\varepsilon|^{1/2}$, $\alpha_1 = \omega/(c|1+\varepsilon|^{1/2})$, we find that the factor $(1-\alpha^2 v_b^2\gamma_0^2/\omega^2)$ in J_{1z} vanishes. We would be interested in retaining only those terms in current density that go as $(\omega - k_z v_b)^{-2}$. Thus

$$\vec{J}_1 \approx -\frac{\partial n_{0b}}{\partial x}\frac{e^2\alpha_1 v_b^2 E_z}{mi\omega\gamma_0(\omega - k_z v_b)^2}\hat{z}. \tag{17}$$

We define $\vec{E}_s$ and $\vec{H}_s$ as the mode structures of the fields of the SPW in the absence of the beam, satisfying Maxwell's equations

$$\vec{\nabla}\times\vec{E}_s = i(\omega/c)\vec{H}_s, \quad \vec{\nabla}\times\vec{H}_s = -i(\omega/c)\underline{\underline{\varepsilon}}'\cdot\vec{E}_s, \tag{18}$$

with appropriate boundary conditions at the interface. Here $\underline{\underline{\varepsilon}}' = \underline{\underline{I}}$ in medium I $(x>0)$ and $\underline{\underline{\varepsilon}}' = \underline{\underline{\varepsilon}}$ in medium II $(x<0)$. In the presence of the beam current, let the fields be, $\vec{E} = A(t)\vec{E}_s$, $\vec{H} = B(t)\vec{H}_s\cdot\vec{E}$ and $\vec{H}$ satisfy the Maxwell's equations

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}, \quad \nabla\times\vec{H} = \frac{4\pi}{c}(\vec{J}_{lb}+\vec{J}_{lP}) + \frac{\varepsilon_L}{c}\frac{\partial\vec{E}}{\partial t}, \tag{19}$$

where $\vec{J}_{lP} = \underline{\underline{\sigma}}\cdot A\vec{E}_s + i(\partial\underline{\underline{\sigma}}/\partial\omega)(\partial A/\partial t)\cdot\vec{E}_s, \underline{\underline{\sigma}}$ is the electrical conductivity of bismuth related to effective permittivity. Outside bismuth, $\varepsilon_L = 1$, $\sigma = 0$ whereas inside bismuth, $\underline{\underline{\varepsilon}} = \varepsilon_L\underline{\underline{I}} + 4\pi i\underline{\underline{\sigma}}/\omega$. From (18) and (19) we obtain

$$\frac{\partial B}{\partial t} = -i\omega(A-B), \tag{20}$$

$$\left[\frac{\partial A}{\partial t}\frac{\partial}{\partial\omega}(\underline{\underline{\varepsilon}}'\omega) - i\omega\underline{\underline{\varepsilon}}'(A-B)\right]\cdot\vec{E}_s = -4\pi\vec{J}_{lb}. \tag{21}$$

Using (20) in (21), assuming $(\partial B/\partial t) \approx (\partial A/\partial t)$, multiplying the resulting equation by $\vec{E}_s^*$ and integrating over x from $-\infty$ to $+\infty$ we obtain

$$\frac{\partial A}{\partial t} + \Gamma A = -\left(2\pi\int_0^\infty J_{IZ}E_{SZ}^*\mathrm{d}x\right)\Big/\left(\int_{-\infty}^\infty \vec{E}_s\cdot\vec{E}_s^*\mathrm{d}x\right) = \frac{P_2}{i(\omega - k_z v_b)^2}A, \tag{22}$$

where

$$P_2 = \frac{\omega_{pb}^2}{\gamma_0}\frac{2\sqrt{\pi}\alpha_1^2 v_b^2\alpha_1 a f}{\omega(1-\varepsilon)(1+1/\varepsilon^2)}$$

and

$$f = \frac{\sqrt{\pi}}{2}(1 - erf(\alpha_1 a - x_0/a))\mathrm{e}^{[\alpha_1^2 a^2 - 2\alpha_1 x_0]}.$$

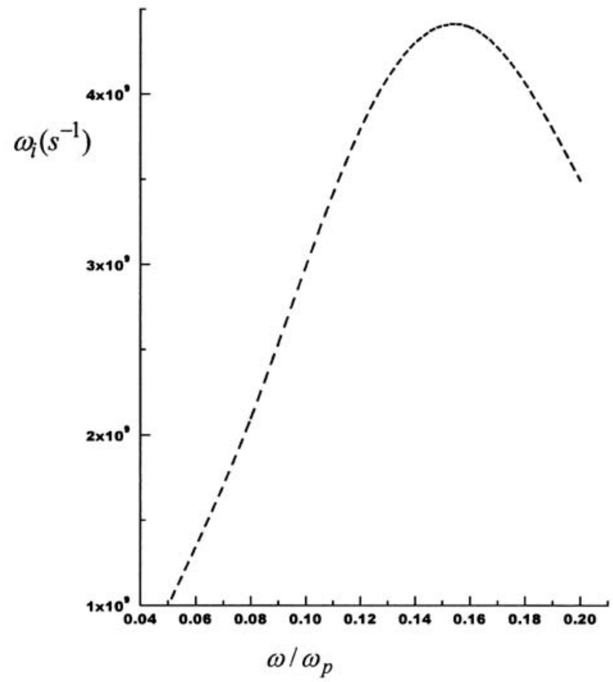

Figure 2. Growth rate of a surface plasma wave as a function of ω/ω_p for the following parameters: $I_b = 100$ mA, $a = 10$ μm, $b = 300$ μm, $x_0 = 300$ μm.

Writing $\omega = k_z v_b + \delta$, $\partial A/\partial t = -i\delta$ we obtain

$$\delta^2(\delta + i\Gamma) = P_2 \mathrm{e}^{i2l\pi}, \quad l = 0, 1, 2, \ldots. \tag{23}$$

If damping coefficient Γ is neglected then (22) gives the growth rate

$$\omega_i = \mathrm{Im}\delta = \left(\sqrt{3}/2\right) P_2^{1/3}. \tag{24}$$

In the other case when $\Gamma > \delta$ the growth rate is

$$\omega_i = \frac{1}{\sqrt{2}}(P_2/\Gamma)^{1/2}. \tag{25}$$

In figure 2 we have plotted the growth rate of SPW as a function of ω/ω_p for $I_b = 100$ mA, $a = 10$ μm, $b = 300$ μm, $x_0 = 300$ μm, $\varepsilon_L = 100$ and $\omega_p = 3 \times 10^{13}$ rad/s. The growth rate peaks at $\omega/\omega_p = 0.15$.

4. Discussion

The dispersion characteristics of surface plasma waves are similar to those of SPWs over the conductors. However they manifest mass anisotropy effects and the frequency cutoff,

ω_{cutoff}, for these waves is two orders of magnitude down due to low carrier density. The thickness of the surface layer, in which they are localized, is larger than in conductors. These waves can be excited by a relativistic electron beam propagating parallel to the interface. The required beam energy for excitation of SPW is $E_b = \gamma_0 mc^2$ where $\gamma_0 \approx |\varepsilon|^{1/2}$, which is quite large unless one is closer to $\omega \approx \omega_p/\sqrt{\varepsilon_2 + 1}$.

These waves can also be excited using submillimeter waves from a free electron laser. This would require a ripple on the surface so as to satisfy the phase matching condition. One may also employ the attenuated total reflection configuration [13] in which bismuth film is deposited on a prism and wave is launched onto the prism–bismuth interface at an angle such that the component of propagation constant of the electromagnetic wave in the glass along the interface equals the propagation constant of the surface. The surface waves thus generated can be employed for annealing the bismuth film or its sputtering. Bismuth may also be employed for fabricating surface wave devices.

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References

[1] P R Willmott and J R Huber, *Rev. Mod. Phys.* **72**, 315 (2000)
[2] R F Xiao, H B Liao, N Cue, X W Sun and H S Kwok, *J. Appl. Phys.* **80**, 4266 (1996)
[3] J M Schneider, S Rohde, W D Sproul and A Matthews, *J. Phys.* **D33**, R173 (2000)
[4] D Grozev, K Kirov, K Makashewa and A Shiraraova, *IEEE Trans. Plasma Sci.* **25**, 415 (1997)
[5] M Ezaki, H Kumagai, K Toyoda and M Obara, *IEEE J. Select. Topics in Quant. Electron.* **1**, 841 (1995)
[6] B Bhat and V K Tripathi, *J. Appl. Phys.* **46**, 2155 (1975)
[7] C S Liu and V K Tripathi, *Interaction of electromagnetic waves with electron beams and plasmas* (World Scientific, Singapore, 1994) p. 32
[8] E C Jordan and K G Balmain, *Electromagnetic waves and radiating systems* (Prentice Hall, India, 1987) p. 635
[9] I Ursu, I N Mihailescu, A M Prokhorov, V I Konov and V N Tokarev, *Physica* **C132**, 395 (1985)
[10] A Zherikhin, V Bagratashvili, V Burimov, E Sobol, G Shubnii and A Sviridov, *Physica* **C198**, 341 (1992)
[11] S A Akhmanov, V I Emelyanov, N I Korotev and V N Seminogov, *Sov. Phys. Usp.* **28**, 1084 (1985)
[12] C S Liu and V K Tripathi, *IEEE J. Quant. Electron.* **34**, 1503 (1998)
[13] I I Smolyaninov, D L Mazzoni and C C Davis, *Phys. Rev. Lett.* **77**, 3877 (1996)
[14] S I Bozhevolnyi and F A Pudonin, *Phys. Rev. Lett.* **78**, 2823 (1997)
[15] S C Kaushik and V K Tripathi, *J. Phys. Chem. Solids* **35**, 521 (1994)
[16] S K Sharma and V K Tripathi, *J. Appl. Phys.* **43**, 2155 (1972)
[17] K Jayachandran and C S Menon, *Pramana – J. Phys.* **50**, 221 (1998); *Indian J. Pure Appl. Phys.* **35**, 417 (1997)
[18] C M Bhandari and V K Agrawal, *Indian J. Pure Appl. Phys.* **28**, 448 (1990)