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Three-dimensional wake potential in a streaming dusty plasma

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Abstract. The oscillatory wake potential for a slowly moving or static test dust particulate in a finite temperature, collisionless and unmagnetized dusty plasma with a continuous flow of ions and dust particles has been studied. The collective resonant interaction of the moving test particle with the low-frequency and low-phase-velocity dust-acoustic mode is the origin of the periodic attractive force between the like polarity particulates along and perpendicular to the streaming ions and dust grains resulting into dust-Coulomb crystal formation. This wake potential can explain the three-dimensional dust-Coulomb crystal formation in the laboratory conditions.

Keywords. Dusty plasma; dust-acoustic mode; wake potential; plasma crystal; electrostatic wave and oscillations.

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After Ikezi's prediction [1], the dust-Coulomb crystal formation has been demonstrated in a number of laboratory experiments [2–10]. Recently, Takahashi *et al* [11] demonstrated experimentally the formation of dust-Coulomb crystals in terms of wake potential in a dusty plasma with a finite ion flow. They showed that a dust particulate located in the up-stream of ion flows caused an attraction on another dust particulate in the lower part explaining vertical periodic structure along the ion flow direction.

Vladimirov and Nambu [12] first showed that the collective interaction of the static test dust particulate with the low-frequency (l-f) oscillations in the ion flow in a dusty plasma can provide the attractive oscillatory wake potential along the ion flow direction. Vladimirov and Oshihara [13] and Oshihara and Vladimirov [14] extended this theory to consider periodic structures along and perpendicular to the ion flow direction.

Nambu *et al* [15] and Shukla and Rao [16] explained the wake potential formation in terms of the resonant interaction of the drifting grains and the extremely l-f dust-acoustic waves in an unmagnetized dusty plasma. They argued that the resonant interaction of the slowly moving or static grains with the extremely l-f dust-acoustic wave involving the dust grains would be more effective in forming the wake potential. However, these theories are valid for predicting one-dimensional crystal structure along the motion of the dust particulates. Here, we study the formation of the oscillatory wake potential along and perpendicular to the drifting direction of the dust particulates in an unmagnetized, finite temperature, and collisionless dusty plasma with continuous flow of ions and slowly moving or static dust grains. The collective resonant interaction of the moving test dust particle with the dust-acoustic mode is the origin of the periodic attractive force between the like polarity particulates along and perpendicular to the streaming ions and dust grains resulting in the dust-Coulomb crystal formation. Thus, this wake potential can explain the three-dimensional dust-Coulomb crystal formation in the laboratory conditions.

We consider a finite temperature, collisionless, unmagnetized, and homogeneous dusty plasma. In the equilibrium, we have $n_{i0} = n_{e0} + Z_d n_{d0}$ where $n_{\alpha 0}$ with $\alpha = e, i, d$ are the equilibrium densities of electrons, ions, dust grains, and Z_d is the number of charges residing on a dust grain surface. We assume constant charge on the dust particles and thereby neglect any damping of the wave mode that may arise due to the grain charge fluctuations. Thermal electrons are assumed stationary, and ions and dust grains are assumed to be streaming with uniform velocities $\underline{u}_0$ and $\underline{v}_0$, respectively in the presence of a self-consistent constant electric field. These uniform drifts of ions and dust particles may be due to an ambient electric field which may be present in the plasma because of a variety of reasons. In the laboratory, the constant electric field may be generated in the sheath region on account of the high mobility of electrons. In the space situation, this constant electric field may be established due to inhomogeneous distribution of charge carriers. Following Shukla and Rao [16] with appropriate conditions, $\omega \ll kv_{te}$, $|\omega - \underline{k}\cdot\underline{u}_0| \gg kv_{ti}$, $\omega_{pi} \leq \underline{k}\cdot\underline{u}_0$, and $|\omega - \underline{k}\cdot\underline{v}_0| \gg kv_{td}$, we write the dielectric response function of the plasma in the presence of a l-f and low-phase-velocity dust-acoustic wave in the dusty plasma as

$$\begin{aligned}\varepsilon(\omega,\underline{k}) = 1 + \frac{1}{k^2\lambda_{\mathrm{D}}^2} - \frac{\omega_{pd}^2}{(\omega - \underline{k}\cdot\underline{v}_0)^2} \\ + i\left[\sqrt{\frac{\pi}{2}}\frac{1}{k^2\lambda_{\mathrm{D}e}^2}\frac{\omega}{kv_{te}}\exp\left\{-\left(\frac{\omega}{\sqrt{2}kv_{te}}\right)^2\right\}\right. \\ \left. + \sqrt{\frac{\pi}{2}}\frac{1}{k^2\lambda_{\mathrm{D}i}^2}\frac{\omega - \underline{k}\cdot\underline{u}_0}{kv_{ti}}\exp\left\{-\left(\frac{\omega - \underline{k}\cdot\underline{u}_0}{\sqrt{2}kv_{ti}}\right)^2\right\}\right],\end{aligned} \tag{1}$$

where $\lambda_{\mathrm{D}j} = (T_j/4\pi e^2 n_{j0})^{1/2}$, $v_{tj} = (T_j/m_j)^{1/2}$, $j = e, i$, and $\lambda_{\mathrm{D}} \equiv \lambda_{\mathrm{D}e}$. Here, T_j is the temperature and m_j is the mass of the species j with charge e. It may be mentioned here that for the usual laboratory dust crystal experiments using a rf-discharge device, the ion drift velocity $\underline{u}_0$ is comparable to the ion-acoustic velocity which is much larger than the ion-thermal velocity ($u_0 \sim \sqrt{T_e/m_i} \gg \sqrt{T_i/m_i}$), so that, $|\omega - \underline{k}\cdot\underline{u}_0| \gg kv_{ti}$ in the presence of the low-phase-velocity dust-acoustic wave. This was wrongly assumed in ref. [16]. However, this does not change their main conclusion except for the Landau damping of the dust-acoustic wave.

The electrostatic potential around a test dust particulate in the presence of the electrostatic mode $(\omega, \underline{k})$ in a uniform dusty plasma whose response function is given by $\varepsilon(\omega,\underline{k})$ (eq. (1)), is given by [17,18]

$$\Phi(\underline{x},t) = \int \frac{q_t}{2\pi^2 k^2}\frac{\delta(\omega - \underline{k}\cdot\underline{v}_t)}{\varepsilon(\omega,\underline{k})}\exp(i\underline{k}\cdot\underline{r})\mathrm{d}\underline{k}\mathrm{d}\omega, \tag{2}$$

where $\underline{r} = \underline{x} - \underline{v}_t t$, $\underline{v}_t$ is the velocity vector of the test dust particulate and q_t is its charge.

Following ref. [16], the Coulomb potential caused by the dust-acoustic oscillations is given by

$$\Phi_w(\rho,\xi) = \left|\left(\frac{q_t\lambda_{\mathrm{D}}^4\omega_{pd}^2}{\pi}\right)\int \frac{J_0(k_\perp\rho)\delta(\omega-k_\parallel v_t)}{(1+k^2\lambda_{\mathrm{D}}^2)^2}\right.$$
$$\times \frac{k^2 \exp ik_\parallel\xi}{[(\omega-k_\parallel v_0)^2-\omega_{sd}^2]} k_\perp \mathrm{d}k_\perp \mathrm{d}k_\parallel \,\mathrm{d}\omega, \tag{3}$$

where $\underline{v}_t \parallel \underline{v}_0 \parallel \hat{z}$, $\xi = z - v_t t$, and (ρ,θ,z) are the cylindrical space coordinates of a point of observation in the dusty plasma.

On carrying out ω- and $k_\parallel$-integrations, we obtain

$$\Phi_w(\rho,\xi) = \left[\frac{-2q_t\lambda_{\mathrm{D}}^2 c_{sd}(v_t-v_0)^2}{\{(v_t-v_0)^2-c_{sd}^2\}^{3/2}}\right]$$
$$\times \int J_0(k_\perp\rho)\sin\left(\frac{k_\perp c_{sd}\xi}{\sqrt{(v_t-v_0)^2-c_{sd}^2}}\right) k_\perp^2 \,\mathrm{d}k_\perp. \tag{4}$$

For $\rho = 0$, in eq. (4), we obtain $\Phi_w(\rho=0,\xi) = (2q_t/\xi)\cos(\xi/L_{sd})$, where $L_{sd} = [(v_t - v_0)^2 - c_{sd}^2]^{1/2}\lambda_{\mathrm{D}}/c_{sd}$, $c_{sd} = \omega_{pd}\lambda_{\mathrm{D}}$. This Φ_w is the same for the wake potential given by eq. (12) [16].

For $\rho \gg \lambda_{\mathrm{D}}$ and $\lambda_{\mathrm{D}}^{-1} \gg k_\perp$, we can take $J_0(k_\perp\rho) \simeq (2/\pi k_\perp\rho)^{1/2}\cos(k_\perp\rho - \pi/4)$, and taking the upper limit of integration as $1/\lambda_{\mathrm{D}}$, the wake potential turns out to be

$$\Phi_w(\rho,\xi) \simeq \left[\frac{2q_t c_{sd}(v_t-v_0)^2}{\{(v_t-v_0)^2-c_{sd}^2\}^{3/2}}\right]$$
$$\times \sqrt{\frac{\lambda_{\mathrm{D}}}{2\pi\rho}}\left[\frac{\cos(x_-/\lambda_{\mathrm{D}}-\pi/4)}{x_-} + \frac{\cos(x_+/\lambda_{\mathrm{D}}-\pi/4)}{x_+}\right], \tag{5}$$

where

$$x_\pm = \rho \pm \frac{c_{sd}\xi}{\sqrt{(v_t-v_0)^2-c_{sd}^2}}. \tag{6}$$

The oscillatory wake potential $\Phi_w(\rho,\xi)$ is a function of $\rho,\xi,v_t,v_0,c_{sd},\lambda_{\mathrm{D}}$ and the dynamics of electrons, ions, and the unmagnetized dust grains. Thus, we obtain the periodic spacing of the wake potential along and perpendicular to the ion flow direction.

For a given ρ, the effective attraction length along the dust flow direction is $L_{s\parallel} = \lambda_{\mathrm{D}}\sqrt{(v_t-v_0)^2-c_{sd}^2}/c_{sd}$. This result was earlier obtained in ref. [16], which predicted one-dimensional periodic alignment of the dust particulates along the ion-flow direction. For a given ξ, the wake potential is periodic in the plane perpendicular to the dust-flow-direction, which would give rise to the formation of quasi-lattice with spacing $L_{s\perp} = \lambda_{\mathrm{D}}$.

Thus, we note that the results of the present investigation can be useful in understanding the force of attraction between two charged particulates in a dusty plasma situation that is common in low temperature laboratory plasmas as well as in space and astrophysical situations [19].

It may be further added here that in a laboratory situation of the dust-crystal experiments, the slowly moving grains can even be assumed static $(v_0 = 0)$. The test particle velocity v_t of any arbitrary dust particulate in the laboratory condition may be due to the thermal motion of the dust grains, or, the relative motion of the grains in the ion-frame with respect to the streaming ions, etc. For the excitation of the wake potential, the test particle velocity v_t must exceed the phase velocity (c_{sd}) of the dust-acoustic wave (cf. eq. (6)). This may cause the three-dimensional dust-Coulomb crystal formation in laboratory conditions.

In summary, the dielectric response function for a finite temperature, collisionless, and unmagnetized dusty plasma has been written down. Electrons are assumed Boltzmann with uniform streams of ions and dust grains. We calculate the potential due to this dielectric function and obtain three parts of the total potential. Besides, the Debye–Hückel potential and the rapidly decreasing far field potential due to the complex part of the dielectric function, we obtain an oscillatory wake potential which can cause periodic attraction of charged dust grains of the same polarity along and perpendicular to the streaming dust grains. This oscillatory wake potential can explain the formation of the dust-Coulomb crystals with periodic spacing along and perpendicular to the ion-flow direction. Thus, this analytical work can explain the formation of the three-dimensional dust-Coulomb crystals in the laboratory conditions [11]. Furthermore, the Wakefield studies should help to understand the origin of attractive forces which can cause coagulation of charged dust grains in the Earth's lower ionosphere (in noctilucent clouds) as well as in molecular clouds [19].

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