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Dynamical properties of a two-level system with arbitrary nonlinearities

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Abstract. We investigate some aspects of a generalized JC-model which include arbitrary forms of non-linearities of both the field and the intensity-dependent atom–field coupling. We obtain an exactly analytic solution of the model, by means of which we identify and numerically demonstrate the region of parameters where significantly large entanglement can be obtained.

Keywords. Two-level system; entanglement.

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1. Overview

The fundamental difference between quantum and classical physics is the possible existence of non-classical correlations between distinct quantum systems. The physical property responsible for the non-classical correlations is called entanglement. From early nineties, the field of quantum information theory opened up and expanded rapidly. Quantum entanglement began to be seen not only as a puzzle, but also as a resource for communication, information processing and quantum computing, such as in the investigation of quantum teleportation, dense coding, decoherence in quantum computers and the evaluation of quantum cryptographic schemes [1]. Entanglement was found to be a manipulable resource. Under certain conditions, states of low entanglement could be purified into more entangled states by acting locally, and states of higher entanglement could be 'diluted' to give larger numbers of less entangled states. A number of entanglement measures have been discussed in the literature, such as the von Neumann reduced entropy, the relative entropy of entanglement [2], the so-called entanglement of distillation and the entanglement of formation [3]. Several authors proposed physically motivated postulates to characterize entanglement measures [2,4,5]. What these postulates (although they vary from author to author in the details) have in common are that they are based on the concepts of operational formulation of quantum mechanics [6]. A method using quantum mutual entropy to measure the degree of entanglement in the time development of the Jaynes–Cummings model has been adopted in [7], which we called degree of entanglement due to mutual entropy

(DEM). We have formulated the entanglement in the time development of the JC-model with squeezed state [8], and then we have shown that the entanglement can be controlled by means of squeezing.

In this communication, we aim at extending the previously cited treatments to study the problem of a two-level atom interacting with a single-mode including acceptable kinds of non-linearities of both the field and the intensity-dependent atom–field coupling. We employ the analytical results obtained to investigate the properties of the entanglement due to the concurrence hierarchy.

2. The entanglement

In the theory of open system or the reduction theory, one often considers two subsystems $\mathscr{H}_1$ and $\mathscr{H}_2$ represented by Hilbert space. Let $S(\mathscr{H}_i)$, $(i = 1,2)$ be the state spaces (the set of all density operators). Also $S(\mathscr{H}_1 \otimes \mathscr{H}_2)$ denotes the state space in the composite system $\mathscr{H}_1 \otimes \mathscr{H}_2$. The following decomposed states in composite system are called disentangled states:

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle, \quad |\psi_1\rangle \in S(\mathscr{H}_1), \quad |\psi_2\rangle \in S(\mathscr{H}_2). \tag{1}$$

However, in general, it is almost impossible to decompose the states in composite system like the above, that is, the following states exist:

$$\begin{aligned} |\psi\rangle = \alpha|\psi_1\rangle \otimes |\psi_2\rangle + \beta|\phi_1\rangle \otimes |\phi_2\rangle,& \\ |\psi_1\rangle, |\phi_1\rangle \in S(\mathscr{H}_1), \quad |\psi_2\rangle, |\phi_2\rangle \in S(\mathscr{H}_2)& \\ |\alpha|^2 + |\beta|^2 = 1, \quad \alpha \neq 0, \quad \beta \neq 0.& \end{aligned} \tag{2}$$

The above states which cannot be described by product states of two subsystems, are called entangled states. In the case of pure quantum states for two subsystems, a number of physically intuitive measures of entanglement have been known for some time. However, for general mixed states of an arbitrary number of subsystems, entanglement measures are still under development.

Consider $\mathscr{H}_1$ and $\mathscr{H}_2$ that interact with each other. How are the entropies of these systems related to the entropy of the composite system that comprises them both? The answer to this question was listed by the Araki–Lieb theorem [9]. Let $S(\mathscr{E}_t^* \rho_{\mathscr{H}_1})$ and $S(\mathscr{E}_t^* \rho_{\mathscr{H}_2})$ denote the entropies of the two interacting systems and $S(\mathscr{E}_t^* \rho)$ denotes the entropy of the composite system. The continuous map $\mathscr{E}_t^*$ describing the time evolution between the ion and the field for this process is defined by the unitary operator generated by the total Hamiltonian H such that

$$\begin{aligned} \mathscr{E}_t^* \; : \; \mathscr{S}(\mathscr{H}_{\mathrm{A}}) \longrightarrow \mathscr{S}(\mathscr{H}_{\mathrm{A}} \otimes \mathscr{H}_{\mathrm{F}}), \\ \mathscr{E}_t^* \rho = U_t\, (\rho \otimes \varpi)\, U_t^*, \quad U_t \equiv \mathrm{e}^{-itH/\hbar}. \end{aligned}$$

Araki and Lieb showed that these entropies satisfy the 'triangle inequalities'

$$|S(\mathscr{E}_t^* \rho_{\mathscr{H}_1}) - S(\mathscr{E}_t^* \rho_{\mathscr{H}_2})| \leq S(\mathscr{E}_t^* \rho) \leq S(\mathscr{E}_t^* \rho_{\mathscr{H}_1}) + S(\mathscr{E}_t^* \rho_{\mathscr{H}_2}). \tag{3}$$

Quantum entropies are generally difficult to compute because they involve the diagonalization of large (and, in many cases, infinite dimensional) density matrices. Thus explicit illustrations of the inequalities (3) are difficult to come by. Phoenix and Knight [4] gave a nice illustration of these inequalities in the context of Jaynes–Cummings model. They considered a two-level atom interacting with an undamped cavity initially in a coherent state. In this case the composite entropy is initially zero and remains zero at all times because the atom-field system is isolated from its environment. Under those circumstances the latter inequality $S(\mathscr{E}_t^*\rho) \leq S(\mathscr{E}_t^*\rho_{\mathscr{H}_1}) + S(\mathscr{E}_t^*\rho_{\mathscr{H}_2})$ is trivially satisfied whereas the former implies that $S(\mathscr{E}_t^*\rho_{\mathscr{H}_1}) = S(\mathscr{E}_t^*\rho_{\mathscr{H}_2})$. It is easy to calculate the atomic entropy $S(\mathscr{E}_t^*\rho_{\mathscr{H}_1})$ but the calculation of the quantum field entropy is more problematic. However, Phoenix and Knight succeeded in evaluating the field entropy in closed form and showed that it did indeed equal the atomic entropy at all times. The entropies of the atom and the field, when treated as a separate system, are defined through the corresponding reduced density operators by

$$S(\mathscr{E}_t^*\rho_{\mathscr{H}_1(\mathscr{H}_2)}) = -Tr_{\mathscr{H}_1(\mathscr{H}_2)}\{\mathscr{E}_t^*\hat{\rho}_{\mathscr{H}_1(\mathscr{H}_2)} \log \mathscr{E}_t^*\hat{\rho}_{\mathscr{H}_1(\mathscr{H}_2)}\}, \tag{4}$$

and thus we have the entanglement degree

$$I_\sigma\left(\mathscr{E}_t^*\rho_{\mathscr{H}_1}, \mathscr{E}_t^*\rho_{\mathscr{H}_2}\right) = 2S\left(\mathscr{E}_t^*\rho_{\mathscr{H}_1}\right). \tag{5}$$

Pure state entanglement of bipartite systems is well-understood in the sense that the relevant parameters for its optimal manipulation by local operations and classical communication have been identified and analysed [5]. From the viewpoint of the Phoenix–Knight [4] entropy formalism we have investigated the properties of the evolution of the entropy and entanglement of the three-level atom in single mode [2] and multimode [10].

For the entangled states $\mathscr{E}_t^*\varrho \in S(\mathscr{H}_1 \otimes \mathscr{H}_2)$, the entanglement degree is defined by the following formula as a distance (difference) from a disentangled state $tr_{\mathscr{H}_1}\mathscr{E}_t^*\varrho \otimes tr_{\mathscr{H}_2}\mathscr{E}_t^*\varrho \in S(\mathscr{H}_1 \otimes \mathscr{H}_2)$:

$$I_{\mathscr{E}_t^*\rho}\left(\mathscr{E}_t^*\rho_t^A, \mathscr{E}_t^*\rho_t^F\right) = tr\left(\mathscr{E}_t^*\rho\left(\log \mathscr{E}_t^*\rho - \log \mathscr{E}_t^*\rho_t^A \otimes \mathscr{E}_t^*\rho_t^F\right)\right). \tag{6}$$

Thus we can obtain the degree of entanglement due to mutual entropy by means of measuring the difference with the disentangled state $\mathscr{E}_t^*\rho_t^A \otimes \mathscr{E}_t^*\rho_t^F$, ($\mathscr{E}_t^*\rho_t^A \equiv tr_F\mathscr{E}_t^*\rho$, $\mathscr{E}_t^*\rho_t^F \equiv tr_A\mathscr{E}_t^*\rho$). On the other hand, the entanglement of formation of the mixed state $\mathscr{E}_t^*\rho$ is defined as the average entanglement of the pure states of the decomposition, minimized over all decompositions of $\mathscr{E}_t^*\rho$:

$$E_f(\mathscr{E}_t^*\rho) = \min\left(\sum_j p_j E_f(\varphi_j)\right), \tag{7}$$

The basic equation (eq. (7)) is justified by the physical interconvertibility of a collection of pairs in an arbitrary pure state $|\psi\rangle$ and a collection of pairs in the standard singlet state, the asymptotic conversion ratio being given by $E_f(\mathscr{E}_t^*\rho)$. In d-dimension, the explicit expression of entanglement of formation is only found for several special types of mixed state. However, the explicit formulas have been found for the two-level quantum system [11]. The entanglement of formation of an arbitrary state is related to a quantity called concurrence $C(\mathscr{E}_t^*\widehat{\rho})$ by a function

$$E_f(\mathscr{E}_t^*\widehat{\rho}) = h\left(0.5 + 0.5\sqrt{1 - C^2(\mathscr{E}_t^*\widehat{\rho})}\right), \tag{8}$$

where $h(x) = -x\log x - (1-x)\log(1-x)$, is the binary entropy function. The entanglement of formation is monotonically increasing with respect to the increasing concurrence. The concurrence is defined by an almost magic formula, $C(\mathscr{E}_t^{*}\widehat{\rho}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where λ_i are the square roots of the eigenvalues of $\mathscr{E}_t^{*}\widehat{\rho}\,\mathscr{E}_t^{*}\widetilde{\widehat{\rho}}$ in descending order, $\mathscr{E}_t^{*}\widetilde{\widehat{\rho}} = (\sigma_y \otimes \sigma_y)\,\mathscr{E}_t^{*}\rho^{*}\,(\sigma_y \otimes \sigma_y)$, where $\widehat{\sigma}_y$ is the Pauli matrix. Because concurrence provides a measure of entanglement in two-level system, it is worth generalizing concurrence to higher dimension. A generalized concurrence by considering arbitrary conjugations acting on arbitrary Hilbert spaces has been presented in ref. [12]. The spin flip operator $\widehat{\sigma}_y$ has been generalized [13] to a universal invertor $\widehat{S}_d$ defined as $\widehat{S}_d(\mathscr{E}_t^{*}\widehat{\rho}) = 1 - \mathscr{E}_t^{*}\widehat{\rho}$, the pure state concurrence in any dimension takes the form

$$C_1(\phi) = \sqrt{2\left[1 - Tr(\mathscr{E}_t^{*}\widehat{\rho}_A^{2})\right]}. \tag{9}$$

To quantify entanglement by concurrence hierarchy is a new idea.

As already noticed and conjectured by many researchers, one quantity perhaps is not enough to measure all aspects of entanglement (see for a review [14]). As the question of separability, Peres-Horodecki's [15] criterion is enough for bipartite two-level quantum system. For higher dimensions, if we want to find whether a bipartite state is entangled, besides partial transposition operation proposed by Peres [15], we need to find other positive but not completely positive maps. Presently, how to find whether a bipartite state in $C^d \otimes C^d$ is entangled is still an open problem.

3. The model

In what follows we will introduce and analyse a general model for the interaction between a two-level atom with a quantized electromagnetic field including all acceptable forms of non-linearities of both the field and the intensity-dependent atom–field coupling. In the rotating wave approximation, the total Hamiltonian can be written as [16]

$$\begin{aligned}\hat{H} &= E_e|e\rangle\langle e| + E_g|g\rangle\langle g| + \hbar\Omega\hat{a}^{\dagger}\hat{a} + \Re(\hat{a}^{\dagger}\hat{a}) \\ &\quad + \hbar\lambda\{f(\hat{a}^{\dagger}\hat{a})\otimes\hat{a}\otimes\sigma^{+} + \hat{a}^{\dagger}\otimes\sigma^{-}\otimes f(\hat{a}^{\dagger}\hat{a})\},\end{aligned} \tag{10}$$

where E_m is the energy for level $|m\rangle$. The frequencies of transition between the levels are defined by the equality $\omega_0 = (E_e - E_g)/\hbar$, and $\hat{a}$ and $\hat{a}^{\dagger}$, are the annihilation and the creation operators, respectively, for the mode of the cavity field satisfying $[\hat{a}, \hat{a}^{\dagger}] = 1$, and Ω is the field frequency. We denote by $\Re(\hat{a}^{\dagger}\hat{a})$ the one-mode field non-linearity and $\hbar\lambda f(\hat{a}^{\dagger}\hat{a})$ represents an arbitrary intensity-dependent atom–field coupling. $\hat{\sigma}^{\pm}$ are the pseudo-spin matrices. The second term of $\hat{H}_{\mathrm{in}}$ describes the dynamic Stark shifts of the ground and excited levels of the atom which depend on the one-photon coupling constant, on the mismatch $|\Delta|$ (the detuning parameter $\Delta = \omega_0 - \Omega$), and on the intensity and statistics of the cavity field. We assume that the atom is initially either in the excited state $|e\rangle$ or in the ground state $|g\rangle$. This can be described from the state $|\varpi, \phi\rangle$ given by

$$|\varpi, \nu\rangle = \cos\left(\frac{\varpi}{2}\right)|e\rangle + \sin\left(\frac{\varpi}{2}\right)\exp(-i\nu)|g\rangle, \tag{11}$$

where ν is the relative phase of the two atomic levels. For the excited state $|e\rangle$ we have to take $\varpi \to 0$ while for $\varpi \to \pi$ the wave function describing the atom in the ground state $|g\rangle$ is obtained. Also, we suppose that the initial state of the field is the coherent state:

$$\hat{\rho}_f(0) = |\theta\rangle\langle\theta|, \quad |\theta\rangle = \sum_{n=0}^{\infty} b_n |n\rangle, \tag{12}$$

for a coherent state $b_n = \exp(\bar{n}/2)\sqrt{\bar{n}^n/n!}$, ($\bar{n} = |\varpi|^2$). In physical terms, their squares b_n^2 correspond to the photon number distribution. If we assume that at $t = 0$ the field-atom system is in a pure state, then the initial density operator of a system can be given by $\hat{\rho}(0) = \hat{\rho}_f(0) \otimes \hat{\rho}_a(0)$, i.e.

$$\hat{\rho}(0) = |\theta\rangle\langle\theta| \otimes |\varpi,\nu\rangle\langle\varpi,\nu|, \tag{13}$$

where $\hat{\rho}_f(0)$ and $\hat{\rho}_a(0)$ describe the initial values for the bimodal field and the atomic density operators. The time evolution of the statistical operator $\hat{\rho}(t)$ at any time $t > 0$ is given by

$$\frac{\mathrm{d}\hat{\rho}(t)}{\mathrm{d}t} = -\frac{i}{\hbar}[\hat{H}(t), \hat{\rho}(t)]. \tag{14}$$

For the initial condition (eq. (13)) and according to eq. (14), the time-dependent analytical solution for the density matrix $\hat{\rho}(t)$ is given by

$$\begin{aligned}
\rho(t) = \cos^2\left(\frac{\varpi}{2}\right) & \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} [A_n A_m^* |n;e\rangle\langle e,m| + B_n B_m^* |n+1;g\rangle\langle g,m+1| \\
& + A_n B_m^* |n;e\rangle\langle g,m-1| + B_n A_m^* |n+1;g\rangle\langle e,m|] + \sin^2\left(\frac{\varpi}{2}\right) \\
& \times \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} [A_{n-1} A_{m-1}^* |n;g\rangle\langle g,m| + B_{n-1} B_{m-1}^* |n-1;e\rangle\langle e,m-1| \\
& + A_{n-2} B_{m-2}^* |n;g\rangle\langle e,m-1| + B_{n-1} A_{m-1}^* |n-1;e\rangle\langle g,m|] \\
& + \cos\left(\frac{\varpi}{2}\right)\sin\left(\frac{\varpi}{2}\right) \mathrm{e}^{i\nu} \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} [A_n A_{m-1}^* |n;e\rangle\langle g,m| + B_n B_{m-1}^* |n;g\rangle \\
& \times \langle e,m-1| + A_n B_{m-1}^* |n;e\rangle\langle e,m-1| + B_n A_{m-1}^* |n+1;g\rangle\langle g,m|] \\
& + \cos\left(\frac{\varpi}{2}\right)\sin\left(\frac{\varpi}{2}\right) \mathrm{e}^{-i\nu} \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} [A_{n-1} A_m^* |n;g\rangle\langle e,m| + B_m^* B_{n-1} |n-1;e\rangle \\
& \times \langle g,m-1| + A_{n-1} B_m^* |n;e\rangle\langle g,m+1| + B_{n-1} A_m^* |n-1;e\rangle\langle e,m|],
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
A_n &= b_n \mathrm{e}^{-i\gamma_n\lambda t}\left(\cos\mu_n\lambda t - iW_n \frac{\sin\mu_n\lambda t}{\mu_n}\right), \\
B_n &= -ib_n \mathrm{e}^{-i\gamma_n\lambda t} f(n+1)\sqrt{(n+1)}\frac{\sin\mu_n\lambda t}{\mu_n}, \\
\gamma_n &= \frac{1}{2\lambda}(\Re(n) + \Re(n+1)),
\end{aligned} \tag{16}$$

$$W_n = \frac{\hbar\Delta}{2\lambda} + \frac{1}{2\lambda}(\Re(n) - \Re(n+1)),$$
$$\mu_n = \sqrt{W_n^2 + (n+1)f^2(n+1)}.$$

Having obtained the explicit form of the density matrix operator $\rho(t)$ for the system under consideration, we are therefore in a position to discuss any phenomenon related to the system.

It is interesting to note that when $\Re(\hat{a}^\dagger\hat{a}) = \hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}, \quad f(\hat{a}^\dagger\hat{a}) = 1$ we get the results of [17,18]. When we put $\Re(\hat{a}^\dagger\hat{a}) = 0$, $f(\hat{a}^\dagger\hat{a}) = \hat{a}\sqrt{N}$ and $N = \hat{a}^\dagger\hat{a}$ we get the results of [16,17], where the case of a single-photon is considered. Another point worth noting in this context is the possibility of manipulating different forms of both non-linearities involved in the present model.

4. Results

Supplemental to the analytical solution presented in the above section, we shall devote the following discussion to analyse the numerical results of the entanglement. Here we would like to point out that in order to ensure an excellent accuracy, the behavior of the concurrence as a measure of entanglement (CME) has been determined with great precision, where a resolution of 10^3 points per unit of scaled time has been employed for regions exhibiting strong fluctuation. In our consideration to the behavior of the CME, we take a dimensionless scaled time λt. In the mean time we have fixed the parameters $\bar{n} = 20$, $(\varpi = \pi, 0, \nu = 0), \Delta = 0$, in order to analyse the effects resulting from variation in the non-linear medium and the intensity of non-linear coupling. It is well-known that the evolution of two-level atoms is governed by the collapse and revival for both slow and fast oscillations, where the atom and the field never stop exchanging energy, so that the convergence to a final state does not take place.

Therefore we shall turn our attention to discuss the effect of the non-linearity of the single-mode field with a Kerr-type medium on CME. To see the influence of the non-linear medium in CME let us take different values of χ/λ keeping all other parameters the same as in figure 1a. The results are presented in figures 1b and 1c. As we have the value of $\chi/\lambda = 0.5$, we find that the value of the maximum field entropy begins to decrease (see figure 1c). Further, we noticed that the degree of entanglement between the field and the atom is also reduced. In the meantime we realize that the amplitude of CME decreases as χ/λ increases further. Here we may mention that if the coupling parameter of a Kerr-like medium is stronger than the atom–field coupling one can see that the system starts to dominate the dynamics (there is nearly decoupling of the atom and field) and there is a repetition for some kind of regularity in the evolution of the system. This is apparent from the regular spikes present in figure 1c. This result is in agreement with the fact that in the limit for strong non-linear interaction of the Kerr-like medium with the field mode, the field and the atom are almost decoupled. Also we can see that the amplitude of the oscillation becomes smaller.

It should be expected that as a result of the long-time evolutions of CME, there are certain amounts of Kerr-like medium which would make the Rabi oscillations of the atom to exhibit a superstructure. This of course means that with increasing coupling constants χ there exists an enhancement of the energy exchange between the atoms and the optical

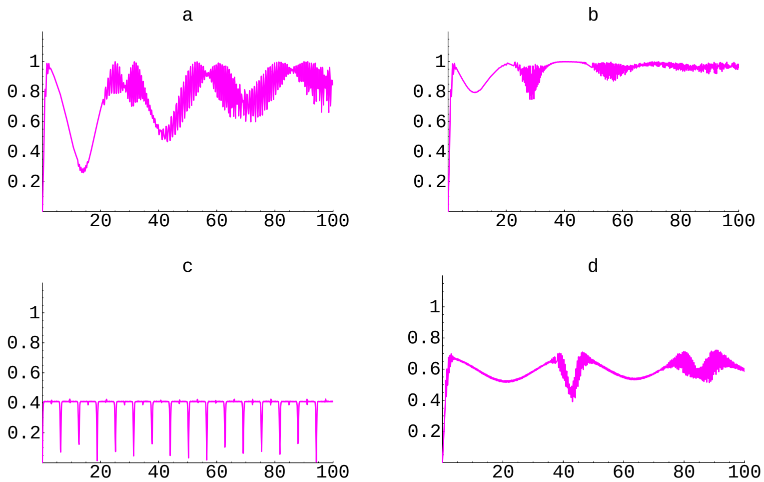

Figure 1. $C_1(\phi)$ as a function of the scaled time λt. Calculations assume that $(\varpi = \pi, \nu = 0)$ and the field in the coherent state with the initial average photon numbers $\bar{n} = 20$. **(a)** $\chi/\lambda = 0.0$, $f(n) = 1$, **(b)** $\chi/\lambda = 0.1$, $f(n) = 1$, **(c)** $\chi/\lambda = 0.5$, $f(n) = 1$, **(d)** $\chi/\lambda = 0.0, \Delta/\lambda = 5$, and $f(n) = 1$.

fields. Finally we see that the amplitude of the oscillation as well as the revival time become smaller but with more revivals in the same period of time besides decreasing in the collapse time. This gives emphasis to the fact that the non-linear medium has a remarkable effect on CME and the system of a two-level atom with non-linear medium can have a potential application in the field of quantum information. The detuning effect has been shown in figure 1d. Figures 2 and 3 represent different values of the intensity coupling, where the values of the parameters $f(n)$ are equal to $\sqrt{n}$, for figure 2 and $1/\sqrt{n}$ for figure 3, and other parameters have the same values as in figure 1. One observes that the CME shows rapid oscillations for $f(n) = \sqrt{n}$ (see figures 2a, 2b). But the situation is completely changed when we consider $f(n) = 1/\sqrt{n}$. Here CME shows small oscillation in a regular manner. This is particularly because of the non-linear nature of the coupling in this model which results in the Rabi frequency being proportional to the photon number, when we consider $f(n) = \sqrt{n}$, as opposed to its square root, as is the case in one-photon JC-model (see figure 2a). Also it is noticed that in the absence of the non-linear medium we see a gradual decrease in the amplitudes of the Rabi oscillations. With increasing non-linear interaction of the Kerr-like medium with field mode and in the presence of the intensity atom–field coupling in the form $f(n) = 1/\sqrt{n}$, CME decreases (see figure 2). In this case the degree of entanglement between the field and the atom reduces and almost pure state is reached. It is evident that the field and the atom are in pure states when the Kerr-like effect increases further and when the intensity coupling constant has taken the form $1/\sqrt{n}$. It is worth noting that the results presented here are almost similar to those obtained in our previous work [2,10] where the von Neumann entropy has been used to quantify the entanglement. Our work suggests, further investigation is worthwhile into the relationship between different measures of entanglement.

In conclusion, we have shown here by exporting numerically in the parameter space of a generalized JC-model that the concurrence hierarchy clearly demonstrates the entangle-

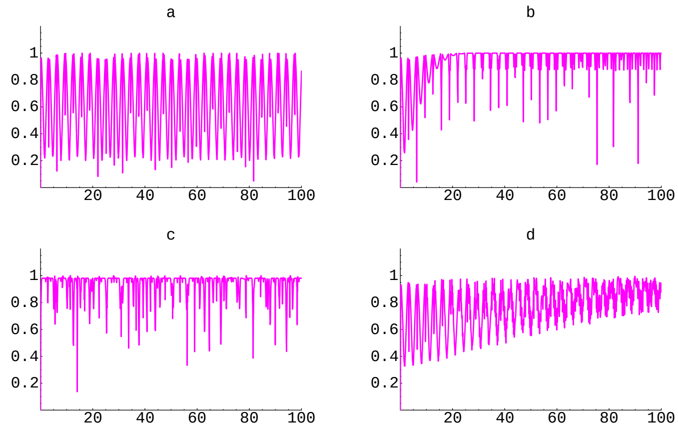

Figure 2. $C_1(\phi)$ as a function of the scaled time λt. Calculations assume that $(\varpi = \pi, v = 0)$ and the field in the coherent state with the initial average photon numbers $\bar{n} = 20$. **(a)** $\chi/\lambda = 0.0$, $f(n) = \sqrt{n}$, **(b)** $\chi/\lambda = 0.1$, $f(n) = \sqrt{n}$, **(c)** $\chi/\lambda = 0.5$, $f(n) = \sqrt{n}$, **(d)** $\chi/\lambda = 0.0, \Delta/\lambda = 5$, and $f(n) = \sqrt{n}$.

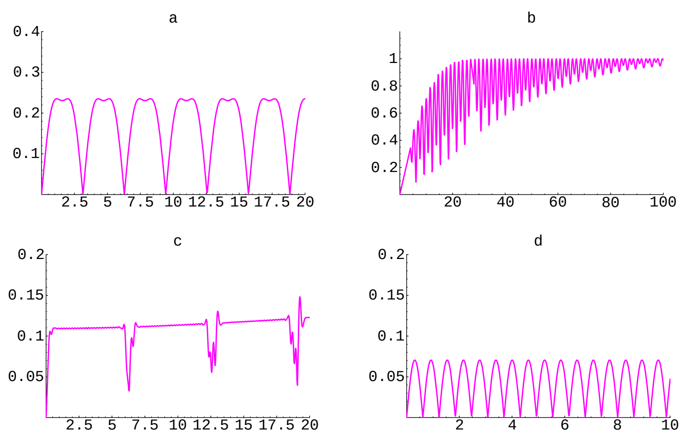

Figure 3. $C_1(\phi)$ as a function of the scaled time λt. Calculations assume that $(\varpi = \pi, v = 0)$ and the field in the coherent state with the initial average photon numbers $\bar{n} = 20$. **(a)** $\chi/\lambda = 0.0$, $f(n) = 1/\sqrt{n}$, **(b)** $\chi/\lambda = 0.1$, $f(n) = 1/\sqrt{n}$, **(c)** $\chi/\lambda = 0.5$, $f(n) = 1/\sqrt{n}$, **(d)** $\chi/\lambda = 0.0, \Delta/\lambda = 5$, and $f(n) = 1/\sqrt{n}$.

ment effects contained in the present model. Whilst the model was quite general, we looked specifically at some special choices of the non-linearities [18–20]. In particular, we have explored the influence of the various parameters of the system on the entanglement. When the non-linear interaction of the Kerr-type medium with the field mode is very strong, it leads to a decrease of concurrence hierarchy. To make the problem more realistic, the atom decay and cavity decay should be taken into account. We hope to report on such issues in a forthcoming paper.

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