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# Dust-cyclotron and dust-lower-hybrid modes in self-gravitating magnetized dusty plasmas

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**Abstract.** Theoretical investigation has been made on two different ultra-low-frequency electrostatic modes, namely, dust-cyclotron mode and dust-lower-hybrid mode, propagating perpendicular to the external magnetic field, in a self-gravitating magnetized two-fluid dusty plasma system. It has been shown that the effect of the self-gravitational force, acting on both dust grains and ions, significantly modifies the dispersion properties of these two electrostatic modes. The implications of these results to some space and astrophysical dusty plasma systems, especially to planetary ring-systems and cometary tails, are briefly mentioned.

**Keywords.** Dusty plasmas; dust-cyclotron waves; dust-lower-hybrid waves.

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There has been a great deal of interest in understanding different types of collective processes in dusty plasmas, because of their vital role in the study of astrophysical and space environments, such as, asteroid zones, planetary atmospheres, interstellar media, circumstellar disks, dark molecular clouds, cometary tails, nebulae, earth's environment, etc. [1–5]. These dust grains are invariably immersed in the ambient plasma and radiative background. The interaction of these dust grains with the other plasma particles (viz. electrons and ions) is due to the charge carried by them. The dust grains are charged by a number of competing processes, depending upon the local conditions, such as, photo-electric emission stimulated by the ultra-violet radiation, collisional charging by electrons and ions, disruption and secondary emission due to the Maxwellian stress, etc. [1–5]. For details of the physics of these dust grain charging processes we refer to chapter two of Shukla and Mamun [5].

It has been found that the presence of static charged dust grains modifies the existing plasma wave spectra [6–8]. Bliokh and Yaroshenko [6] studied electrostatic waves in dusty plasmas and applied their results in interpreting spoke-like structures in Saturn's rings (revealed by Voyager space mission [9]). Angelis *et al* [7] investigated the propagation of ion-acoustic waves in a dusty plasma, in which a spatial inhomogeneity is created by the distribution of immobile dust particles [10]. They [7] applied their results in interpreting the low frequency noise enhancement observed by the *Vega and Giotto* space probes in the dusty regions of Halley's comet [11].

On the other hand, it has been shown both theoretically [12–15] and experimentally [16,17] that the dust charge dynamics introduces different new eigenmodes, such as, dust-acoustic mode, dust-ion-acoustic mode, dust-drift mode, etc. [12–17]. These collective processes or wave phenomena in a dusty plasma can be studied in either of the three possible regimes, namely: (i) the electromagnetic force is much greater than the gravitational force, (ii) the electromagnetic force is of the same order of magnitude as the gravitational force, and (iii) the gravitational force is much greater than the electromagnetic force. Case (i) corresponds to the usual laboratory plasma situations where Coulombic interaction is primarily responsible for the plasma dielectric behavior. Case (ii) corresponds to planetary atmospheres and interstellar media [1, 18–20] where the thickness of the Jovian ring, spoke formation in Saturn's rings, etc. are thought to be due to the balance of these two forces. Case (iii) generally corresponds to astrophysical plasmas where the formation of large-scale structure is attributed to gravitational condensation [21].

Most of these studies [12–17] on these new modes (associated with extremely massive dust grains) are concerned with situation (i), but not with cases (ii) and (iii). Recently, a number of investigations [22–25] have been made of dust-acoustic waves in a self-gravitating dusty plasma system. Mahanta *et al* [22], Mamun [23], and Mamun and Alam [24] have studied the effect of the self-gravitational field on dust-acoustic waves by ignoring the ion dynamics, whereas Avinash and Shukla [25] have investigated dust-acoustic waves in a self-gravitating unmagnetized dusty plasma, taking into account the dynamics of dust-grains and ions. Since most of the dusty plasmas in laboratory and space environments are confined in an external magnetic field, it is of practical interest to examine the properties of dusty plasma waves in a magnetized dusty plasma [5]. (Very recently we have studied ultra-low-frequency dust-electromagnetic waves in a self-gravitating magnetized two-fluid dusty plasma taking into account the dynamics of dust-grains and ions [26]). In this brief report, we extend our earlier investigation to ultra-low-frequency dust-electrostatic waves, namely, dust-cyclotron mode and dust-lower-hybrid mode, propagating perpendicular to the external magnetic field.

We consider a two-component, self-gravitating, warm, magnetized dusty plasma system consisting of negatively charged (extremely massive) dust and positively charged ion fluids. Thus, at equilibrium we have $Z_i n_{i0} = Z_d n_{d0}$, where $n_{i0}$ ($n_{d0}$) is the equilibrium ion (dust) number density and $Z_d$ ($Z_i$) is the number of electrons (protons) residing on the dust grains (ions). This plasma system is assumed to be immersed in an external static magnetic field. It is also assumed here that the electron number density is highly depleted due to the attachment of almost all electrons to the surface of the extremely massive dust grains. This model is relevant to planetary ring-systems (e.g., Saturn's F-ring [1,14]) and laboratory experiments [16,17]. The macroscopic state of this self-gravitating, warm, magnetized dusty plasma system may be described by [12,23,25,26]:

$$\frac{\partial N_s}{\partial t} + \nabla\cdot(N_s\mathbf{U}_s) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{U}_s\cdot\nabla\right)\mathbf{U}_s = -\frac{q_s}{m_s}\nabla\Phi + \frac{q_s}{m_s c}(\mathbf{U}_s\times\mathbf{B}_0) - \nabla\Psi - \frac{1}{N_s m_s}\nabla P_s, \tag{2}$$

$$\nabla^2\Psi = 4\pi G\sum_s m_s N_s, \tag{3}$$

$$\nabla^2\Phi = 4\pi\sum_s q_s N_s, \tag{4}$$

where $m_s$, $q_s$ and $N_s$ are, respectively, mass, charge and number density of the species $s$ (dust grains and ions); $\mathbf{U}_s$ is the hydrodynamic velocity, $P_s = \gamma_s N_s k_B T_s$ with $k_B T_s$ being the thermal energy and $\gamma_s$ being the adiabatic constant; $\Phi$ is the electrostatic wave potential; $G$ is the universal gravitational constant; $c$ is the speed of light in vacuum. We are interested in looking at different extremely low-frequency electrostatic modes $(\omega, \mathbf{k})$ propagating perpendicular to the external magnetic field $\mathbf{B}_0$ (we assume that $\mathbf{B}_0$ is along the $x$-axis, i.e., $\mathbf{B}_0 \parallel \hat{\mathbf{x}}$ and propagation vector $\mathbf{k}$ is along the $y$-axis, i.e., $\mathbf{k} \parallel \hat{\mathbf{y}}$). We now carry out a normal mode analysis given in ref. [26], i.e., we first express our dependent variables $N_s$, $U_s$, $\Psi$ and $\Phi$ in terms of their equilibrium and perturbed parts, linearize eqs (1)–(4) to a first-order approximation and derive the general dispersion relation for low-frequency electrostatic mode

$$1 - \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2 - k^2 v_{td}^2 + \omega_{Jd}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2 - k^2 v_{ti}^2 + \omega_{Ji}^2} - \frac{\omega_{pd}^2 \omega_{Jd}^2 + \omega_{pi}^2 \omega_{Ji}^2 + \omega_{Jd}^2 \omega_{Ji}^2}{(\omega^2 - \omega_{cd}^2 - k^2 v_{td}^2 + \omega_{Jd}^2)(\omega^2 - \omega_{ci}^2 - k^2 v_{ti}^2 + \omega_{Ji}^2)} = 0, \tag{5}$$

where $\omega_{ps} = \sqrt{4\pi n_{s0} q_s^2 / m_s}$, $\omega_{Js} = \sqrt{4\pi G m_s n_{s0}}$ and $\omega_{cs} = |q_s| B_0 / m_s c$. Our present interest is to study low-frequency electrostatic modes of two different frequency limits, namely, $\omega \sim \omega_{cd}$ (where $\omega_{Ji}, \omega_{ci} < k v_{ti}$ is valid) and $\omega_{cd} < \omega < \omega_{ci}$ (where $\omega_{Ji}, k v_{ti} < \omega_{ci}$ is valid). The low-frequency electrostatic mode corresponding to the former case is termed as dust-cyclotron mode (analogous to ion-cyclotron mode) and corresponding to the latter case is termed as dust-lower-hybrid mode (analogous to ion-lower-hybrid mode).

### *A. Dust-cyclotron mode*

To study dust-cyclotron mode we use the approximations $\omega_{Ji}, \omega_{ci} \ll k v_{ti}$. This approximation reduces the general dispersion relation to a simple form:

$$\omega^2 = \omega_{cd}^2 + k^2 v_{td}^2 + \frac{k^2 C_d^2}{1 + k^2 \lambda_{Di}^2} - \omega_{Jd}^2 \left[1 + \left(\frac{Z_d m_i}{Z_i m_d}\right)\left(\frac{1}{1 + k^2 \lambda_{Di}^2}\right)\right] - \frac{\omega_{Ji}^2}{1 + k^2 \lambda_{Di}^2}\left[1 + G\left(\frac{m_i m_d}{Z_i Z_d e^2}\right)\right], \tag{6}$$

where $C_d = (\gamma_i Z_d k_B T_i / Z_i m_d)^{1/2}$ and $\lambda_{Di} = (\gamma_i k_B T_i / 4\pi n_{i0} Z_i^2 e^2)^{1/2}$. This equation represents the dispersion relation for the dust-cyclotron mode, in which the effects of self-gravitational field (acting on both dust particles and ions), thermal pressures of dust and ion fluids, and ion dynamics are included. If we consider the unmagnetized case and neglect the effects of the self-gravitating field, ion dynamics and dust fluid temperature (i.e. $\omega_{cd} \to 0$, $\omega_{Jd,i} \to 0$, $k^2 \lambda_{Di}^2 \ll 1$, and $v_{td} \to 0$), this becomes the dispersion relation for the dust-acoustic mode which was studied by Rao *et al* [12]. On the other hand, if we neglect the effects of external magnetic field and dust fluid temperature, but not of the self-gravitational field and ion-dynamics, our dispersion relation reduces to that obtained by Avinash and Shukla [25].

It is shown from our dispersion relation for the dust-cyclotron mode that due to the effect of the self-gravitational force acting on dust grains and ions, this mode becomes unstable if

$$\left(\omega_{cd}^2 + k^2 v_{td}^2 + \delta k^2 C_d^2\right) < \left(\omega_{Jd}^2 \left[1 + \delta\left(\frac{Z_d m_i}{Z_i m_d}\right)\right] + \delta\omega_{Ji}^2 \left[1 + G\left(\frac{m_i m_d}{Z_i Z_d e^2}\right)\right]\right), \tag{7}$$

where $\delta = 1/(1+k^2\lambda_{Di}^2)$. The criterion for this gravitational instability, for $k^2\lambda_{Di}^2 \ll 1$ and $(Z_d m_i/Z_i m_d) \ll 1$, can be simplified as $S_a > 0$, where $S_a \simeq \omega_{Jd}^2/\omega_{pd}^2 - V_{Ad}^2/c^2 - (2\pi\lambda_{Di}/\lambda)^2$ and $V_{Ad} = B_0/\sqrt{4\pi n_{d0} m_d}$. The growth rate $\gamma_a$ of this unstable mode becomes $\gamma_a \simeq \sqrt{S_a}$. To have some numerical appreciations of our results we have plotted $S_a = 0$ curves (showing stable and unstable regions) and obtained numerical values of the growth rate $\gamma_a$ for $\lambda/\lambda_{Di} \simeq 10^5$–$10^6$, $\omega_{Jd}/\omega_{pd} \simeq 10^{-5}$–$10^{-6}$, $V_{Ad}/c = 1$–$3.0\times 10^{-5}$. These values correspond to the usual space dusty plasma parameters [1–5], viz., $n_{d0} \simeq 10^{-7}$–$10$ cm$^{-3}$, $Z_d \simeq 10$–$10^4$, $T_i \simeq T_d \simeq 10^4$–$10^6$ K, $m_d \simeq 10^{-12}$–$10^{-7}$ g, $B_0 \simeq 10^{-3}$–$10$ G, etc. Our analytical and numerical calculations predict that (i) the dust-cyclotron mode may become unstable due to the effect of the self-gravitational force acting on dust grains, (ii) the effects of the external magnetic field and the thermal pressures of both the dust and ion fluids try to stabilize this dust-cyclotron mode and counter the gravitational condensation of the dust grains, and (iii) the growth rate $\gamma_a$ of this unstable dust-cyclotron mode increases with increasing values of $\omega_{Jd}/\omega_{pd}$ and $\lambda/\lambda_{Di}$, but decreases with increasing values of $V_{Ad}/c$.

### *B. Dust-lower-hybrid mode*

To examine dust-lower-hybrid mode we use the approximations $\omega_{cd} < \omega < \omega_{ci}$ and $\omega_{Ji}$, $kv_{ti} \ll \omega_{ci}$. These approximations reduce the general dispersion relation for the dust-lower-hybrid mode

$$\omega^2 = \omega_{cd}\omega_{ci}\left(1 + \frac{\omega_{cd}\omega_{ci}}{\omega_{pd}^2}\right)^{-1} + k^2 v_{td}^2 - \omega_{Jd}^2\left[1 + \frac{Z_d m_i}{Z_i m_d}\left(1 + \frac{\omega_{cd}\omega_{ci}}{\omega_{pd}^2}\right)^{-1}\right] - \omega_{Ji}^2\left(1 + \frac{\omega_{cd}\omega_{ci}}{\omega_{pd}^2}\right)^{-1}\left[1 + G\left(\frac{m_i m_d}{Z_i Z_d e^2}\right)\right]. \tag{8}$$

It is shown from dispersion relation (8) that the effects of the self-gravitational field and dust fluid temperature modify this dust-lower-hybrid mode significantly, and that due to the effect of this self-gravitational field, the dust-lower-hybrid mode becomes unstable if

$$\left(\mu\omega_{cd}\omega_{ci} + k^2 v_{td}^2\right) < \left(\omega_{Jd}^2\left[1 + \mu\left(\frac{Z_d m_i}{Z_i m_d}\right)\right] + \mu\omega_{Ji}^2\left[1 + G\left(\frac{m_i m_d}{Z_i Z_d e^2}\right)\right]\right), \tag{9}$$

where $\mu = (1 + \omega_{cd}\omega_{ci}/\omega_{pd}^2)^{-1}$. The criterion for this instability, for $\omega_{ci}\omega_{cd}/\omega_{pd}^2 \ll 1$ and $Z_d m_i/Z_i m_d \ll 1$, can be simplified as $S_b > 0$, where $S_b \simeq \omega_{Jd}^2/\omega_{pd}^2 - V_{Ai}^2/c^2 - (2\pi\lambda_{Dd}/\lambda)^2$, and $V_{Ai} = B_0/\sqrt{4\pi n_{i0} m_i}$. The growth rate $\gamma_b$ of this unstable mode is given by $\gamma_b = \sqrt{S_b}$. To have some numerical appreciations of our results we have plotted $S_b = 0$ curves (showing stable and unstable regions) and obtained numerical values of the growth rate $\gamma_b$ for $\lambda/\lambda_{Dd} = 10^2$–$10^4$, $\omega_{Jd}/\omega_{pd} \simeq 0.01$–$0.1$, $V_{Ai}/c = 0.01$–$0.03$. These values also correspond to the usual space dusty plasma parameters given in our previous case. Our analytical and numerical calculations predict that (i) the effect of the self-gravitational field and the increase in the wavelength of the mode try to destabilize the dust-lower-hybrid mode, (ii) the effects of the external magnetic field and the thermal pressure of the dust fluid try to stabilize this mode and counter the gravitational condensation of the dust grains, and (iii) the growth rate $\gamma_b$ of this unstable dust-lower-hybrid mode increases with increasing values of $\omega_{Jd}/\omega_{pd}$ and $\lambda/\lambda_{Dd}$, but decreases with increasing values of $V_{Ai}/c$.

To compare this dust-cyclotron (dust-lower-hybrid) mode with ion-cyclotron (ion-lower-hybrid) mode [27], it can be shown from our dispersion relations that the phase velocity of the dust-cyclotron mode is approximately $Z_d m_i/Z_i m_d$ (whose value may range from $10^{-4}$ to $10^{-8}$) times smaller than that of the ion-cyclotron mode, whereas the phase velocity of the dust-lower-hybrid mode is approximately $Z_d m_e/m_d$ (where $m_e$ is the mass of an electron) times smaller than that of the ion-lower-hybrid mode.

It is important to mention here that magnetized dusty plasma is significantly different from unmagnetized one due to the interaction of dust grain charge with the external magnetic field [5]. The other basic properties of these modes or of other dust-associated modes, which are addressed somewhere else [4,5,15,28–30], are also important but beyond the scope of the present brief report.

It may be stressed here that the results of the present investigation may be useful for understanding the electrostatic disturbances in a number of astrophysical dusty plasma systems, such as, planetary ring systems (viz. Saturn's rings [1,6,9]), cometary environments (viz. Halley's comet [7,11]), interstellar medium [1–5], etc., where negatively charged dust particulates and positively charged ions are the major plasma species.

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