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Jeans instability of an inhomogeneous streaming dusty plasma

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Abstract. The dynamics of a self-gravitating unmagnetized, inhomogeneous, streaming dusty plasma is studied in the present work. The presence of the shear flow causes the coupling between gravitational and electrostatic forces. In the absence of self-gravity, the fluctuations in the plasma may grow at the expense of the density inhomogeneity and for certain wavelengths, such an unstable mode may dominate the usual streaming instability. However, in the presence of self-gravity, the plasma inhomogeneity causes an overlap between Jeans and streaming modes and collapse of the grain will continue at all wavelengths.

Keywords. Jeans instability; inhomogeneous plasma; polarization field.

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1. Introduction

Interstellar grains are mainly composed of graphite, silicate and metallic compounds and they comprise about 1% of the mass of the interstellar medium. Significant fraction of these grains are charged [1]. Due to the low ionization fraction (10^{-7}), these grains are generally embedded in a weakly ionized plasma, and may lose or acquire an electron or may remain neutral. It is not unusual to consider the grains of different sizes (micron or less) and compositions, but only with $\pm 2e, \pm e$ or 0 electronic charges [2–4]. Umebayashi and Nakano [4] indicated that electrically charged grains are rare in the dense molecular clouds as these clouds are shielded from the UV radiation. However, in the HII region, which are formed when a significant fraction of radiation from a new born star, or an associated emission nebula, escapes without being absorbed by the in-falling matter from the accretion disk, dust grain may pick up, depending on the thermal velocities of the background plasma particles, between 10 to 100 electronic charges [1]. These charged grains are not monochromatic in size. The size distribution of the interstellar grain has been investigated by comparing the observed extinction curves with the theoretical one. The observations do not fit to a single size and single composition [1]. Therefore, size, mass and electronic charge of the grain may vary over a wide range. Further, it can be argued that at some stage of the condensation from gas to dust, circumstellar shells and protoplanetary disks must have predominantly large dust grains.

The existence of large, massive, charged dust grain gives rise to the possibility of overlap of gravitational and electromagnetic forces. For example, for grains of mass $m_d \sim 10^{-5}$ g, with $Z_d \sim$ 10–100 electronic charges, two forces may become comparable, i.e., $R = G m_d^2/(Z_d e)^2 \sim O(1)$. Here R is the ratio of the gravitational and the electrostatic forces, m_d is the grain mass, $Z_d e$ is the grain charge and G is the gravitational constant. The overlap of electrostatic and gravitational forces may radically change the way the collapse of the matter proceeds [5]. We anticipate that the gravito-electrodynamics of the dusty plasma may play an important role at some stage of the star formation.

The dynamics of the self-gravitating dusty plasma have recently been studied by several authors [5–14]. It has been shown that when self-gravity of the grain is balanced by the electric polarization field of the plasma, condensation of the grain may occur due to the background plasma properties [5]. However, background flow of the plasma is assumed to be absent in most of these studies although interaction of the moving plasma with the dust is a common feature of the interstellar medium. For example, the hot stellar wind emanating from the star, interacts with the ambient partially ionized medium. However, mixing of various constituents of the interstellar matter is a very slow process ($\sim 10^9$ years) [15]. Thus, the interstellar medium should be characterized by the large scale inhomogeneities. Furthermore, such an inhomogeneity can lead to the existence of a large scale polarization electric field [16]. Such a polarization field in turn will give rise to flows in the medium. Therefore, it is important to study the dynamics of a self-gravitating dusty plasma in the presence of equilibrium flows and fields.

It is well-known that the charge on the dust fluctuates, e.g., due to the attachment of electrons and ions from the ambient background plasma, photoelectron emission, sputtering etc., and generally, is not constant. However, the variation of the grain charge is described by the high-frequency ($\sim$ mega-Hertz) kinetic processes whereas gavitational instability is a low frequency process ($\sim$ Hz). Therefore, we shall ignore the charge dynamics in the present investigation. The negligible effect of the charge fluctuation on the Jeans collapse has been reported in the past [12].

The existence of a micro-Gauss ($\sim 10^{-6}$) magnetic field in the interstellar medium is well-known [17]. The charged grain will gyrate around the magnetic field. However, simple estimates for the dust charge equal to 100e, with the particle mass 10^{-5} g yield $\sim 10^{13}$ year gyration period, which is larger than the age of the galaxy ($\sim 10^{10}$ years). Therefore, grains may be assumed unmagnetized, though under certain conditions, the magnetic field may effect the grain dynamics, e.g. via ambipolar diffusion, etc.

2. Basic equations

A three-component, inhomogeneous dusty plasma consisting of electrons, ions and charged dust grains is considered. The grains are assumed to have equal radius with identical charges. We shall assume that the plasma particle density is much smaller than the dust density, i.e., $m_e n_e \ll m_i n_i \ll m_d n_d$ and thus, the gravitational potential is solely determined by the grain density. Then, the dynamics of the dusty plasma is described by the following equations: the continuity equation

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{v}_\alpha) = 0, \tag{1}$$

for electron, ion and dust grains. The equation of motion for electrons and ions are,

$$m_\beta n_\beta \left[\frac{\partial \vec{v}_\beta}{\partial t} + \vec{v}_\beta \cdot \nabla \vec{v}_\beta \right] = -T_\beta \nabla n_\beta - q_\beta n_\beta \nabla \phi - m_\beta n_\beta \nabla \psi. \tag{2}$$

Here $\beta = e, i$ and v, P, T, ϕ, ψ denote the velocity, pressure, temperature, electrostatic and gravitational potential respectively. For cold dust grains, we have

$$m_d n_d \left[\frac{\partial \vec{v}_d}{\partial t} + \vec{v}_d \cdot \nabla \vec{v}_d \right] = Z_d e n_d \nabla \phi - m_d n_d \nabla \psi. \tag{3}$$

Here $q_d = -Z_d n_d$ is the grain charge, which for definiteness, has been assumed negative. The potential fields are defined by the following pair of Poisson's equations

$$\nabla^2 \phi = -4\pi e \left[n_i - n_e - Z_d n_d \right], \tag{4}$$

$$\nabla^2 \psi = 4\pi G m_d n_d. \tag{5}$$

The above set of eqs (1)–(5) completely describes the dynamics of a self-gravitating dusty plasma.

2.1 *Equilibrium*

The equilibrium of a self-gravitating dusty plasma may be defined by balancing the non-linear convective term $(\vec{v} \cdot \nabla)\vec{v}$ with the self-gravity term or by balancing the self-gravity against the electric field [5,16]. However, *a priori*, the existence of a non-zero equilibrium electric field and the consequent flow in a quasi-neutral plasma is unclear. In a self-gravitating dusty plasma in the hydrostatic equilibrium, when pressure gradient term balances the self-gravity term, the plasma number density scale is proportional to the mass of the plasma particle, i.e., $n(x) \sim \exp(-m\psi/T)$. In an electron–ion plasma, such a scale difference may not be significant. However, in a dusty plasma, the scale separation between the dust number density distribution and electron and ion number density distribution is very large. For example, for the interstellar grains with $m_d \sim 10^{-12}$ g and $T_d \sim 0.0001 T_e$, the ratio of the dust to the electron scale height ($L_d/L_e \sim m_d T_e / m_e T_d$) and the dust density scale turns out to be few parsec (1 parsec $\sim 3 \times 10^{18}$ cm) for a few km electron scale-length. This scale separation will give rise to a large scale electric field in a dusty plasma medium [16]. We anticipate that such a field could exist in the interstellar medium, e.g., around early as well as late type stars, around proto-planetary disks, etc. It is physically plausible that such a large scale field may induce large scale flows in the medium.

The overall charge neutrality of such a polarized medium is preserved. In order to derive the scale over which the quasi-neutrality condition is respected, we first note that in the presence of electric field, plasma number densities are given as $n(x) \sim \exp(-((m\psi - q\phi)/T))$. Then, ignoring the electron and the ion inertia in comparison with the dust inertia, and imposing quasi-neutrality condition $n_i - n_e - Z_d n_d = 0$, one gets, $\phi/\psi \approx m_d / Z_d e$

for a cold dust particle. Further, making use of $n_{e,i}(x) \sim \exp(\pm\Phi)$, where $\Phi = e\phi/T_{e,i}$, and solving in terms of dust density, from eqs (4) and (5), one gets

$$n_d(r) = \frac{1}{R-1}\frac{n_e}{Z_d}\left[\exp(\tau\Phi) - \frac{n_i}{n_e}\exp(-\Phi)\right], \tag{6}$$

where $R = Gm_d^2/(Z_d e)^2\,\Phi = e\phi/T_i$ and $\tau = T_i/T_e$.

Making use of eq. (6) and approximating electron and ion number densities by $n_{e,i} = n_0(1 \pm \Phi)$ in $m_{e,i} \to 0$ limit, we can derive the scale of non-neutrality of a self-gravitating dusty plasma. From eq. (6), one may write $n_d \approx (R-1)^{-1}(n_0/Z_d)2\Phi$, where we have assumed $\tau = 1$. Then, from Poisson's equation (4), one gets $L^{-2}\Phi \sim 2\lambda_{\rm D}^{-2}\Phi(R/(R-1))$. We see that the quasi-neutrality condition $L^{-2}\lambda_{\rm D}^2 \ll 1$ is valid only if $R \ll 1$. We recall that $R \ll 1$ corresponds to a situation when self-gravity is much weaker than the electrostatic interaction between plasma particles. Since self-gravity is the prime cause behind the polarization of the medium, its feebleness is consistent with the quasi-neutrality condition. In the $R \gg 1$ limit, the dust density vary as $1/R$ for $\Phi =$ constant, suggesting that the difference between the electron and ion number densities has to increase as $\sim R$ in order to have a proper equilibrium force balance. For a given R, the dust number density varies as $\sim \exp(\Phi)$, i.e., it becomes a function of the plasma potential which in turn, has been generated by the polarization of the medium.

In the light of the above discussion, we shall assume a one-dimensional (along x) steady state, with an equilibrium between the electrostatic and the gravitational potentials in the presence of shear flow. From eqs (2) and (3), for cold ions and dust, after integration,

$$\frac{v_{ix}^2}{2} + \frac{e\phi}{m_i} = C_i, \tag{7}$$

$$\frac{v_{dx}^2}{2} - \frac{Z_d e\phi}{m_d} + \psi = C_d. \tag{8}$$

For inertia-less electrons,

$$n_e = n_{e0}\exp\left(\frac{e\phi}{T_e}\right), \tag{9}$$

where n_{e0} is the electron density at $\phi = 0$. From continuity equation (1), after integration, one gets

$$n_i v_{ix} = n_{i0} v_{i0} = f_i, \tag{10}$$

$$n_d v_{dx} = n_{d0} v_{d0} = f_d, \tag{11}$$

$$n_e v_{ex} = n_{e0} v_{e0} = f_e, \tag{12}$$

where f_α and $n_{\alpha 0}$ are fluxes and densities of the αth species at some initial moment. Making use of eqs (7) and (8) in (10) and (11), the ion and dust densities can be expressed as

$$n_i = f_i\left[2\left(C_i - \frac{e\phi}{m_i}\right)\right]^{-1/2} \tag{13}$$

and

$$n_d = f_d \left[2 \left(C_d + \frac{Z_d e \phi}{m_d} - \psi \right) \right]^{-1/2}. \tag{14}$$

Using eqs (9), (12) and (13), Poisson's equation (4) can be written as

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} = -4\pi e \Bigg\{ \frac{n_{i0} v_{i0}}{\left[2\left(C_i - \frac{e}{m_i}\phi\right)\right]^{1/2}} - \frac{Z_d n_{d0} v_{d0}}{\left[2\left(C_d + \frac{Z_d e}{m_d}\phi - \psi\right)\right]^{1/2}} - n_{e0} \exp\left(\frac{e\phi}{T_e}\right) \Bigg\}. \tag{15}$$

We see from eq. (15) that in the absence of a background flow of ions and dust particles, thermal motion of electrons is responsible for the generation of a finite electric field.

It is interesting to investigate the relaxation of a polarized self-gravitating medium. In order to study the relaxation, we shall expand the equilibrium in the asymptotic limit of vanishing electrostatic potential. Expanding the above expression around $\phi = 0$, in the lowest order, one obtains

$$n_{i0} = Z_d n_{d0} + n_{e0}, \tag{16}$$

and, in the next order

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} = 4\pi e^2 \left(\frac{n_{e0}}{T_e} - \frac{n_{i0}}{m_i v_{i0}^2} - \frac{Z_d^2 n_{d0}}{m_d v_{d0}^2} \right) \phi = \left(k_{\mathrm{D}e}^2 - k_{\mathrm{D}i}^2 - k_{\mathrm{D}d}^2\right) \phi. \tag{17}$$

Here

$$k_{\mathrm{D}e}^2 \left(= \lambda_{\mathrm{D}e}^{-2}\right) = \frac{4\pi n_{e0} e^2}{T_e}, \quad k_{\mathrm{D}i}^2 = \frac{4\pi n_{i0} e^2}{m_i v_{i0}^2} = \frac{\omega_{pi}^2}{v_{i0}^2},$$
$$k_{\mathrm{D}d}^2 = \frac{4\pi n_{d0} (Z_d e)^2}{m_d v_{d0}^2} = \frac{\omega_{pd}^2}{v_{d0}^2},$$

where ω_{pi} and ω_{pd} are the ion and dust plasma frequencies respectively. The role of the right-hand side terms in eq. (17) varies. Whereas the first term represents the usual Debye shielding, the remaining two terms represent the 'anti-shielding' [18]. Defining ion and dust acoustic speed as

$$C_s^2 = \lambda_{\mathrm{D}e}^2 \omega_{pi}^2, \quad C_{ds}^2 = \lambda_{\mathrm{D}e}^2 \omega_{pd}^2$$

and

$$\chi^{-2} = \lambda_{\mathrm{D}e}^{-2} \left[1 - \left(\frac{C_s^2}{v_{i0}^2} + \frac{C_{ds}^2}{v_{d0}^2} \right) \right],$$

eq. (18) can be written as

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} - \frac{\phi}{\chi^2} = 0. \tag{18}$$

The solution of eq. (18) is

$$\phi = A \exp\left(-\frac{x}{\chi}\right). \tag{19}$$

The condition

$$\left(\frac{C_s^2}{v_{i0}^2} + \frac{C_{ds}^2}{v_{d0}^2}\right) = 1, \tag{20}$$

corresponds to exact cancellation of the Debye shielding by the anti-shielding. This condition arises when the bulk flow speed becomes sonic. In a quasi-neutral plasma, such a situation arises at the plasma-charged layer boundary and the plasma potential is determined by its value at the boundary. When $\chi^2 = k_{\mathrm{D}e}^2 - k_{\mathrm{D}i}^2 - k_{\mathrm{D}d}^2 < 0$, i.e., when bulk flow speed in the plasma becomes supersonic (a situation when anti-shielding will dominate the shielding), the solution of eq. (18) becomes oscillatory and potential will have alternating sign.

Since the self-gravitating dusty plasma is a polarized medium, the resultant flow induced by the polarization electric field is responsible for the anti-shielding. The effect of the anti-shielding will be felt by the gravitational potential as well. Expanding gravitational potential $\psi = \psi_0 + \psi_1 + \cdots$ around $\phi = 0$, from Poisson's equation (5) we have,

$$\frac{\mathrm{d}^2\left(\psi = \psi_0 + \psi_1 + \cdots\right)}{\mathrm{d}x^2} = 4\pi G m_d \frac{n_{d0} v_{d0}}{\left[2\left(C_d + \frac{Z_d e}{m_d}\phi - \psi\right)\right]^{1/2}}. \tag{21}$$

Then to the lowest order

$$\frac{\mathrm{d}^2\psi_0}{\mathrm{d}x^2} = 4\pi G m_d n_{d0}, \tag{22}$$

and in the next order,

$$\frac{\mathrm{d}^2\psi_1}{\mathrm{d}x^2} = -4\pi G m_d n_{d0}\left(\frac{Z_d e\phi}{m_d v_{d0}^2}\right). \tag{23}$$

Equation (23) indicates the coupling between the gravitational and electrostatic potentials. Such a coupling could have been anticipated as dust density is responsible for both the existence of the large scale polarization field and also acts as a source term for the gravitational field. For the supersonic flow, when the electrostatic potential ϕ becomes oscillatory, the gravitational collapse of the dust will alternate between condensation and dispersion, depending upon the sign of ϕ.

We note that the expansion around $\phi = 0$ does not imply 'Jeans swindle' as the swindle involves leaving self-gravity unbalanced in the equilibrium. A self-gravitating plasma can be visualized as a medium in which, finite electrostatic field is generated due to the presence of the self-gravity and such a field in turn, induces a large scale flow in the medium. Therefore, in the asymptotic limit $\phi \to 0$, self-gravity is balanced by the flow.

We shall assume a scenario where Debye shielding is exactly cancelled by the anti-shielding, i.e., ($\chi^2 = 0$). This will amount to the presence of a constant electric field $E_0 = -\partial\phi/\partial x$. It is clear that the choice of such an electric field intimately relates the background plasma particle density to the dust particle density [5]. The departure from perfect cancellation of the Debye shielding by the anti-shielding will lead to the generation of an inhomogeneous electric field. For the present analysis, we shall assume a homogeneous electric field. Using eq. (11), one can define the inhomogeneity scale as

$$\frac{1}{n_{d0}}\frac{\mathrm{d}n_{d0}}{\mathrm{d}x} = -\frac{1}{v_{d0}}\frac{\mathrm{d}v_{d0}}{\mathrm{d}x} = K_d. \tag{24}$$

In a self-gravitating dusty plasma, gravitational and electrostatic forces may become comparable [$R \sim O(1)$], resulting in an asymptotic, homogeneous background [5]. $R \sim O(1)$ corresponds to $K_d \to 0$. Another consequence of the perfect balance of the two forces is the shrinkage of the shear scale, since shear scale is a measure of the force imbalance. Therefore, the presence of a finite, non-zero electric field, on the one hand, removes the necessity of a 'Jeans swindle' and, on the other hand, makes the equilibrium flows uniform ($K_d \to 0$).

3. Stability analysis

We shall perturb the physical variables around equilibrium quantities, $n_\alpha(x) = n_{\alpha 0}(x) + \tilde{n}_\alpha$, $\vec{v}_{(i,d)} = v_{(i0,d0)}(x) + \tilde{v}_{(i,d)}$, $\phi = \phi_0 + \tilde{\phi}$, $\psi = \psi_0 + \tilde{\psi}$. Here subscript zero denotes the equilibrium quantities and tilde denotes their perturbed values. The linearized ion and dust continuity eqs (1) are

$$\left[\frac{\partial}{\partial t} + v_{(i0,d0)}\frac{\partial}{\partial x}\right]\tilde{n}_{i,d} + \tilde{n}_{i,d}\frac{\mathrm{d}v_{(i0,d0)}}{\mathrm{d}x} + n_{(i0,d0)}\left[\frac{\partial \tilde{v}_{(ix,dx)}}{\partial x} + \frac{\partial \tilde{v}_{(iy,dy)}}{\partial y}\right] + \tilde{v}_{(ix,dx)}\frac{\mathrm{d}n_{(i0,d0)}}{dx} = 0. \tag{25}$$

The linearized x and y component of the cold ion momentum eq. (2) is

$$\left(\frac{\partial}{\partial t} + v_{i0}\frac{\partial}{\partial x} + \frac{\mathrm{d}v_{i0}}{\mathrm{d}x}\right)\tilde{v}_{ix} = -\frac{e}{m_i}\nabla_x\tilde{\phi}, \tag{26}$$

$$\left(\frac{\partial}{\partial t} + v_{i0}\frac{\partial}{\partial x}\right)\tilde{v}_{iy} = -\frac{e}{m_i}\nabla_y\tilde{\phi}. \tag{27}$$

The linearized dust momentum equations (eq. (5)) are

$$\left(\frac{\partial}{\partial t} + v_{d0}\frac{\partial}{\partial x} + \frac{\mathrm{d}v_{d0}}{\mathrm{d}x}\right)\tilde{v}_{dx} = \frac{Z_d e}{m_d}\nabla_x\tilde{\phi} - \nabla_x\tilde{\psi}, \tag{28}$$

$$\left(\frac{\partial}{\partial t} + v_{dx0}\frac{\partial}{\partial x}\right)\tilde{v}_{dy} = \frac{Z_d e}{m_d}\nabla_y\tilde{\phi} - \nabla_y\tilde{\psi}. \tag{29}$$

The Poisson's eqs (4) and (5) becomes

$$\nabla^2\tilde{\phi} = -4\pi\left[e(\tilde{n}_i - \tilde{n}_e) - Z_d\tilde{n}_d\right]. \tag{30}$$

$$\nabla^2\tilde{\psi} = 4\pi G m_d \tilde{n}_d. \tag{31}$$

From eq. (9) we have

$$\frac{\tilde{n}_e}{n_{e0}} = \frac{e\tilde{\phi}}{T_e}. \tag{32}$$

Equations (25)–(32) are a complete set of linearized equations. We recall that the perfect balance between the electric and gravitational forces ($R = 1$) will lead to a homogeneous dust density distribution (eq. (24)). If the ion density inhomogeneity scale is much larger than the wavelength of the fluctuations, i.e., $K_i\lambda_x \ll 1$, where $K_i = (1/n_{i0})(\mathrm{d}n_{i0}/\mathrm{d}x)$, $\lambda_x = 2\pi/k_x$ then in the $R \to 1$ limit, we can Fourier analyze the perturbations as $\exp^{-i\left(\omega t - k_x x - k_y y\right)}$. Then the continuity equation (25) can be written as

$$\frac{\tilde{n}_{(i,d)}}{n_{(i,d)0}} = \frac{\left[k_x - iK_{(i,d)}\,\tilde{v}_{(i,d)x}\right] + k_y\tilde{v}_{(i,d)y}}{D^*_{(i,d)}}. \tag{33}$$

Here

$$D^*_\alpha = D_\alpha + i\,\frac{\mathrm{d}v_{\alpha 0}}{\mathrm{d}x}, \tag{34}$$

with $D_\alpha = \omega - k_x v_{\alpha 0}$ as the Doppler-shifted frequency. The ion momentum equation for $\tilde{v}_{ix}$ and v_{iy} yields

$$D^*_i\tilde{v}_{ix} = \frac{e}{m_i}k_x\tilde{\phi}, \quad D_i\tilde{v}_{iy} = \frac{e}{m_i}k_y\tilde{\phi}. \tag{35}$$

Similarly, eliminating $\tilde{\psi}$ in terms of $\tilde{n}_d$, and making use of

$$\tilde{v}_{dy} = -\frac{k_y}{D_d}\left[\left(\frac{Z_d e}{m_d}\right)\tilde{\phi} + \frac{\omega_J^2}{k^2}\frac{\tilde{n}_d}{n_{d0}}\right] \tag{36}$$

in eq. (33), and using the resulting expression for the perturbed density in terms of $\tilde{v}_{dx}$ and $\tilde{\phi}$ the dust momentum equation for $\tilde{v}_{dx}$ can be written as

$$D^*_d\tilde{v}_{dx} = -\left(\frac{Z_d e}{m_d}\right)k_x\tilde{\phi} - \frac{k_x\omega_J^2}{k^2\left(D^*_d + \dfrac{k_y^2\omega_J^2}{D_d k^2}\right)} \times\left[\left(k_x - iK_d\right)\tilde{v}_{dx} - \left(\frac{Z_d e}{m_d}\right)\frac{k_y^2}{D_d}\tilde{\phi}\right], \tag{37}$$

where $k^2 = k_x^2 + k_y^2$. Eliminating $\tilde{v}_{ix,iy}$ and $\tilde{v}_{dx}$ in terms of $\tilde{\phi}$ in eq. (30), and using the resulting expression in the quasi-neutrality condition

$$Z_d \tilde{n}_d + \tilde{n}_e - \tilde{n}_i = 0, \tag{38}$$

one gets the following dispersion relation,

$$k_{\mathrm{De}}^2 = \omega_{pi}^2 \left[\frac{\left(k^2 - i k_x K_i\right) D_i + i k_y^2 \frac{\mathrm{d}v_{i0}}{\mathrm{d}x}}{D_i \left(D_i^*\right)^2} + \beta \frac{\left(k^2 - i k_x K_d\right) D_d + i k_y^2 \frac{\mathrm{d}v_{d0}}{\mathrm{d}x}}{D_d \left[\left(D_d^*\right)^2 + \omega_{\mathrm{J}}^2 \left[1 + \frac{i}{k^2 D_d}\left(k_y^2 \frac{\mathrm{d}v_{d0}}{\mathrm{d}x} - k_x K_d D_d\right)\right]\right]} \right]. \tag{39}$$

Here $\beta = Z_d^2 (m_i/m_d)(n_d/n_i)$. The dispersion relation, eq. (39) has been derived in the $k\lambda_{\mathrm{De}} \ll 1$ limit. When $G \to 0$, the above dispersion relation reduces to eq. (28) of ref. [18], except for their misprinted term with E_0. We note that $(k_y^2\, \partial v_{ix}/\partial x)/(k^2 D_i) \sim (k_y/k)^2 (K_i/k_x) \ll 1$ and $((k_y^2\, \partial v_{dx})/\partial x)/(k^2 D_d) \sim (k_y/k)^2 (K_d/k_x) \ll 1$ in the local approximation (i.e., $k_x \gg K_i, K_d$) and hence, the dispersion relation eq. (39) can be written as

$$k_{\mathrm{De}}^2 = \omega_{pi}^2 \left[\frac{\left(k^2 - i k_x K_i\right)}{\left(D_i^*\right)^2} + \beta \frac{\left(k^2 - i k_x K_d\right)}{\left[\left(D_d^*\right)^2 + \omega_{\mathrm{J}}^2 \left(1 - \frac{i k_x K_d}{k^2}\right)\right]} \right]. \tag{40}$$

The above dispersion relation can be analyzed in different limiting cases. A detailed perturbative analysis of the above dispersion relation, in the absence of self-gravity, has been carried out by Varma [18]. Thus, we shall restrict ourselves to the perturbative analysis of a simple case before subjecting the dispersion relation to a numerical analysis in the different limits.

Defining $V = (v_{i0} + v_{d0})/2$, $v = (v_{i0} - v_{d0})/2$, $\Omega = (\omega - k_x V)/2$, $\nu = (k_x v)/k$, $\lambda = \Omega/\nu$, one can write $D_i = k(\Omega - \nu) = k\nu(\lambda - 1)$, and $D_d = k\nu(\lambda + 1)$, and thus, eq. (40) can be recast as

$$1 = \left(\frac{C_*}{\nu}\right)^2 \left[\frac{1}{(\lambda - 1)^2} + \frac{\beta^*}{(\lambda + 1)^2 + \left(\frac{\omega_{\mathrm{J}}}{k\nu}\right)^2 \left(1 - \frac{i k_x K_d}{k^2}\right)} \right]. \tag{41}$$

In writing eq. (41), we have assumed $K_i, K_d \ll k_x$. Here, $C_*^2 = C_s^2 (1 - i k_x K_i/k^2)$, $\beta^* = \beta (1 - i k_x K_d/k^2)/(1 - i k_x K_i/k^2)$.

First we analyze the above dispersion equation (41) in the absence of self-gravity, $\omega_{\mathrm{J}} = 0$, and $K_i = K_d = K$, i.e., when the ion and the dust inhomogeneity scales are equal. Then, the above equation becomes

$$1 = \left(\frac{C_*}{\nu}\right)^2 \left[\frac{1}{(\lambda - 1)^2} + \frac{\beta^*}{(\lambda + 1)^2} \right]. \tag{42}$$

Solving eq. (42) perturbatively in the vicinity of $\lambda = -1 + \delta$ in $\delta \ll 1$ limit, one gets

$$\left[\left(\frac{C_*}{v}\right)^2 - 4\right]\delta^2 - 4\beta^*\left(\frac{C_*}{v}\right)^2\delta + 4\beta^*\left(\frac{C_*}{v}\right)^2 = 0. \tag{43}$$

The roots, $\delta_{1,2}$ are

$$\delta_{1,2} = \frac{2\beta^*\left(\frac{C_*}{v}\right)^2}{\left(\frac{C_*}{v}\right)^2 - 4}\left[1 \pm \sqrt{1 - \frac{\left(\frac{C_*}{v}\right)^2 - 4}{\beta^*\left(\frac{C_*}{v}\right)^2}}\right]. \tag{44}$$

For real β^* and C_* the root becomes unstable, if

$$\left(\frac{C_*}{v}\right)^2 > \beta^*\left(\frac{C_*}{v}\right)^2 + 4. \tag{45}$$

We note that $C_*/v \sim C_s/v$ for $n_e \sim n_i$ and $k_x \approx k$. Thus, when the ion-acoustic speed C_s is greater than the relative drift speed between ion and dust particles $v \sim \left(v_{i0} - v_{d0}\right)$, the wave becomes unstable. This is the usual streaming instability. Next, we solve the dispersion relation, eq. (39) numerically for three cases: (a) when $k_x K_d/k^2 = k_x K_i/k^2 = 0$, (b) when $K_d = K_i = 0.5$ and (c) when $K_d \neq K_i$. In figure 1, curve 1 corresponds to $k_x K_d/k^2 = k_x K_i/k^2 = 0$ and curve 2 corresponds to $k_x K_d/k^2 = 0.5$ and $k_x K_i/k^2 = 0$. We note that the growth rate of streaming instability in the presence of homogeneous ion and

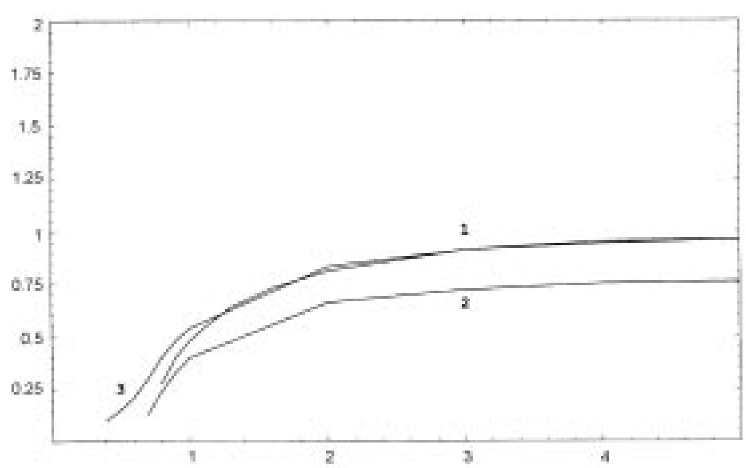

Figure 1. Growth rate $\mathrm{Im}(\lambda)$ is plotted for three cases: (a) $K_d = K_i = 0$, (b) $K_d \neq K_i, K_d = 0.5, K_i = 0$ and (c) $K_d = K_i = 0.5$, against $x = C/v$. Curve 1 in this figure corresponds to the usual streaming instability in the absence of any inhomogeneity (case (a)); curve 2 shows the growth rate when inhomogeneity scales are unequal (case (b)); and curve 3 corresponds to case (c) when both scales are non-zero and equal. The presence of both ion and dust inhomogeneities (curve 3) do not affect the growth rate of the streaming instability. However, presence of only dust inhomogeneity, curve 2 reduces the growth rate of streaming instability.

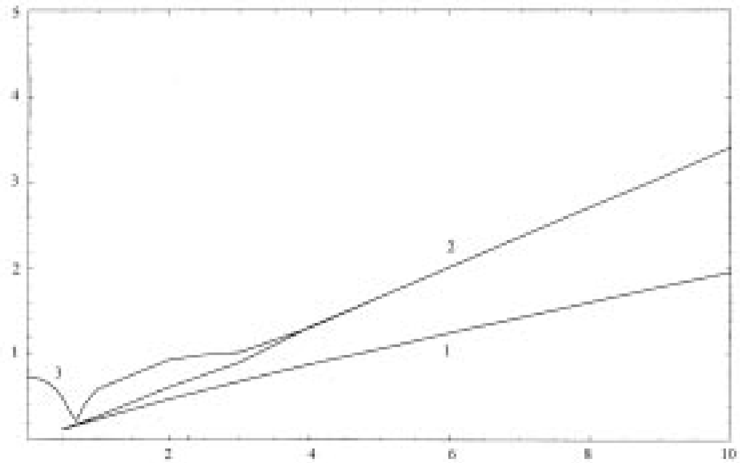

Figure 2. Curves 1 and 2 correspond to the cases $k_x K_d/k^2 = 0.5$, $k_x K_i/k^2 = 0$ and $k_x K_d/k^2 = k_x K_i/k^2 = 0.5$ in the absence of potential fields. We see that inhomogeneity introduces a new unstable mode in a streaming dusty plasma and after $x > 3$, this mode starts dominating over the streaming mode. If only one type of inhomogeneity is present (curve 2), the growth rate is smaller than when both dust and ion density distributions are inhomogeneous. When the gravitational field is present ($(\omega_\mathrm{J}/k_x v)^2 = 0.5$), the growth rate is enhanced (curve 3). The Jeans mode couples with the streaming mode for $x \sim 1$ and keeps growing after a dip in the curve.

dust density distribution (curve 1) is larger than when dust density distribution is inhomogeneous and ion density distribution is homogeneous (curve 2). When both ion and dust density distribution is inhomogeneous and their inhomogeneity scale equals each other, then the growth rate of the instability (curve 3) remains similar to the homogeneous case (curve 1). When electric and gravitational fields are switched on, the streaming instability remains almost unchanged.

In figure 2, curves 1 and 2 correspond to the cases when $k_x K_d/k^2 = 0.5$, $k_x K_i/k^2 = 0$ and $k_x K_d/k^2 = k_x K_i/k^2 = 0.5$ in the absence of potential fields. We see that the inhomogeneity introduces a new unstable mode in a streaming dusty plasma, and after $x > 3$ this new mode even starts dominating over the streaming mode. If only one type of inhomogeneity is present (curve 1), the growth rate is smaller than when both dust and ion density distributions are inhomogeneous (curve 2). The free energy available for this instability comes from the inhomogeneity of the plasma density distributions. When the gravitational field is present $(\omega_\mathrm{J}/k_x v)^2 = 0.5$, first we see a coupling between the Jeans and the streaming modes (curve 3). The coupling of the Jeans and the streaming mode is caused by the plasma inhomogeneity. For small x (≤ 0.7), Jeans mode dominates. This is consistent with the fact that at large wavelength, gravitational instability will operate. However, when $x > 0.7$, streaming instability is excited. This has an interesting consequence for the accretion of matter. The self-gravity can trigger the collapse at the long wavelength and then, streaming instability takes over and facilitates the condensation at small wavelengths. Therefore, in an inhomogeneous plasma, the collapse of the matter may proceed uninhibited at all wavelengths.

4. Summary

In the present work, we have analyzed the dynamics of a self-gravitating, inhomogeneous dusty plasma. We note that due to large scale differences between plasma particles, the existence of an electric field and the resultant shear flow is a natural dynamical consequence. The presence of flow has non-trivial effect on the plasma potential. When the mean bulk flow speed of the plasma particles becomes sonic, the Debye shielding is completely removed. Furthermore, when the bulk flow speed becomes supersonic, the plasma potential becomes oscillatory. The presence of oscillatory plasma potential will also make the gravitational potential oscillatory.

The linear stability analysis of such a plasma shows that, apart from the usual streaming instability, one will have an unstable mode caused by the inhomogeneity of the plasma. In the absence of self-gravity, the inhomogeneity-induced mode may not be dominant because its growth rate is smaller than that of the streaming instability. However, when self-gravity is present, this mode couples to the Jeans mode and, as a consequence, such a plasma is unstable at all wavelengths. This may lead to the condensation of matter at all scales. Therefore, the filamentation of the matter at small scales could be triggered by the inhomogeneity-induced modes. Our results suggest that self-gravity and inhomogeneity may play a complementary role in the condensation of matter, and may cause the matter to condense at all scales.

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