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On the study of ion-acoustic solitary waves and double-layers in a drift multicomponent plasma with electron-inertia

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Abstract. Using the pseudopotential method, theoretical investigation has been made on the first-order Korteweg-deVries ion-acoustic solitons in a multicomponent plasma consisting of warm positive ions, negative ions and isothermal electrons. The effects of electron-inertia and drift motion of the ions on the amplitudes and widths of the solitons have been studied in a plasma having (H^+, Cl^-), (H^+, O^-), (He^+, H^-) and (He^+, O^-) ions. Ion-acoustic double-layers have also been investigated for such plasmas. It has been found that drift velocity and electron-inertia have significant contribution on the formation of double-layers in multicomponent plasma.

Keywords. Ion-acoustic solitary wave; drift motion; double-layers.

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1. Introduction

The evolution of small-amplitude ion-acoustic solitary waves in plasmas has been a highly active subject of theoretical and experimental investigation in recent years. Ion-acoustic solitons and double-layers have been theoretically studied by various authors in plasmas consisting of cold ions [1], warm ions [2], negative ions [3-5], two-temperature electrons [6,7] and in the presence of magnetic field [8,9], Landau damping [10], density gradient [11], temperature gradient [12] etc. Propagation of electrostatic waves, particularly ion-acoustic waves through ionospheric plasma consisting of negative ions in addition to electrons and positive ions has been found to be more interesting specially for the formation of solitary waves and double-layers in the plasma. In this context, the work of Das and Tagare [13] are very important. They showed that even a small amount of negative ions may be important for the formation of ion-acoustic solitary wave. Subsequently, other authors [14–18] considered the effect of negative ions in the plasma and discussed the excitation of solitary wave and double-layer in the plasma. In a negative-ion plasma, compressive and rarefactive solitons are observed.

Drift motion of the ions also plays an important role for the existence of ion-acoustic solitary waves. Recently Chattopadhyaya *et al* [19] found that drift motion of the ions have significant contribution on the excitation of ion-acoustic solitary wave and double-layers in a negative-ion plasma.

The effect of electron-inertia on small-amplitude ion-acoustic soliton has not been widely discussed except by a few authors [20–23]. Khuel and Zhang [24] have studied the effect of ion-drift on small-amplitude ion-acoustic soilton considering the electron-inertia in the plasma. They have shown that ion-acoustic soliton solutions exist only if ion-drift velocity is less than the electron thermal velocity. However, the contribution of both the electron-inertia and drift motion has not been critically studied for the ion-acoustic solitons and double-layers in multicomponent plasma consisting of electrons, positive ions and negative ions.

In the present paper, we intend to study the characteristics of ion-acoustic solitons and double-layers in a plasma having different types of positive ions and negative ions using pseudopotential method. The effects of drift motion of the ions and the electron-inertia have also been taken into consideration in our present study. We obtain the critical values of ionic temperatures for the existence of solitary wave in the plasma with different values of stream velocities of ions. The profiles of the first-order K-dV soliton and higher order mK-dV soliton and also of double-layers have been drawn for the multicomponent plasma having (H^+, Cl^-), (H^+, O^-), (He^+, H^-) and (He^+, O^-) ions with streaming motion of ions and also the effect of electron-inertia.

2. Formulation

We consider a three-component plasma consisting of warm isothermal electrons, positive and negative ions. We further assume that the plasma is collisionless and unmagnetised, and the electron and the ions are non-relativistic. Moreover, the electrons and ions have constant streaming velocities in the equilibrium state. We consider the electron dynamics in full detail to study the effect of electron inertia. Assuming that the usual hydrodynamic description is possible, we observe that the unidirectional fluid equations of motion governing the collisionless unmagnetised plasma can be expressed by the following equations:

For positive and negative ions:

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0 \tag{1}$$

$$\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} + \frac{\sigma_s}{n_s}\frac{\partial p_s}{\partial x} = \frac{Q_s}{\mu_s}\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial p_s}{\partial t} + u_s \frac{\partial p_s}{\partial x} + 3p_s \frac{\partial u_s}{\partial x} = 0 \tag{3}$$

where $\sigma_s = T_s/T_e$, the subscript s stands for positive (i) ions and negative (j) ions and $Q_s = -1, Z$ for $s = i, j$ and $\mu_s = 1, \mu$ for $s = i, j$.

For electron:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = 0 \tag{4}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{1}{\mu_e}\frac{\partial \phi}{\partial x} - \frac{1}{\mu_e n_e}\frac{\partial n_e}{\partial x} \tag{5}$$

where $\mu_e = m_e/m_i$.

The Poission's equation is

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + \sum_{s=i,j} Q_s n_s. \tag{6}$$

In these equations (eqs (1)–(6)), u_s, u_e are the velocities of positive, negative ion and electron; n_s, n_e their respective number densities; p_s the thermal pressures of positive and negative ions; m_s, m_e the masses of positive, negative ion and electron, ϕ the electrostatic potential and σ_s the temperature of positive and negative ions.

In these equations we have normalised the velocities by the characteristic velocity $\sqrt{KT_e/m}$, all the densities by the equilibrium value $n^{(0)}$, and the length by the Debye-length $\sqrt{KT_e/4\pi e^2 n^{(0)}}$ whereas the potential is normalised by KT_e/e, so that the equations appear in totally dimensionless form. Here K is the Boltzmann's constant and T_e the electron temperature. For charge neutrality condition, $1 + Zn_j^{(0)} = n_i^{(0)}$, where Z is the number of negative charges in negative ions.

We study the solitary wave solution using Sagdeev's pseudopotential approach. For this purpose we make the dependent variable depend on an independent variable η defined by $\eta = x - Vt$ where V is the soliton velocity. In terms of η, eqs (1)–(6) reduce to

$$-V\frac{\partial n_s}{\partial \eta} + \frac{\partial}{\partial x}(n_s u_s) = 0 \tag{7}$$

$$(u_s - V)\frac{\partial u_s}{\partial \eta} + \frac{\sigma_s}{n_s}\frac{\partial p_s}{\partial \eta} = \frac{Q_s}{\mu_s}\frac{\partial \phi}{\partial x} \tag{8}$$

$$(u_s - V)\frac{\partial p_s}{\partial \eta} + 3p_s\frac{\partial u_s}{\partial \eta} = 0 \tag{9}$$

$$-V\frac{\partial n_e}{\partial \eta} + \frac{\partial}{\partial \eta}(n_e u_e) = 0 \tag{10}$$

$$(u_e - V)\frac{\partial u_e}{\partial \eta} = \frac{1}{\mu_e}\frac{\partial \phi}{\partial \eta} - \frac{1}{\mu_e n_e}\frac{\partial n_e}{\partial \eta} \tag{11}$$

$$\frac{\partial^2 \phi}{\partial \eta^2} = n_e + \sum_{s=i,j} Q_s n_s. \tag{12}$$

Further, we assume that the basic equations are supplemented by the following boundary conditions as $|x| \longrightarrow \infty$, i.e., $\eta \longrightarrow \infty$

(i) $n_e \to 1,\ \phi \to 0$

(ii) $n_s \to n_s^{(0)}$

(iii) $u_s \to u_s^{(0)}$

(iv) $u_e \to 0$

(v) $p_s \to p_s^{(0)}$. (13)

Now integrating eq. (7) and using the boundary conditions (13) we get

$$n_s = \frac{n_s^{(0)}(u_s^{(0)} - V)}{(u_s - V)}. \tag{14}$$

Again from (9) and (8) and using (13) we get similarly

$$p_s = \frac{p_s^0(u_s^{(0)} - V)^3}{(u_s - V)^3} \tag{15}$$

$$u_s = \frac{n_s^{(0)^{3/2}}}{\sqrt{6\sigma_s p_s^{(0)}}}\Bigg\{(V^2 + a_{0s} + 2(Q_s/\mu_s)\phi) - \left[(V^2 + a_{0s} + 2(Q_s/\mu_s)\phi)^2 - \frac{12\sigma_s p_s^{(0)}}{n_s^{(0)}}(u_s^{(0)} - V)^2\right]^{1/2}\Bigg\}^{1/2} \tag{16}$$

where

$$a_{0s} = u_s^{(0)^2} - 2Vu_s^{(0)} + \frac{3\sigma_s p_s^{(0)}}{n_s^{(0)}}. \tag{17}$$

Integrating eq. (10) and using (13) we get

$$n_e = \frac{V}{V - u_e}. \tag{18}$$

Again integrating eq. (11) and using (18) and (13) we get finally

$$\phi = \log\left(\frac{V}{V - u_e}\right) + \frac{1}{2}\mu_e[(V - u_e)^2 - V^2]. \tag{19}$$

From (19), expanding and taking the first-order term we get

$$n_e = \frac{1}{2}[(g_1 + g_2 + 2\phi)^{1/2} - (g_1 - g_2 + 2\phi)^{1/2}] \tag{20}$$

where

$$g_1 = 1 + \mu_e V^2, \quad g_2^2 = 4\mu_e V^2.$$

Now the Sagdeev potential ψ satisfies the relation

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = -\frac{\partial\psi}{\partial\phi}. \tag{21}$$

Again ψ can be written as the sum of three terms

$$\psi(\phi) = \psi_e(\phi) + \psi_s(\phi) \tag{22}$$

where s stands for positive (i) and negative (j) ions. From (12) we see that

$$\psi_e(\phi) = -\int n_e\,\mathrm{d}\phi \tag{23}$$

and

$$\psi_s(\phi) = -\int Q_s n_s\,\mathrm{d}\phi \tag{24}$$

Now from (23) and using (20) and (13) we get

$$\psi_e(\phi) = \frac{1}{6}[\{(g_1+g_2)^{3/2} - (g_1-g_2)^{3/2}\} \\ + \{(g_1-g_2+2\phi)^{3/2} - (g_1+g_2+2\phi)^{3/2}\}]. \tag{25}$$

Similarly from (24) by using (16), (17) and (13) we obtain

$$\psi_s(\phi) = \sum_s \frac{Q_s\mu_s n_s^{(0)^{3/2}}}{3\sqrt{12\sigma_s p_s^{(0)}}}\left[\{(b_{1s}+b_{2s})^{3/2} - (b_{1s}-b_{2s})^{3/2}\}\right. \\ \left. - \left\{\left(b_{1s}+b_{2s}+\frac{2Q_s}{\mu_s}\phi\right)^{3/2} - \left(b_{1s}-b_{2s}+\frac{2Q_s}{\mu_s}\phi\right)^{3/2}\right\}\right] \tag{26}$$

where

$$b_{1s} = V^2 + \sum_s a_{0s} \\ b_{2s} = \sum_s \frac{12\sigma_s p_s}{n_s^{(0)}(V-u_s^{(0)})^2}. \tag{27}$$

Thus we get finally from (22) by using (25) and (26)

$$\psi(\phi) = A + F(\phi) \tag{28}$$

where,

$$A = \frac{1}{6}\left[(g_1+g_2)^{3/2} - (g_1-g_2)^{3/2}\right. \\ \left. + \sum_s \frac{Q_s\mu_s n_s^{(0)^{3/2}}}{\sqrt{3\sigma_s p_s^{(0)}}}\{(b_{1s}+b_{2s})^{3/2} - (b_{1s}-b_{2s})^{3/2}\}\right] \tag{29}$$

and

$$F(\phi) = \frac{1}{6}\Bigg[(g_1 - g_2 + 2\phi)^{3/2} - (g_1 + g_2 + 2\phi)^{3/2} + \sum_s \frac{2Q_s \mu_s n_s^{(0)^{3/2}}}{\sqrt{12\sigma_s p_s^{(0)}}}\left\{\left(b_{1s} - b_{2s} + \frac{2Q_s}{\mu_s}\phi\right)^{3/2} - (b_{1s} + b_{2s})^{3/2} + \frac{2Q_s}{\mu_s}\phi)^{3/2}\right\}\Bigg]. \tag{30}$$

3. Ion-acoustic soliton

(a) *Conditions for the existence of soliton solution*

The form of the pseudopotential ψ would determine whether a soliton-like solution of eq. (22) will exist or not. The condition for existence of soliton solution is

$$\left.\frac{\mathrm{d}\psi}{\mathrm{d}\phi^2}\right|_{\phi=0} < 0 \tag{31}$$

or

$$[(g_1 + g_2)^{1/2} - (g_1 - g_2)^{-1/2}] + \sum_s \frac{Q_s^3 n_s^{(0)^{3/2}}}{\mu_s (3\sigma_s p_s^{(0)})^{1/2}} \times [(b_{1s} + b_{2s})^{-1/2} - (b_{1s} - b_{2s})^{-1/2}] < 0.$$

This is the condition for the existence of a potential well. Another condition is

$$\psi(\phi_c) \geq 0 \tag{32}$$

where

$$\phi_c = \min(\phi_{c_1}, \phi_{c_2}, \phi_{c_3}, \phi_{c_4}). \tag{33}$$

Again from (28) it is apparent that ψ becomes complex if the following inequalities are satisfied:

$$g_1 - g_2 + 2\phi < 0 \tag{34}$$

$$g_1 + g_2 + 2\phi < 0 \tag{35}$$

$$b_{1s} - b_{2s} + \sum_s \frac{2Q_s}{\mu_s}\phi < 0 \tag{36}$$

and

$$b_{1s} + b_{2s} + \sum_s \frac{2Q_s}{\mu_s}\phi < 0. \tag{37}$$

From (34) and (35) we get successively

$$\phi_{c_2} = \phi < -\frac{1}{2}(g_1 - g_2) \tag{38}$$

and

$$\phi_{c_1} = \phi < -\frac{1}{2}(g_1 + g_2). \tag{39}$$

Also from (36) and (37) we get similarly

$$\phi_{c_4} = \phi < -\sum_s \frac{\mu_s}{2Q_s}(b_{1s} - b_{2s}) \tag{40}$$

and

$$\phi_{c_3} = \phi < -\sum_s \frac{\mu_s}{2Q_s}(b_{1s} + b_{2s}). \tag{41}$$

From (31) the existence of soliton solution, there are some critical values of negative-ion density (n_{jc}) and ionic temperatures (σ_c) which may be obtained from the following equations:

$$A_3 n_{jc}^3 + A_2 n_{jc}^2 + A_1 n_{jc} + A_0 = 0 \tag{42}$$

$$B_2 \sigma_c^2 + B_1 \sigma_c + B_0 = 0 \tag{43}$$

where

$$A_0 = 3\sigma\mu \left[\frac{V\mu_e^{1/2}}{1 - V^2\mu_e} \{3\sigma - (V - u_i^{(0)})^2 - 1\} \right]$$

$$A_1 = \mu(V - u_j^{(0)})^2 + \frac{V\mu_e^{1/2}}{1 - V^2\mu_e}[\mu(V - u_j^{(0)})^2\{(V - u_i^{(0)})^2 - 3\sigma\}$$
$$-3\sigma\mu Z\{(V - u_i^{(0)})^2 + 2\}]$$

$$A_2 = Z^2[(V - u_i^{(0)})^2 - 3\sigma(\mu + 1)] + \mu Z(V - u_j^{(0)})^2$$
$$\times \left[2 + (V - u_i^{(0)})^2 \left(\frac{V\mu_e^{1/2}}{1 - V^2\mu_e} \right) \right]$$

$$A_3 = Z^2[\mu(V - u_j^{(0)})^2 + Z(V - u_i^{(0)})^2]$$

and

$$B_0 = Z^2[\mu(V-u_j^{(0)})^2 + Z(V-u_i^{(0)})^2]n_j^{(0)^3}$$
$$+Z\left[Z(V-u_i^{(0)})^2 + 2\mu(V-u_j^{(0)})^2 + \mu(V-u_j^{(0)})^2\right.$$
$$\left.\times(V-u_i^{(0)})^2\left(\frac{V\mu_e^{1/2}}{1-V^2\mu_e}\right)\right]n_j^{(0)^2}$$
$$+\mu(V-u_j^{(0)})^2\left[1+(V-u_j^{(0)})^2(V-u_i^{(0)})^2\left(\frac{V\mu_e^{1/2}}{1-V^2\mu_e}\right)\right]n_j^{(0)}$$
$$B_1 = -3\left[(\mu+1)Z^2 n_j^{(0)^2} + \mu\{Z(V-u_i^{(0)})^2 + (V-u_j^{(0)})^2 + 2Z\}\right.$$
$$\left.\times\left(\frac{V\mu_e^{1/2}}{1-V^2\mu_e}\right)n_j^{(0)} + \mu\left\{1+(V-u_i^{(0)})^2\cdot\left(\frac{V\mu_e^{1/2}}{1-V^2\mu_e}\right)\right\}\right]$$
$$B_2 = 9\mu\left(\frac{V\mu_e^{1/2}}{1-V^2\mu_e}\right).$$

(b) *K-dV soliton and mK-dV soliton*

For not so large amplitude waves we assume $\phi < 1$. Therefore, an analytical solution of eq. (21) for the ion-acoustic soliton can be obtained by expanding $\partial\psi/\partial\phi$ in terms of ϕ. After some calculation and using boundary condition we get from (21),

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = P_1\phi - P_2\phi^2 + P_3\phi^3 + \cdots \qquad (44)$$

where

$$P_1 = \frac{1}{2}\left[(h_1^{-1/2} - h_2^{-1/2}) + \sum_s \frac{Q_s^3 n_s^{(0)^{3/2}}}{\mu_s(3\sigma_s p_s^{(0)})^{1/2}}(h_3^{-1/2} - h_4^{-1/2})\right]$$
$$P_2 = \frac{1}{2}\left[(h_1^{-3/2} - h_2^{-3/2}) + \sum_s \frac{Q_s^4 n_s^{(0)^{3/2}}}{\mu_s(3\sigma_s p_s^{(0)})^{1/2}}(h_3^{-3/2} - h_4^{-3/2})\right]$$
$$P_3 = \frac{1}{2}\left[(h_1^{-5/2} - h_2^{-5/2}) + \sum_s \frac{Q_s^5 n_s^{(0)^{3/2}}}{\mu_s(3\sigma_s p_s^{(0)})^{1/2}}(h_3^{-5/2} - h_4^{-5/2})\right]$$
$$h_1 = (1+\mu_e^{1/2}V)^2$$
$$h_2 = (1-\mu_e^{1/2}V)^2$$
$$h_3 = \sum_s[(V-u_s^{(0)}) + (3\sigma_s p_s^{(0)}/n_s^{(0)})^{1/2}]^2$$
$$h_4 = \sum_s[(V-u_s^{(0)}) - (3\sigma_s p_s^{(0)}/n_s^{(0)})^{1/2}]^2.$$

Now, we take the terms up to ϕ^2 in (44) and get

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = P_1\phi - P_2\phi^2. \tag{45}$$

From (45), the first-order K-dV soliton solution is obtained as

$$\phi_{1(\text{K-dV})} = \frac{3P_1}{2P_2}\sec h^2\theta \quad \text{where} \quad \theta = (P_1/4)^{1/2}\cdot\eta. \tag{46}$$

The width of the K-dV soliton is given by

$$\delta_1 = 2P_1^{-1/2}. \tag{47}$$

From (46) and (47) it is observed that the electron inertia and drift motion of positive ions and negative ions have important contribution on the amplitude and the width of the soliton.

Again by taking the terms up to ϕ^3 we get from (44)

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right)^2 = \alpha_1\phi^2 - \alpha_2\phi^3 + \alpha_3\phi^4 \tag{48}$$

where

$$\alpha_1 = P_1, \quad \alpha_2 = 2P_2/3 \quad \text{and} \quad \alpha_3 = -P_3/2.$$

Integrating again we get finally the solution of higher order mK-dV soliton

$$\phi_{2(\text{mK-dV})} = \frac{2\alpha_1}{(\alpha_2^2 - 4\alpha_1\alpha_3)^{1/2}[2\cosh^2(\theta) - 1] + \alpha_2} \tag{49}$$

where $\theta = \eta\sqrt{\alpha_1}/2$.

The width of the soliton is

$$\delta_2 = \frac{2}{\sqrt{\alpha_1}}\cosh^{-1}\left\{\left[\frac{0.6905\,\alpha_2}{(\alpha_2^2 - 4\alpha_1\alpha_3)^{1/2}} + 1.6905\right]^{1/2}\right\}. \tag{50}$$

In (49) and (50) we see that both the drift motion and the electron inertia have important contribution on the amplitude and width of the ion-acoustic solitary wave in the plasma.

From (46), (47), (49) and (50) we see that drift motion of both the positive and negative ions as well as electron inertia have important contribution on the formation of ion-acoustic soliton in the plasma. However, to get the physical insight of the solitary wave, we have depicted the profiles of the solitary waves in figures 1–4 for the plasma having (H^+, Cl^-), (H^+, O^-), (He^+, O^-) and (He^+, H^-) ions. In our numerical estimation, we have taken the value of ion temperatures greater than the critical values of it, which are given in tables 1–4. In all the cases, it indicates the first-order K-dV equation gives rarefactive soliton where as second-order mK-dV solution gives compressive solitary waves. Moreover, we observed that due to increase or decrease of the stream velocity, soliton amplitude changes significantly. The variation of width with the negative ion concentration for both the first-order and higher-order solitons are shown in figure 5. It is observed that for the plasma

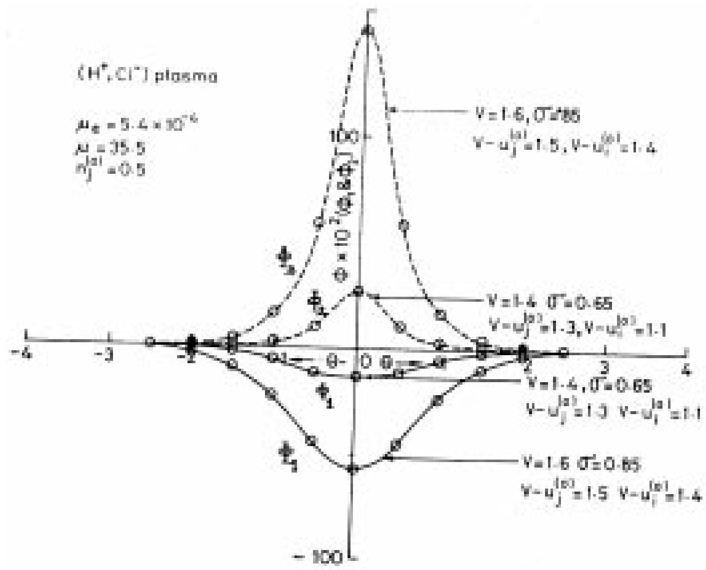

Figure 1. The profile of K-dV and mK-dV solitons in (H^+, Cl^-) plasma for different values of stream velocity and temperature of ions (— first order, - - - second order).

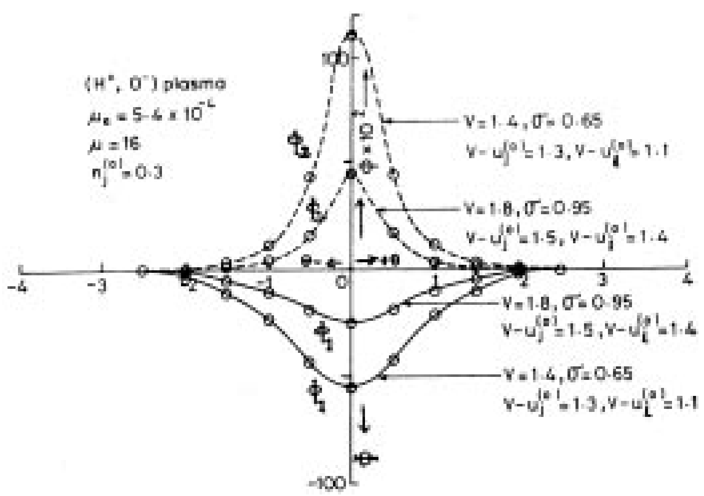

Figure 2. The profiles of K-dV and mK-dV solitons in (H^+, O^-) plasma for different values of temperature and stream velocity of the ions.

having all types of negative ions, the width decreases with the increase of negative ion concentration up to the values of ≈0.6–0.7. Moreover, the variation of width for heavy negative ion plasma is more interesting than that for the light negative ion plasma.

Table 1. For (He^+, O^-) plasma.

$V - u_i^{(0)} = 0.9, V - u_j^{(0)} = 1.1$			$V - u_i^{(0)} = 1.2, V - u_j^{(0)} = 1.3$			$V - u_i^{(0)} = 1.2, V - u_j^{(0)} = 1.3$		
V	$n_j^{(0)}$	σ_c	V	$n_j^{(0)}$	σ_c	V	$n_j^{(0)}$	σ_c
1.3	0.1	0.0451	1.5	0.1	0.0556	1.7	0.1	0.0556
	0.2	0.1059		0.2	0.1338		0.2	0.1337
	0.3	0.1791		0.3	0.2310		0.3	0.2305
	0.4	0.2608		0.4	0.3414		0.4	0.3406
	0.5	0.3467		0.5	0.4595		0.5	0.4585
	0.6	0.4334		0.6	0.5805		0.6	0.5791

Table 2. For (H^+, O^-) plasma.

$V - u_i^{(0)} = 1.1, V - u_j^{(0)} = 1.3$			$V - u_i^{(0)} = 1.4, V - u_j^{(0)} = 1.5$			$V - u_i^{(0)} = 1.4, V - u_j^{(0)} = 1.5$		
V	$n_j^{(0)}$	σ_c	V	$n_j^{(0)}$	σ_c	V	$n_j^{(0)}$	σ_c
1.4	0.1	0.0545	1.6	0.1	0.0656	1.8	0.1	0.1461
	0.2	0.1296		0.2	0.1564		0.2	0.4785
	0.3	0.2223		0.3	0.2745		0.3	0.9947
	0.4	0.3285		0.4	0.4092		0.4	1.6946
	0.5	0.4434		0.5	0.5569		0.5	2.5800
	0.6	0.5627		0.6	0.7118		0.6	3.6567

Table 3. For (He^+, H^-) plasma.

$V - u_i^{(0)} = 1.1, V - u_j^{(0)} = 0.9$			$V - u_i^{(0)} = 1.3, V - u_j^{(0)} = 1.2$			$V - u_i^{(0)} = 1.3, V - u_j^{(0)} = 1.2$		
V	$n_j^{(0)}$	σ_c	V	$n_j^{(0)}$	σ_c	V	$n_j^{(0)}$	σ_c
1.3	0.1	0.0508	1.5	0.1	0.0716	1.7	0.1	0.0707
	0.2	0.1337		0.2	0.1896		0.2	0.1891
	0.3	0.2296		0.3	0.3291		0.3	0.3285
	0.4	0.3226		0.4	0.4660		0.4	0.4657
	0.5	0.4069		0.5	0.5917		0.5	0.5907
	0.6	0.4825		0.6	0.7035		0.6	0.7035

Table 4. For (H^+, Cl^-) plasma.

$V - u_i^{(0)} = 1.3, V - u_j^{(0)} = 1.1$			$V - u_i^{(0)} = 1.5, V - u_j^{(0)} = 1.4$			$V - u_i^{(0)} = 1.5, V - u_j^{(0)} = 1.4$		
V	$n_j^{(0)}$	σ_c	V	$n_j^{(0)}$	σ_c	V	$n_j^{(0)}$	σ_c
1.4	0.1	0.0544	1.6	0.1	0.0654	1.7	0.1	0.0655
	0.2	0.1292		0.2	0.1575		0.2	0.1572
	0.3	0.2215		0.3	0.2729		0.3	0.2720
	0.4	0.3274		0.4	0.4070		0.4	0.4051
	0.5	0.4423		0.5	0.5541		0.5	0.5513
	0.6	0.5620		0.6	0.7089		0.6	0.7052

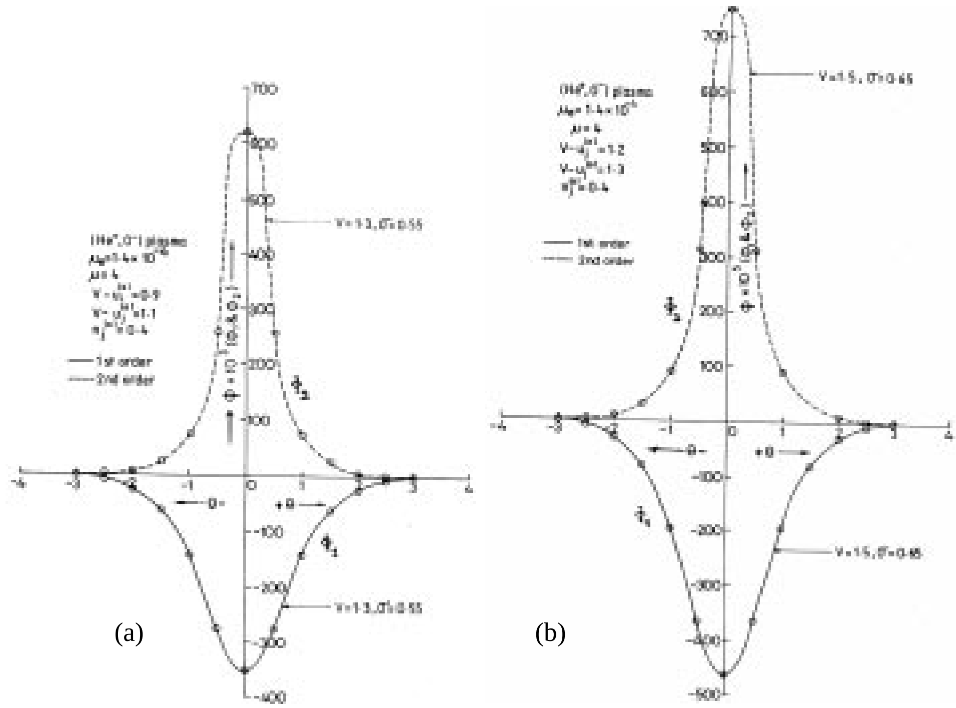

Figure 3. (**a**) The profiles of K-dV and mK-dV solitons in (He^{+}, O^{-}) plasma. (**b**) The profiles of K-dV and mK-dV solitons in (He^{+}, O^{-}) plasma for different values of ion-temperature and phase velocity of the wave.

4. Ion-acoustic double-layers

From eq. (44), the solution for double-layers in a negative-ion plasma can also be obtained. Double-layers will be formed under following conditions:

$$\text{(i)} \quad \psi = 0 \quad \text{at} \quad \phi = 0 \quad \text{and} \quad \phi = \phi_{\max}$$
$$\text{(ii)} \quad d\psi/d\phi = 0 \quad \text{at} \quad \phi = 0 \quad \text{and} \quad \phi = \phi_{\max}. \tag{51}$$

Now integrating eq. (44) we obtain

$$(d\phi/d\eta)^2 + \psi(\phi) = 0 \tag{52}$$

where

$$\psi(\phi) = -\frac{1}{2}P_3\phi^4 + \frac{2}{3}P_2\phi^3 - P_1\phi^2 - K_1. \tag{53}$$

K_1 is the integration constant which is zero by using (51).

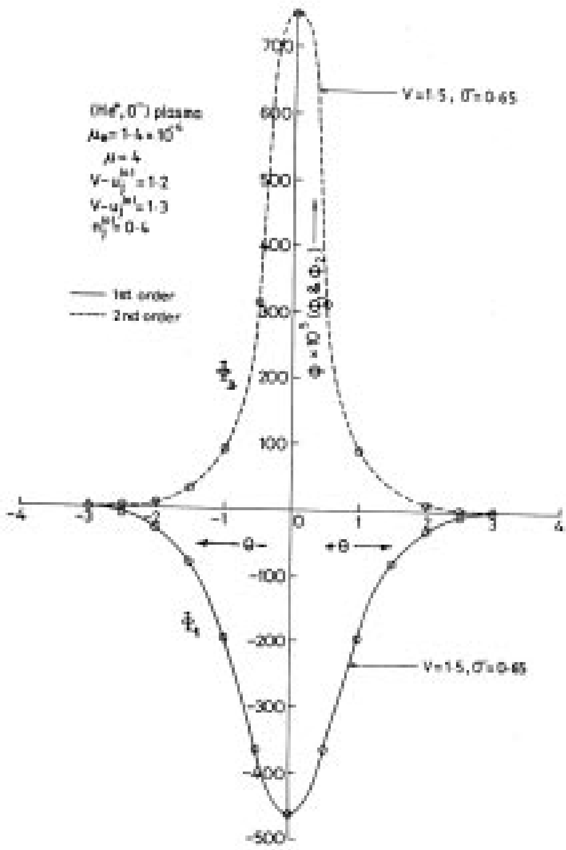

Figure 4. The profiles of K-dV and mK-dV solitons in (He^+, O^-) plasma.

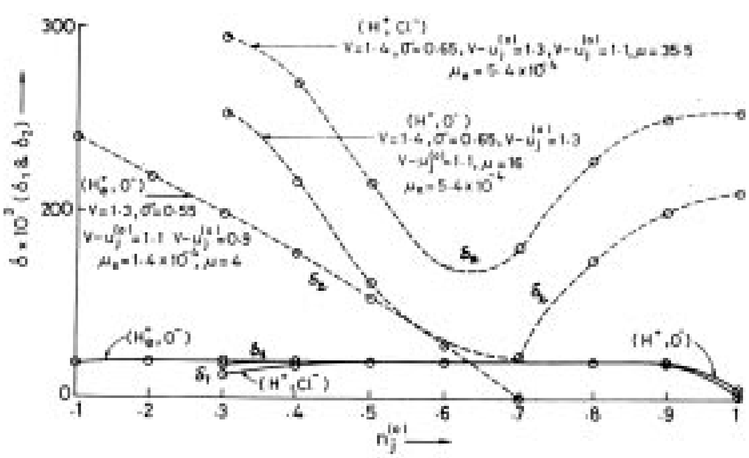

Figure 5. The variation widths of solitons in plasma having (H^+, O^-), (He^+, O^-), (H^+, Cl^-) ions for different negative ion concentrations.

Therefore

$$\psi(\phi) = \frac{1}{12}H_1\phi^4 + \frac{1}{3}H_2\phi^3 - 2H_3\phi^2 \tag{54}$$

or

$$\psi(\phi,\phi_m) = \frac{1}{12}H_1\phi_1^2(\phi-\phi_m)^2$$

where

$$H_1 = -6P_3, \quad H_2 = 2P_2, \quad H_3 = P_1/2$$

and

$$\phi_m = -2H_2/H_1.$$

The solution of eq. (52) using eq. (54) is given by

$$\phi_{\mathrm{DL}}(H_2,H_1,\phi_m) = \frac{1}{2}f(H_2)\cdot\phi_m[1-\tanh\{(-(1/24)H_1)^{1/2}\cdot\phi_m\cdot\theta'\}] \tag{55}$$

where

$$\theta' = \eta - \eta_0.$$

It is to be noted that for compressive double-layers $H_2 > 0$ which gives $f(H_2) = +1$. For the rarefactive double-layers $H_2 < 0$ from which the parameter $f(H_2) = -1$.

From the expression (55), it is seen that ϕ_m depends on the plasma parameters, i.e., density of negative ions, stream velocities of the ions, temperature of the ions and also the electron-inertia. To see the characteristics of the structure of double-layers in multicomponent plasma having the negative ions (H^+, O^-), (He^+, O^-) and (He^+, H^-), we have drawn figures 6–8 for different values of negative ion concentration, ionic temperatures and stream velocities and considering electron-inertia. It is observed that the potential of

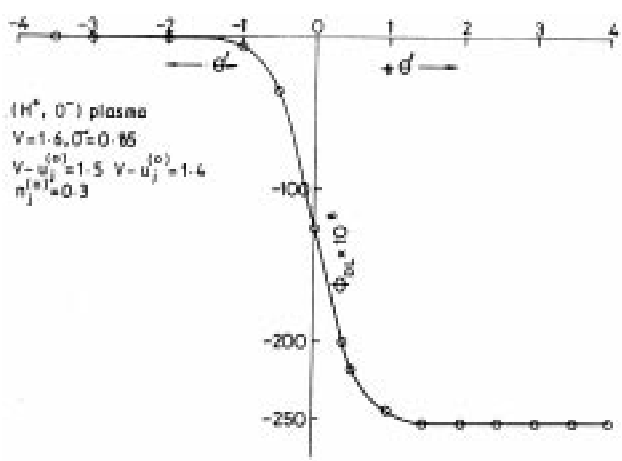

Figure 6. The structure of the double-layers in (H^+, O^-) plasma.

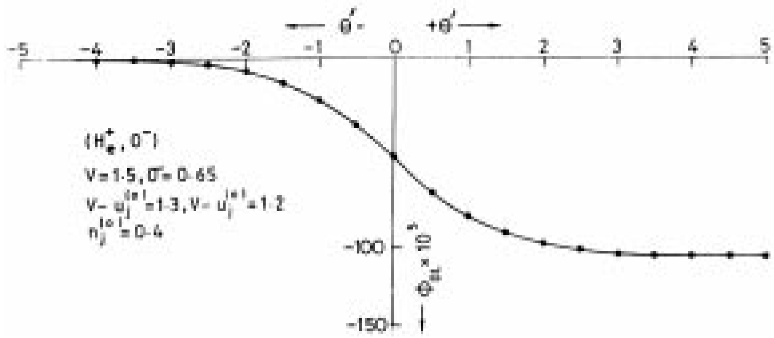

Figure 7. The structure of the double-layers in (He^{+}, O^{-}) plasma.

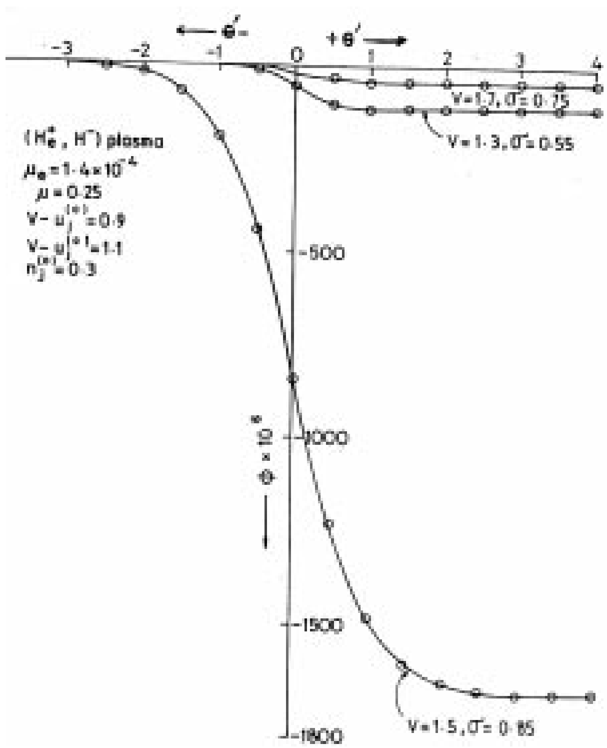

Figure 8. The structure of the double-layers in (He^{+}, H^{-}) plasma.

double-layers (ϕ_{DL}) is larger for (He^{+}, H^{-}) plasma than that of (H^{+}, O^{-}) and (He^{+}, O^{-}) plasmas, i.e., when the mass-ratio of negative ion and positive ion is small, the double-layers are more prominent. Moreover, it is seen that ϕ_{DL} is negative in the above cases which predicts that only rarefactive double-layers exist in the negative-ion plasma in the presence of electron-inertia.

5. Concluding remarks

From our present study we observed that drift motion, ionic temperature and electron-inertia have significant role on the formation of ion-acoustic soliton and double-layers in a negative ion plasma. It is to be noted that negative ions are present in space and it can also be produced in the laboratory. Nakamura *et al* [25,26], Ludwig *et al* [27] and other authors experimentally observed the ion-acoustic soliton and double-layers in a negative ion plasma. However, in the presence of drift motion, ion-acoustic soliton and double-layers have not yet been investigated in the laboratory. So, we are unable to compare our present theoretical results with the experimentally observed values.

For our present analysis, we have considered the plasma to be non-relativistic. But, consideration of relativistic effect would give more important results for the ion-acoustic solitons as indicated by Das and Paul [28], Nejoh [29], Chakraborty *et al* [30], Mondal *et al* [31] and other authors which would be applicable to astrophysical plasmas and laser-induced laboratory plasma.

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