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Ground state structure of some double-λ hypernuclei by a three-body model using a simple coordinate space approach

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Abstract. New experimental data on the binding energy $B_{\lambda\lambda}$ of ${}^{6}_{\lambda\lambda}$He, reported very recently, come up with the values $B_{\lambda\lambda} = 7.25 \pm 0.14$ MeV and $\Delta B_{\lambda\lambda} = 1.01 \pm 0.2$ MeV which are substantially lower than the old data $B_{\lambda\lambda} = 10.9 \pm 0.8$ MeV and $\Delta B_{\lambda\lambda} = 4.7 \pm 1.0$ MeV in use in literature since 1966. In view of the new data we decided to undertake a re-study of the ${}^{6}_{\lambda\lambda}$He hypernucleus using the same three-body model (α–λ–λ) with a simple coordinate space variational approach which was employed earlier with the old data on ${}^{6}_{\lambda\lambda}$He. After fitting different λ–λ potentials to the new data of ${}^{6}_{\lambda\lambda}$He we have applied our method to study some double-λ hypernuclei in light, medium and heavy mass regions and have determined the structural quantities like $B_{\lambda\lambda}$, the r.m.s. values of core-λ ($\langle r_{\text{core}-\lambda}\rangle$) and λ–λ ($\langle r_{\lambda-\lambda}\rangle$) distances theoretically. The core-λ interaction considered is of Woods–Saxon type. The strength and the range of the core-λ potential have been adjusted to reproduce the λ-binding energy (B_{λ}). These are in good agreement with the relativistic mean field (RMF) results. Our study shows that the λ–λ bonding energy $\Delta B_{\lambda\lambda}$ decreases with increasing mass number from ${}^{10}_{\lambda\lambda}$Be to ${}^{210}_{\lambda\lambda}$Pb of a double-$\lambda$ hypernucleus.

Keywords. Three-body model; effective potential; scaling parameter; double-λ hypernuclei; bond energy.

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1. Introduction

We undertake a study of various double-λ hypernuclei in low, medium and high mass regions using a three-body model which we employed earlier [1] for ${}^{6}_{\lambda\lambda}$He. A double-λ hypernucleus is treated in an asymmetric three-body model consisting of the core and two λ particles. Structural information in respect of r.m.s. distances of core-λ and λ–λ subsystems are derived in the present work. An efficient computer code to achieve convergence in the two-λ separation energy has been developed by us. Recently Takahashi *et al* [2] reported a double-hypernucleus event observed in an emulsion stack exposed to the 1.66 GeV/c K^- meson beam at the KEK proton synchroton. The event is uniquely interpreted as the sequential decay of ${}^{6}_{\lambda\lambda}$He emitted from a Ξ^- hyperon nuclear capture at rest. The λ–λ separation energy $B_{\lambda\lambda}$ and the λ–λ interaction energy $\Delta B_{\lambda\lambda}$ of ${}^{6}_{\lambda\lambda}$He have been determined without the ambiguities arising from the possibilities of excited states. The reported new data are $B_{\lambda\lambda} = 7.25 \pm 0.14$ MeV and $\Delta B_{\lambda\lambda} = 1.01 \pm 0.2$ MeV. These are substantially lower than the old data [3] $B_{\lambda\lambda} = 10.9 \pm 0.8$ MeV, $\Delta B_{\lambda\lambda} = 4.7 \pm 1.0$ MeV in use in literature since 1966. The report of the new experimental data on ${}^{6}_{\lambda\lambda}$He is a sufficient reason for us to study ${}^{6}_{\lambda\lambda}$He anew employing the earlier model [1]. Different λ–λ potentials have been fitted with the new $B_{\lambda\lambda}$ value of ${}^{6}_{\lambda\lambda}$He which acts as a constraint on the potentials. Using two three-term Gaussian (3-G) λ–λ potentials thus determined, we employed our three-body model (core-λ–λ) to study double-λ hypernuclei ${}^{10}_{\lambda\lambda}$Be, ${}^{14}_{\lambda\lambda}$C, ${}^{18}_{\lambda\lambda}$O, ${}^{22}_{\lambda\lambda}$Ne, ${}^{42}_{\lambda\lambda}$Ca, ${}^{92}_{\lambda\lambda}$Zr and ${}^{210}_{\lambda\lambda}$Pb covering light to heavy mass regions with even–even cores. Details are considered in §3.

2. Theory

In the present model the particles are labelled as 1 (core) and 2,3 (λ particles). The mass of the core is M_1 and the mass of a λ particle is M_2. For simplicity we take the λ particle to be spinless. r_1, r_2, r_3 are the distances between the particle pairs (1–2), (1–3) and (2–3) respectively. The variational principle for the Schrödinger equation is given by

$$\delta\langle\psi|H - E|\psi\rangle = 0, \tag{2.1}$$

where H, E, ψ refer to the Hamiltonian, energy and wave function for the internal motion. An even–even core in the ground state has $J = 0$. At this stage we make the simplification that ψ depends only on r_1, r_2, r_3. We further assume following Feshbach and Rubinow [4] and Ren [5] that ψ is represented simply by

$$\psi = \phi(R), \tag{2.2}$$

where R is a new space coordinate defined by $R(\eta) = (r_1 + r_2 + \eta r_3)/2$. η is a scaling parameter and it controls the way the wave function depends on r_1, r_2 and r_3. In addition, two new coordinates $R_2 = r_2$ and $R_3 = (1+\eta)r_3/2$ are introduced. After some calculations (2.1) finally yields an effective two-body equation in R which is

$$\frac{\mathrm{d}^2}{\mathrm{d}R^2}G(R) + \left[4\frac{(\eta^2 + 5\eta + 8)}{D'}E - \frac{V_{\mathrm{eff}}(R)}{D} - \frac{15}{4R^2}\right]G(R) = 0, \tag{2.3}$$

where $G(R) = R^{5/2}\phi(R)$. D and D' are functions of η and the masses of the core and λ particles [1]. $V_{\mathrm{eff}}(R)$ is a long range attractive potential which emerges through the R_2, R_3 integrations of the actual potentials $U(r_1), U(r_2), V(r_3)$. It is defined as follows:

$$V_{\mathrm{eff}}(R) = \frac{1}{R^5}\int_0^R \mathrm{d}R_3 \times \int_{R-R_3}^{R-\beta R_3} \mathrm{d}R_2 (2R - R_2 - \nu R_3) R_2 R_3 \{U(r_1) + U(R_2) + V(r_3)\}, \tag{2.4}$$

where $\beta = (\eta - 1)/(\eta + 1)$ and $\nu = 2\eta/(\eta + 1)$. The r.m.s. values of core-λ and λ–λ separation distances are calculated using the relations

$$\langle r_{\text{core-}\lambda} \rangle = \left[\frac{(2\eta^4 + 14\eta^3 + 42\eta^2 + 70\eta + 64)}{7(1+\eta)^2(\eta^2 + 5\eta + 8)} \bar{R^2} \right]^{1/2}, \tag{2.5}$$

where

$$\bar{R^2} = \frac{\int_0^\infty R^2 G^2(R) \mathrm{d}R}{\int_0^\infty G^2(R) \mathrm{d}R}, \tag{2.6}$$

$$\langle r_{\lambda\lambda} \rangle = \left[\frac{8(\eta^2 + 7\eta + 16)}{7(1+\eta)^2(\eta^2 + 5\eta + 8)} \bar{R^2} \right]^{1/2}. \tag{2.7}$$

The numerical wave function $G(R)$ is obtained from the solution of eq. (2.3)

3. Application

3.1 ${}^{6}_{\lambda\lambda}He$

As stated earlier our present investigation of ${}^{6}_{\lambda\lambda}$He is based on the new experimental data of $B_{\lambda\lambda}$. The α–λ potential to be used has to satisfy the experimental binding energy 3.04 MeV of ${}^{5}_{\lambda}$He hypernucleus. For the different α–λ potentials employed by us we have checked this consistency condition numerically by solving the Schrödinger equation in the centre of mass of α and λ particles. The α–λ and the λ–λ potentials used are:

(a) Gaussian potential B of Tang and Herndon [6] which is

$$V_{\alpha\lambda}(r) = -60.17 \exp(-r^2/1.273^2) \text{ MeV}.$$

This gives the binding energy of ${}^{5}_{\lambda}$He as 2.97 MeV. The corresponding λ–λ singlet potential employed is the Volkov potential with a Gaussian shape having an attractive part V_{A} and a repulsive part V_{R} [7] which are adjusted to obtain the best fit with the experimental $B_{\lambda\lambda}$ value. In this way we have found that $V_{\lambda\lambda}(r) = V_{\text{A}} \exp(-r^2/\alpha^2) + V_{\text{R}} \exp(-r^2/\beta^2)$, where $\alpha = 1.2$ fm, $\beta = 0.82$ fm, $V_{\text{A}} = -83.0$ MeV and $V_{\text{R}} = 165.0$ MeV. In comparison, the earlier calculation [1] fitting the old $B_{\lambda\lambda}$ value requires $V_{\text{A}} = -93.75$ MeV and $V_{\text{R}} = 148.0$ MeV. The present $V_{\lambda\lambda}$ potential in the ${}^{1}S_{0}$ state corresponds to a scattering length $= -1.03$ fm and an effective range $= 5.16$ fm. The value of the scaling parameter η for minimum separation energy is found to be 0.3 and the corresponding λ–λ separation energy is 7.451 MeV.

(b) The other α–λ potential considered here is the single-folding potential worked out in detail in [1]. It is as follows:

$$\text{Folding potential: } V_{\alpha\lambda}(r) = 209.037 \exp(-a_1 r^2) - 178.379 \exp(-b_1 r^2) - 18.317 \exp(-c_1 r^2) \text{ MeV},$$

where $a_1 = 0.5915059$ fm^{-2}, $b_1 = 0.4443259$ fm^{-2} and $c_1 = 0.2709581$ fm^{-2}. With this potential the binding energy of ${}^5_\lambda$He is 2.94 MeV. Three different sets of three term Gaussian potentials (3-G) of the form $V_{\lambda\lambda}(r) = \sum_{i=1}^{3} \omega_i(\lambda\lambda)\exp[-(r/\beta_i)^2]$, each consisting of a repulsive and two attractive terms have been considered for the spin-singlet λ–λ potential. The η is varied till minimum λ–λ separation energy is obtained. Each λ–λ potential is taken from the literature and then the strengths $\omega_i(\lambda\lambda)$ are adjusted till the best fit with the new experimental $B_{\lambda\lambda}$ value is obtained.

(i) Potential 3-G1 is taken from [8a]. After final adjustment, $V_{\lambda\lambda}(r) = 1300.3\exp(-r^2/0.5^2) - 305.3\exp(-r^2/0.9^2) - 10.5\exp(-r^2/1.5^2)$ MeV with scattering length $= -35.59$ fm and effective range $= 2.13$ fm. Here $B_{\lambda\lambda} = 7.237$ MeV.

(ii) 3-G2 is the potential taken from [8b] and after final adjustment, $V_{\lambda\lambda}(r) = 751\exp(-r^2/0.5^2) - 202\exp(-r^2/0.9^2) - 10.7\exp(-r^2/1.5^2)$ MeV.

Here the ranges are the same as that of 3-G1 but the strengths are different. Scattering length $= -5.292$ fm, effective range $= 2.34$ fm and $B_{\lambda\lambda} = 7.476$ MeV.

(iii) 3-G3 is the potential taken from [8c]. It is an OBE-simulating λ–λ interaction. After final adjustment, $V_{\lambda\lambda}(r) = 2103\exp(-r^2/0.35^2) - 284.7\exp(-r^2/0.777^2) - 17.9\exp(-r^2/1.342^2)$ MeV.

Scattering length $= -45.48$ fm, effective range $= 1.79$ fm and $B_{\lambda\lambda} = 7.425$ MeV. Our theoretical results on ${}^{6}_{\lambda\lambda}$He are given in table 1.

3.2 *Some double-λ hypernuclei*

After fitting different sets of λ–λ potentials with the new experimental $B_{\lambda\lambda}$ value of ${}^{6}_{\lambda\lambda}$He we next embark upon a study of the following double-λ hypernuclei ${}^{10}_{\lambda\lambda}$Be, ${}^{14}_{\lambda\lambda}$C, ${}^{18}_{\lambda\lambda}$O, ${}^{22}_{\lambda\lambda}$Ne, ${}^{42}_{\lambda\lambda}$Ca, ${}^{92}_{\lambda\lambda}$Zr and ${}^{210}_{\lambda\lambda}$Pb covering different mass regions in the three-body core-λ–λ model. The core is even–even for all the double-λ hypernuclei listed here. Excepting ${}^{9}_{\lambda}$Be and ${}^{13}_{\lambda}$C, the λ-separation energy B_λ has been calculated using the empirical formula [9]

$$B_\lambda(A) = ([27.0 - 81.9A^{-2/3}) \pm 1.5] \text{ MeV}. \tag{3.1}$$

Core-λ potential is considered to be Woods–Saxon [10].

Table 1. Study of ${}^{6}_{\lambda\lambda}$He.

α–λ Potential	α–λ Binding energy (MeV)	$\lambda-\lambda$ Potential parameters for the best fit $B_{\lambda\lambda}$ value: Type	Scattering length (fm)	Effective range (fm)	η for E_{min}	Present work: $\langle r_{\alpha\lambda}\rangle$ (fm)	$\langle r_{\lambda\lambda}\rangle$ (fm)	$B_{\lambda\lambda}$ (MeV)
Gaussian potential B [6]	2.97	Volkov	-1.03	5.16	0.3	2.25	2.875	7.451
Folding potential	2.94	3-G1	-35.59	2.13	0.45	2.87	3.479	7.237
		3-G2	-5.292	2.34	0.5	2.815	3.353	7.476
		3-G3	-45.48	1.79	0.6	2.787	3.204	7.425

Table 2. Best fit values of V_0 and a of Woods–Saxon (core-λ) potential for different λ-hypernuclei.

Hypernucleus	Woods–Saxon (core-λ) potential parameters V_0 (MeV)	a (fm)	B_λ in MeV from empirical formula	B_λ (MeV) used in the present work	B_λ (expt.) (MeV)
${}^{9}_{\lambda}$Be	-24.27	0.54	–	6.735	6.71 [13]
${}^{13}_{\lambda}$C	-27.2	0.51	–	11.21	11.69 ± 0.12 [14] 11.22 ± 0.08 [15]
${}^{17}_{\lambda}$O	-28.8	0.44	14.61 ± 1.5	14.60	–
${}^{21}_{\lambda}$Ne	-29.1	0.44	16.24 ± 1.5	16.23	–
${}^{41}_{\lambda}$Ca	-29.2	0.44	20.11 ± 1.5	20.11	–
${}^{91}_{\lambda}$Zr	-28.7	0.44	22.95 ± 1.5	22.95	–
${}^{209}_{\lambda}$Pb	-28.14	0.44	24.67 ± 1.5	24.63	–

$$V_{\text{core-}\lambda} = -\frac{V_0}{1+\mathrm{e}^{(r-c)/a}}, \tag{3.2}$$

where $c = r_0 A^{1/3}$, A is the mass number of the core, $r_0 = 1.128 + 0.439A^{-2/3}$ (fm).

Using this potential in the Schrödinger equation for the core and the λ particle, the differential equation is numerically solved to determine the λ-separation energy. V_0 and a are adjusted till the λ-separation energy so obtained tallies with the value determined from the empirical formula in (3.1). The values of V_0 and a thus obtained lie within the limits of the best fit values predicted by the RMF calculation [11]. For ${}^{9}_{\lambda}$Be and ${}^{13}_{\lambda}$C we have used experimental λ-separation energies [13,14] in fixing V_0 and a. Core mass is calculated using the relation [12]

$$\text{Mass excess} = (\text{Mass} - A)\chi_0,$$

where $\chi_0 = (931481 \pm 5)$ KeV and the mass excess is tabulated. The best fit values of V_0 and a for the different λ-hypernuclei considered in the present study are given in table 2. From the four sets of λ–λ interaction employed to fit the new $B_{\lambda\lambda}$ data on ${}^{6}_{\lambda\lambda}$He we have chosen the 3-term Gaussian potentials 3-G2 and 3-G3 which differ in strength and range in the present structural investigation on the double-λ hypernuclei.

With the core-λ and the λ–λ potentials η is varied to get $E_{\min}$. The value of the λ–λ separation energy for a double-λ hypernucleus is $B_{\lambda\lambda} = -E_{\min}$. For each hypernucleus we have achieved convergence in the two λ separation energy. The λ–λ bond energy is calculated using the relation [16]

$$\Delta B_{\lambda\lambda} = B_{\lambda\lambda}({}^{A}_{\lambda\lambda}X) - 2B_\lambda({}^{A-1}_{\lambda}X). \tag{3.3}$$

The r.m.s. values of core-λ and λ–λ separation distances $\langle r_{\text{core-}\lambda}\rangle$ and $\langle r_{\lambda\lambda}\rangle$ respectively are calculated using the relations (2.5), (2.6) and (2.7).

The numerical wave function $G(R)$ is obtained from the solution of eq. (2.3).

Table 3. Calculated values of $B_{\lambda\lambda}$, $\Delta B_{\lambda\lambda}$, $\langle r_{\text{core-}\lambda}\rangle$ and $\langle r_{\lambda\lambda}\rangle$ for different double-λ hypernuclei.

Hypernucleus	λ–λ Potential	η	$B_{\lambda\lambda}$ (MeV)	$\Delta B_{\lambda\lambda}$ (MeV)	$\langle r_{\text{core-}\lambda}\rangle$ (fm)	$\langle r_{\lambda\lambda}\rangle$ (fm)
${}^{10}_{\lambda\lambda}$Be	3-G2	0.40	16.262	2.792	2.368	2.92
	3-G3	0.40	16.341	2.871	2.357	2.908
${}^{14}_{\lambda\lambda}$C	3-G2	0.45	24.951	2.531	2.244	2.72
	3-G3	0.40	25.093	2.673	2.230	2.752
${}^{18}_{\lambda\lambda}$O	3-G2	0.45	31.513	2.313	2.210	2.679
	3-G3	0.40	31.646	2.446	2.198	2.711
${}^{22}_{\lambda\lambda}$Ne	3-G2	0.45	34.529	2.069	2.248	2.725
	3-G3	0.40	34.640	2.18	2.237	2.759
${}^{42}_{\lambda\lambda}$Ca	3-G2	0.45	41.517	1.297	2.469	2.999
	3-G3	0.50	41.526	1.306	2.468	2.940
${}^{92}_{\lambda\lambda}$Zr	3-G2	0.55	46.420	0.52	2.909	3.404
	3-G3	0.50	46.433	0.533	2.90	3.454
${}^{210}_{\lambda\lambda}$Pb	3-G2	0.45	49.281	0.021	3.52	4.267
	3-G3	0.45	49.27	0.01	3.518	4.264

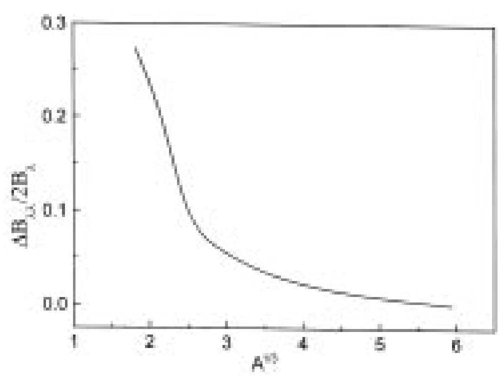

Figure 1. $\Delta B_{\lambda\lambda}/2B_{\lambda}$ as a function of $A^{1/3}$ for the potential 3-G2.

4. Conclusion

For the double-λ hypernuclei ${}^{10}_{\lambda\lambda}$Be, ${}^{14}_{\lambda\lambda}$C, ${}^{18}_{\lambda\lambda}$O, ${}^{22}_{\lambda\lambda}$Ne, ${}^{42}_{\lambda\lambda}$Ca, ${}^{92}_{\lambda\lambda}$Zr and ${}^{210}_{\lambda\lambda}$Pb our results regarding $B_{\lambda\lambda}, \langle r_{\text{core-}\lambda}\rangle, \langle r_{\lambda\lambda}\rangle$ and $\Delta B_{\lambda\lambda}$ are given in table 3 and the results are quite close for both the 3-G λ–λ potentials. We find that the λ–λ bond energy $\Delta B_{\lambda\lambda}$ decreases with increasing mass number of double-λ hypernucleus. Variation of $\Delta B_{\lambda\lambda}/2B_{\lambda}$ with $A^{1/3}$ is shown graphically in figure 1 for the potential 3-G2. For the 3-G3 potential our data indicate that the corresponding graph will be very similar and it will almost overlap the graph for 3-G2. The results are new as we have fitted the λ–λ interaction with the new experimental data on ${}_{\lambda\lambda}^{6}$He. It will be interesting to compare our results with that of other methods. Recently Filikhin and Gal [17] have done Faddeev–Yakubovsky calculations on ${}_{\lambda\lambda}^{6}$He $(\alpha\lambda\lambda)$ and ${}_{\lambda\lambda}^{10}$Be hypernuclei $(\alpha\alpha\lambda\lambda)$ employing the Gaussian soft core simulations of the *s*-wave OBE Nijmegen model NSC 97. With the NSC 97 versions e and f, the $B_{\lambda\lambda}({}_{\lambda\lambda}^{6}\text{He})$ values are 6.82 MeV and 6.60 MeV respectively which are close to the new data on ${}_{\lambda\lambda}^{6}$He. $B_{\lambda\lambda}({}_{\lambda\lambda}^{10}\text{Be})$ values are 15.5 MeV and 15.3 MeV respectively which are slightly lower than our values 16.262 MeV for the 3-G2 potential and 16.341 MeV for the 3-G3 potential.

The present three-body model works on a single variational parameter η and the results are quickly obtained. In spite of some simplifying assumptions the model gives reasonably good, physically interesting structural information pertaining to double-λ hypernuclei in this study and to the halo nuclei considered earlier [1]. Where a physical system may be approximated as a three-body system the present model is useful to carry out an approximate study before embarking upon a laborious elaborate calculation.

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