PRAMANA
— journal of
physics

Vol. 60, No. 6
June 2003
pp. 1161–1169

Renormalized energy-momentum tensor of $\lambda\Phi^4$ theory in curved space-time

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MS received 26 March 2002; revised 28 August 2002; accepted 20 December 2002

Abstract. Divergenceless expression for the energy-momentum tensor of scalar field is obtained using the momentum cut-off regularization technique. We consider a scalar field with quartic self-coupling in a spatially flat (3+1)-dimensional Robertson–Walker space-time, having arbitrary mass and coupled to gravity. As special cases, energy-momentum tensor for conformal and minimal coupling are also obtained. The energy-momentum tensor is observed to exhibit trace anomaly in curved space-time.

Keywords. Curved space-time; scalar field; enery-momentum tensor; effective potential.

PACS Nos 04.62.+v; 11.10.Gh

1. Introduction

Quantum gravity, a complete quantised theory of gravity – still being a distant dream – to study the effects of gravity on quantum fields we must opt for some semi-classical treatment in which gravity is treated classically using Einstein's equations whereas matter fields are quantized in the usual way. Though not completely, quantum field theory (QFT) as applied to curved space-times [1–16] has emerged as a useful alternative to quantum gravity. In the earlier days, works were generally focussed on free fields in the presence of gravity. But, due to the realization that no field in the nature is free from interactions, nowadays theoretical physicists have turned their attention towards the interacting fields [14–16].

Major breakthroughs have been achieved when this was used in cosmological and black hole space-times. In cosmology, interactions are important in the study of phase transitions in the early universe and phenomena like inflation [4–10]. In the case of black holes, phenomena like black hole evoparation were studied. Apart from these, condensed matter physics is an area were these type of interactions play a crucial role.

The particle concept of QFT [1] in curved space does not match with the very physical picture of particles in the sub-atomic level. One problem with this particle interpretation is that it is defined globally in terms of the field modes and is very sensitive to large scale structure of space-time. In contrast, particle detectors used are at least quasi-local in nature. So it is better to investigate this, with the help of a quantity, defined locally in space. Such a quantity is the energy-momentum tensor [1,11–16] defined at a point x, $T_{\mu\upsilon}(x)$. It not only describes the physical structure of the quantum field but acts as a source of gravity in Einstein's field equations. A complete description of the theory demands a finite value for the energy-momentum tensor.

In field theory, the concept of energy-momentum tensor comes through action principle. In flat space-time,

$$T_{\mu\upsilon} = \frac{\partial L}{\partial \varphi_{,\nu}} \varphi_{,\mu} - L g_{\mu\nu}.$$

While moving on to QFT in curved space-time, we must be able to accommodate both general relativistic and field theory aspects of the energy-momentum tensor under a unified framework.

In the present work, we discuss the method of momentum cut-off regularization technique as applied to scalar field having $\lambda\Phi^4$ self-interaction, arbitrarily coupled to a spatially flat Robertson–Walker space-time in 3+1 dimensions. In §2, we obtain exact expression for the renormalized energy-momentum tensor. We deduce the same in the case of conformal and minimal coupling as special cases. The conclusions are presented in the last section.

2. Renormalization of the energy-momentum tensor for $\lambda\Phi^4$ theory

Our aim is to obtain finite expression for the energy-momentum tensor of a quantized scalar field interacting with classical Einstein gravitational field using momentum cut-off regularization technique. We have chosen the $\lambda\Phi^4$ model of self-interaction, and the background space-time to be spatially flat RW metric and the coupling considered is arbitrary. Several authors have studied similar problem using the method of adiabatic regularization [11–16]. We feel that the present method is simple and we have obtained an exact expression for the energy-momentum. We replace the divergent quantities by well-defined expressions in such a manner that it is consistent with the physical basis of the theory. We start with the effective potential obtained from the equation of motion. Replacing the bare parameters with the renormalized quantities expressed in terms of the effective potential, we end up with a finite expression for $T_{\mu\nu}$.

Consider the action [16]

$$S = -\frac{1}{2} \cdot \int \mathrm{d}^4 x (-g)^{\frac{1}{2}} \left[\Phi(\Box\Phi + m^2 + \xi R)\Phi + \frac{\lambda\Phi^4}{4!} \right], \tag{1}$$

where $\Box$ is the De' Alemberts operator, m the mass of the field, ξ and λ are coupling constants and R, the scalar curvature. The corresponding equation of motion is

$$[\Box + m^2 + \xi R + \lambda\Phi^2]\Phi = 0 \tag{2}$$

and its energy-momentum tensor is [1]

$$\begin{aligned} T_{\mu\nu} = {} & (1-2\xi)\partial_\mu\Phi\partial_\nu\Phi + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu}\partial_\alpha\Phi\partial^\alpha\Phi - 2\xi\Phi\nabla_\mu\nabla_\nu\Phi \\ & + 2\xi g_{\mu\nu}\Phi\Box\Phi - \xi G_{\mu\nu}\Phi^2 + \frac{m^2}{2} g_{\mu\nu}\Phi^2 + \frac{\lambda}{4!} g_{\mu\nu}\Phi^4, \end{aligned} \tag{3}$$

where $G_{\mu\nu}$ is the Einstein tensor.

In terms of conformal time η, the RW metric is given by

$$\mathrm{d}s^2 = a(\eta)^2\left[\mathrm{d}\eta^2 - \frac{\mathrm{d}r^2}{(1-\kappa r^2)} - r^2\mathrm{d}\Omega\right], \tag{4}$$

where $\kappa = +1, 0, -1$ correspond to positive, zero or negative spatial curvature respectively. In the present work we consider the $\kappa = 0$, spatially flat case.

It is useful to break Φ into a mean field part ϕ and a part ψ representing quantum fluctuation about the mean field and thus has a vanishing expectation value, i.e., $\langle\psi\rangle = 0$. Hence, $\Phi = \phi + \psi$ and $\langle\Phi\rangle = \phi$. Equations of motion corresponding to the classical and quantum parts are

$$\left(\Box + m^2 + \xi R\right)\phi + \frac{\lambda\phi^3}{3!} + \frac{1}{2!}\lambda\phi\langle\psi^2\rangle = 0, \tag{5}$$

$$\left(\Box + m^2 + \xi R\right)\psi + \lambda\phi^2\psi = 0. \tag{6}$$

In a similar fashion, the expectation value of the energy-momentum tensor can be written as sum of classical and quantum parts. The classical part can be obtained by substituting ϕ for Φ in equation (3). Quantum part is given by

$$\begin{aligned}\langle T_{\mu\nu}\rangle^Q &= (1-2\xi)\langle\partial_\mu\psi\partial_\nu\psi\rangle + (2\xi-1)g_{\mu\nu}\langle\partial^\alpha\psi\partial_\alpha\psi\rangle - 2\xi\langle\psi\nabla_\mu\nabla_\nu\psi\rangle \\ &+2\xi g_{\mu\nu}\langle\psi\Box\psi\rangle - \xi G_{\mu\nu}\langle\psi^2\rangle + \frac{m^2}{2}g_{\mu\nu}\langle\psi^2\rangle + \frac{\lambda}{4!}g_{\mu\nu}\phi^2\langle\psi^2\rangle.\end{aligned} \tag{7}$$

The 0–0 component of $\langle T_{\mu\nu}\rangle^Q$ is

$$\begin{aligned}\langle T_0^0\rangle &= (1-2\xi)\left\langle\left(\frac{\partial\psi}{\partial t}\right)^2\right\rangle + \left(2\xi - \frac{1}{2}\right)\left\langle\left(\frac{\partial\psi}{\partial t}\right)^2\right\rangle - 2\xi\left\langle\psi\frac{\partial^2\psi}{\partial t^2}\right\rangle \\ &+ \frac{2\xi}{a^2}\left\langle\psi\left(\frac{\partial^2\psi}{\partial t^2} + \frac{2\dot{a}}{a}\frac{\partial\psi}{\partial t}\right)\right\rangle + \left[\frac{3\xi}{a^2}\left(\frac{\dot{a}^2}{a^2}\right) + \frac{m^2}{2} + \frac{\lambda\phi^2}{4}\right]\langle\psi^2\rangle,\end{aligned} \tag{8}$$

where $\dot{a}$ denotes differentiation of a with respect to t. By the conformal transformation $t = a\eta$, where η is called the conformal time,

$$\langle T_\eta^\eta\rangle = \frac{1}{2a^2}\left\langle\left(\frac{\partial\psi}{\partial\eta}\right)^2\right\rangle + 6\xi\frac{a'}{a^3}\left\langle\psi\frac{\partial\psi}{\partial\eta}\right\rangle + \left[\frac{m^2}{2} + \frac{\lambda\phi^2}{4} + \frac{3\xi}{a^2}\left(\frac{a'^2}{a^2}\right)\right]\langle\psi^2\rangle, \tag{9}$$

where $'$ denotes differentiation with respect to η. Let the mode expansion of $\psi(x)$ be [1,4,11,16]

$$\psi(x) = \frac{1}{a(\eta)}\int\mathrm{d}\mu(k)\left[a_k y_k(x)\chi_k(\eta) + \mathrm{h.c.}\right], \tag{10}$$

where $\chi_k(\eta)$ satisfies the equation

$$\chi_k''(\eta) + \left[\omega_k^2 + \frac{\lambda a^2 \phi^2}{2} + \left(\xi - \frac{1}{6}\right) a^2 R\right] \chi_k(\eta) = 0. \tag{11}$$

The solution to the above equation can be expressed in the WKB form as [1,2]

$$\chi_k(\eta) = \frac{1}{\sqrt{2w(\eta)}} \exp\left[-i \int w(\eta) \mathrm{d}\eta\right], \tag{12}$$

where

$$w(\eta) = \left[\omega_k^2 + \frac{\lambda a^2 \phi^2}{2} + \left(\xi - \frac{1}{6}\right) a^2 R\right]^{1/2}$$

and also

$$\omega_k^2 = k^2 + m^2 a^2.$$

In the case of spatially flat RW metric,

$$y_k(x) = \frac{\mathrm{e}^{ikx}}{(2\pi)^{3/2}}$$

Using eqs (9) and (10) we get the expression for $\langle T_\eta^\eta \rangle^Q$ as

$$\langle T_\eta^\eta \rangle^Q = \frac{B^2}{16} \log B - \frac{B^2}{16} \log 2 + \frac{a' A}{4a}(1 + \log 2)$$
$$+ \frac{1}{4}(B \log 2) P - \frac{3}{2} \xi \frac{a'}{a} A (1 + \log 2), \tag{13}$$

where

$$P = \left[\frac{m^2 a^2}{2} + \frac{\lambda a^2 \phi^2}{2} - 3\xi \left(\frac{a'^2}{a^2}\right) + \frac{a'^2}{2a^2}\right], \tag{14}$$

$$B = \left[m_r^2 a^2 + \frac{\lambda_r a^2 \phi^2}{2} + \left(\xi_r - \frac{1}{6}\right) a^2 R\right], \tag{15}$$

$$A = \left[m_r^2 + \frac{1}{2} \lambda_r \phi^2 + \left(\xi_r - \frac{1}{6}\right) R\right]. \tag{16}$$

To get the expression for the renormalized energy-momentum tensor, we need to 'dress' the 'bare' parameters m, ξ and λ. From the classical equation of motion, we get [4]

$$\frac{\partial V_{\text{eff}}}{\partial \phi} = \left[\left(m_r^2 + \delta m^2\right) + (\xi_r + \delta \xi) R\right] \phi$$
$$+ (\lambda_r + \delta \lambda) \frac{1}{3!} \phi^3 + (\lambda_r + \delta \lambda) \frac{1}{2!} \phi \langle \psi^2 \rangle, \tag{17}$$

where δm^2, $\delta \xi$ and $\delta \lambda$ are renormalization counter-terms.

The renormalized parameters are given by

$$m_r^2 = \left(\frac{\partial^2 V_{\rm eff}}{\partial\phi^2}\right)_{R=\phi=0}, \tag{18}$$

$$\xi_r = \left(\frac{\partial^3 V_{\rm eff}}{\partial\phi^2\partial R}\right)_{R=\phi=0}, \tag{19}$$

$$\lambda_r = \left(\frac{\partial^4 V_{\rm eff}}{\partial\phi^4}\right)_{R=\phi=0}. \tag{20}$$

Evaluation of the renormalization counter-terms gives

$$\delta m^2 = \frac{-(\lambda_r+\delta\lambda)}{16\pi a^2}\left[\frac{\Lambda^2}{2}+\frac{m_r^2a^2}{4}+\frac{m_r^2a^2}{4}\log(m_r^2a^2)-m_r^2a^2\log 4+2\log\Lambda\right], \tag{21}$$

$$\delta\xi = \frac{-(\lambda_r+\delta\lambda)(\xi_r-\frac{1}{6})}{16\pi}\left[\frac{1}{2}+\frac{1}{4}\log(m_r^2a^2)-\frac{1}{4}(\log 4)+2\log\Lambda\right] \tag{22}$$

and

$$\delta\lambda = \frac{-3\lambda_r^2(1+\log m_ra)}{32\left(1+\frac{3\lambda_r}{32}\log m_ra\right)}, \tag{23}$$

where we have put a momentum cut-off Λ to regularize the k integration. Using (21)–(23) in (13), the final expression for the renormalized $\langle T_\eta^\eta\rangle^Q$ is obtained as

$$\langle T_\eta^\eta\rangle^Q = \frac{B^2\log(\frac{B}{2})}{16}+\frac{a'A}{4a}(1+\log 2)+\frac{1}{4}BP^{\rm ren}(\log 2) -\frac{3}{2}\frac{a'A}{a}(1+\log 2)\left[\xi_r-\frac{(\xi_r-\frac{1}{6})\frac{\lambda_r^2}{\left[1+\frac{3}{32}\log m_ra\right]}\left[\frac{1}{2}+\frac{1}{4}\log(m_r^2a^2)-\frac{1}{4}(\log 4)\right]}{16\pi}\right], \tag{24}$$

where B and A are defined by (15) and (16) and

$$P^{\rm ren} = \left[m_r^2-\frac{\frac{1}{16\pi a^2}\frac{\lambda_r^2}{\left[1+\frac{3}{32}\log m_ra\right]}}{1}+\lambda_r+\lambda_r^2\frac{\phi^2}{\left(1+\frac{3\lambda_r}{32}\log m_ra\right)}\right]\frac{a^2}{2}+\frac{a'^2}{2a^2} +3\left[\xi_r-\left(\xi_r-\frac{1}{6}\right)\frac{\lambda_r^2\left(\frac{1}{2}+\frac{1}{2}\log m_ra-\frac{1}{4}\log 4\right)}{16\pi\left[1+\frac{3}{32}\log m_ra\right]}\right]\left(\frac{a'^2}{a^2}\right). \tag{25}$$

The above expression gives the divergenceless energy density of the scalar field with quartic self-coupling in a spatially flat RW metric. Now we move on to the value of the trace.

We adopt a similar treatment to evaluate the trace of the energy-momentum tensor. Our aim is to find

$$\langle T\rangle^Q = \langle T_\eta^\eta\rangle^Q + \langle T_1^1\rangle^Q + \langle T_2^2\rangle^Q + \langle T_3^3\rangle^Q.$$

Assuming spatial isotropy, and using

$$G_\mu^\mu = -\left(\frac{6a''}{a^3} + \frac{6\kappa}{a^2}\right),$$

we get

$$\langle T\rangle^Q = \frac{-3I}{32\pi^2 a^2}\left[1 + 4\xi\left(1 - \frac{2}{a^2}\right)\right] - \xi G_\mu^\mu + \left(\frac{m^2}{2} + \frac{\lambda\phi^2}{4!}\right) 3\langle\psi^2\rangle + \langle T_\eta^\eta\rangle^Q, \tag{26}$$

where

$$I = \int_0^\infty \frac{k^4 \mathrm{d}k}{\sqrt{k^2 + B}}.$$

Substituting from (24) for $\langle T_\eta^\eta\rangle$, the final expresion for the quantum part of the trace is obtained as [17]

$$\begin{aligned}
\langle T\rangle^Q = {}& 3B^2 \frac{\log 2}{4}\left[1 + \frac{4(1 - \frac{2}{a^2})}{16\pi}\left(\frac{\lambda_r^2}{1 + \frac{3}{32}\log m_r a}\right)\left(\frac{1}{2} + \frac{\log m_r^2 a^2}{4} - \frac{1}{4}\log 4\right)\right] \\
& - \left[\xi_r - \frac{(\xi_r - \frac{1}{6})\lambda_r^2}{16\pi\left(1 + \frac{3}{32}\log m_r a\right)}\left(\frac{1}{2} + \frac{\log m_r^2 a^2}{4} - \frac{1}{4}\log 4\right)\right]\left(\frac{-6a''}{a^3}\right) \\
& + \frac{3}{2}\left[m_r^2 - \frac{\lambda_r^2\{\frac{m_r^2 a^2}{4} + \frac{m_r^2 a^2 \log m_r^2 a^2}{4} - m_r^2 a^2 \log 4\}}{16\pi a^2\left(1 + \frac{3}{32}\log m_r a\right)}\right]\frac{B}{32\pi a^2}(1 + \log B) \\
& + \frac{3}{4!}\left[\lambda_r^2 \phi^2 \frac{1}{\left(1 + \frac{3}{32}\log m_r a\right)}\right]\frac{B}{32\pi a^2}(1 + \log B) \\
& + \frac{B^2 \log B}{16} - \frac{B^2 \log 2}{16} + \frac{a'A}{4a}(1 + \log 2) + \frac{1}{4}BP^{\mathrm{ren}}(\log 2) \\
& - \frac{3}{2}\frac{a'A}{a}(1 + \log 2)\left[\xi_r - \frac{(\xi_r - \frac{1}{6})\frac{\lambda_r^2}{\left[1 + \frac{3}{32}\log m_r a\right]}\left[\frac{1}{2} + \frac{1}{4}\log(m_r^2 a^2) - \frac{1}{4}(\log 4)\right]}{16\pi}\right],
\end{aligned} \tag{27}$$

where B and A are defined by eqs (15) and (16) and P^{ren} is defined by eq. (25). Thus we have got the exact expressions for the $\eta - \eta$ component and trace of the energy-momentum tensor, which completely specifies the energy-momentum tensor due to spatial isotropy.

In the case of conformal coupling, $\xi_r = \frac{1}{6}$, using (24),

$$\begin{aligned}
\langle T_\eta^\eta\rangle^Q = {}& \frac{B_1^2 \log(\frac{B}{2})}{16} + \frac{a'A_1}{4a}(1 + \log 2) + \frac{1}{4}B_1 P_1^{\mathrm{ren}}(\log 2) \\
& - \frac{3}{12}\frac{a'A_1}{a}(1 + \log 2)
\end{aligned} \tag{28}$$

and (27) gives

$$\langle T\rangle^Q = 3B_1^2\frac{\log 2}{4}\left[1+\frac{4(1-\frac{2}{a^2})}{16\pi}\left(\frac{\lambda_r^2}{1+\frac{3}{32}\log m_r a}\right)\left(\frac{1}{2}+\frac{\log m_r^2a^2}{4}-\frac{1}{4}\log 4\right)\right]+\left(\frac{a''}{a^3}\right)$$
$$+\frac{3}{2}\left[m_r^2-\frac{\lambda_r^2\{\frac{m_r^2a^2}{4}+\frac{m_r^2a^2\log m_r^2a^2}{4}-m_r^2a^2\log 4\}}{16\pi a^2\left(1+\frac{3}{32}\log m_r a\right)}\right]\frac{B_1}{32\pi a^2}(1+\log B_1)$$
$$+\frac{3}{4!}\left[\frac{\lambda_r^2\phi^2}{\left(1+\frac{3}{32}\log m_r a\right)}\right]\frac{B_1}{32\pi a^2}(1+\log B_1)$$
$$+\frac{B_1^2\log B_1}{16}-\frac{B_1^2\log 2}{16}+\frac{a'A_1}{4a}(1+\log 2)$$
$$+\frac{1}{4}B_1P_1^{\rm ren}(\log 2)-\frac{3}{12}\frac{a'A_1}{a}(1+\log 2), \tag{29}$$

where

$$B_1 = \left[m_r^2a^2+\frac{\lambda_r a^2\phi^2}{2}\right], \tag{30}$$

$$A_1 = \left[m_r^2+\frac{1}{2}\lambda_r\phi^2\right], \tag{31}$$

$$P_1^{\rm ren} = \left[m_r^2-\frac{1}{16\pi a^2}\frac{\lambda_r^2}{\left[1+\frac{3}{32}\log m_r a\right]}+\lambda_r+\lambda_r^2\frac{\phi^2}{\left(1+\frac{3\lambda_r}{32}\log m_r a\right)}\right]\frac{a^2}{2}+\frac{a'^2}{a^2}. \tag{32}$$

Equations (28) and (29) completely specify the energy-momentum tensor in the case of conformal coupling.

Next we consider the case of minimal coupling, $\xi_r = 0$. In this case, the components of the energy-momentum tensor are obtained as

$$\langle T_\eta^\eta\rangle^Q = \frac{B_2^2\log B_2}{16}-\frac{B_2^2\log 2}{16}+\frac{a'A_2}{4a}(1+\log 2)+\frac{1}{4}B_2P_2^{\rm ren}(\log 2)$$
$$-\frac{3}{2}\frac{a'A_2}{a}(1+\log 2)\left[\frac{\frac{1}{6}\frac{\lambda_r^2}{\left[1+\frac{3}{32}\log m_r a\right]}\left[\frac{1}{2}+\frac{1}{4}\log(m_r^2a^2)-\frac{1}{4}(\log 4)\right]}{16\pi}\right] \tag{33}$$

and

$$\langle T\rangle^Q = 3B_2^2\frac{\log 2}{4}\left[1+\frac{4(1-\frac{2}{a^2})}{16\pi}\left(\frac{\lambda_r^2}{1+\frac{3}{32}\log m_r a}\right)\left(\frac{1}{2}+\frac{\log m_r^2a^2}{4}-\frac{1}{4}\log 4\right)\right]$$
$$+\left[\frac{(\xi_r-\frac{1}{6})\lambda_r^2}{16\pi\left(1+\frac{3}{32}\log m_r a\right)}\left(\frac{1}{2}+\frac{\log m_r^2a^2}{4}-\frac{1}{4}\log 4\right)\right]\left(\frac{-6a''}{a^3}\right)$$

$$+\frac{3}{2}\left[m_r^2-\frac{\lambda_r^2\{\frac{m_r^2a^2}{4}+\frac{m_r^2a^2\log m_r^2a^2}{4}-m_r^2a^2\log 4\}}{16\pi a^2\left(1+\frac{3}{32}\log m_r a\right)}\right]\frac{B_2}{32\pi a^2}(1+\log B_2)$$
$$+\frac{3}{4!}\left[\lambda_r^2\phi^2\frac{1}{\left(1+\frac{3}{32}\log m_r a\right)}\right]\frac{B_2}{32\pi a^2}(1+\log B_2)$$
$$+\frac{B_2^2\log B_2}{16}-\frac{B_2^2\log 2}{16}+\frac{a'A_2}{4a}(1+\log 2)+\frac{1}{4}BP_2^{\text{ren}}(\log 2)$$
$$-\frac{3}{12}\frac{a'A_2}{a}(1+\log 2)\left[\frac{\frac{\lambda_r^2}{\left[1+\frac{3}{32}\log m_r a\right]}\left[\frac{1}{2}+\frac{1}{4}\log(m_r^2a^2)-\frac{1}{4}(\log 4)\right]}{16\pi}\right], \tag{34}$$

where

$$B_2=\left[m_r^2a^2+\frac{\lambda_r a^2\phi^2}{2}-\frac{1}{6}a^2R\right], \tag{35}$$

$$A_2=\left[m_r^2+\frac{1}{2}\lambda_r\phi^2-\frac{1}{6}R\right], \tag{36}$$

$$P_2^{\text{ren}}=\left[m_r^2-\frac{1}{16\pi a^2}\frac{\lambda_r^2}{\left[1+\frac{3}{32}\log m_r a\right]}+\lambda_r+\lambda_r^2\frac{\phi^2}{\left(1+\frac{3\lambda_r}{32}\log m_r a\right)}\right]\frac{a^2}{2}+\frac{a'^2}{2a^2}$$
$$+\frac{1}{2}\left[\frac{\lambda_r^2\left(\frac{1}{2}+\frac{1}{2}\log m_r a-\frac{1}{4}\log 4\right)}{16\pi\left[1+\frac{3}{32}\log m_r a\right]}\right]\left(\frac{a'^2}{a^2}\right). \tag{37}$$

3. Conclusion

We have computed a finite expression for the energy-momentum tensor of a quartically self-coupled scalar field of arbitrary mass and coupling with the spatially flat Robertson–Walker space-time using the method of effective potential renormalization. We have got finite expressions for η–η component and the trace of the energy-momentum tensor. As special cases, we have deduced the same if the external coupling with the gravitational field is minimal ($\xi=0$) or conformal ($\xi=\frac{1}{6}$) in nature. The trace of the energy-momentum tensor when evaluated gives non-zero vacuum expectation value. This in QFT is called trace anomaly, which has wide implications [1–3].

In the case of free scalar field, we can make the vacuum expectation value of the Hamiltonian zero, by normal ordering. But here the scalar field is having $\lambda\phi^4$ self-interaction as well as arbitrary coupling with the external metric. So, here we employ renormalization to get a finite expression out of the divergent one. Still we find the vacuum expectation value to be non-zero. This can be related to creation of particles in the curved space-time [1–3,12].

Acknowledgements

One of the authors (KGA) acknowledges the use of library and computational facilities of IUCAA Reference Centre, Cochin. MJ would like to thank UGC, New Delhi for financial support in the form of SRF. VCK acknowledges Associateship of IUCAA, Pune.

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