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Spinodal decomposition: An alternate mechanism of phase conversion

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Abstract. The scenario of homogeneous nucleation is investigated for a first-order quark–hadron phase transition in a rapidly expanding background of quark gluon plasma. It is found that significant supercooling is possible before hadronization begins. This study also suggests that spinodal decomposition competes with nucleation and may provide an alternative mechanism for phase conversion.

Keywords. Phase transition; nucleation; hydrodynamics.

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1. Introduction

If quark gluon plasma (QGP) is formed in relativistic heavy-ion collisions, then a phase transition from QGP to normal hadron matter must take place during some time of its expansion. Below critical temperature $T_{\rm c}$ (supercooled), the QGP phase becomes metastable and phase transition can be initiated by the nucleation of critical size hadron bubbles in a homogeneous QGP phase. The amount of supercooling depends on the strength of the transition being more for a strong first-order transition. As the system supercools, the barrier separating the metastable QGP phase from the stable hadron phase decreases and the system may reach the point of spinodal instability $T_{\rm s}$, at which the barrier vanishes. Hence, for a strong enough supercooling, the spinodal decomposition may provide an alternate path for phase conversion. In the case of homogeneous nucleation, the system requires to cool up to a temperature $T_{\rm m}$ at which appreciable nucleation is reached and system will be reheated due to the release of latent heat. But if the temperature $T_{\rm m}$ is less than $T_{\rm s}$ then the phase transition may proceed via spinodal decomposition.

For the case of QGP produced in heavy-ion collision, the route to hadronization either through nucleation or through spinodal decomposition depends sensitively on several factors like nucleation rate and expansion scenario. In this work we study the amount of supercooling and rate of hadronization by solving self-consistently the nucleation rate along with hydrodynamic equation corresponding to both longitudinal and spherical expansion.

2. Spinodal and supercooling temperature

As the order as well as the strength of the quark–hadron phase transition is still an unsettled issue, we consider a more generic form of the potential which covers a wide range from very strong to very weak first-order phase transition [1]

$$V(\phi,T) = a(T)\,\phi^2 - b\,T\,\phi^3 + c\,\phi^4, \tag{1}$$

where b and c are positive constants which can be expressed in terms of surface tension σ and correlation length ξ. The requirement that at all temperatures, the difference between the two minima should be equal to the pressure difference between the two phases fixes the third parameter $a(T)$. The spinodal temperature T_{s} where the quark phase becomes unstable and there exists only one minimum corresponding to hadron phase, can be obtained as [2]

$$T_{\mathrm{s}} = \left[\frac{B}{B + 81\sigma/16\xi}\right]^{1/4} T_{\mathrm{c}}, \tag{2}$$

where B is the bag constant. If the QGP supercools up to this point, it will become unstable and may go to hadron phase by spinodal decomposition. For a strong enough transition, σ/ξ is large and T_{s} is low as compared to the case when the transition is weak. We are interested to know whether the system cools down to the temperature T_{s}. For comparison, we denote the minimum temperature reached during the supercooling as T_{m}. Both T_{s} and T_{m} depend on the strength of the transition and need to be evaluated properly. While T_{s} can be estimated directly from eq. (2), T_{m} requires a self-consistent solution of a set of equations involving the nucleation rate and energy momentum conserving hydrodynamical equations as described in ref. [2]

The evolution of the energy density is given by [3]

$$\frac{\mathrm{d}e}{\mathrm{d}\tau} + \frac{D\,\omega}{\tau} = \frac{4\eta/3 + \zeta}{\tau^2}, \tag{3}$$

where $D = 1$ for the expansion in $(1+1)$ dimension. The factors η and ζ are the shear and the bulk viscosity of the medium. For non-viscous plasma (for zero viscosity), the above equation follows the Bjorken's scaling solution where $T^3\tau$ is a constant. The energy momentum equation needs to be solved numerically for expansion in $(3+1)$ dimensions. However, retaining the simplicity, we can still use eq. (3) for spherical expansion [4], with the choice of $D = 3$.

3. Results and discussions

The critical temperature is fixed at $T_{\mathrm{c}} = 160$ MeV. The correlation length is fixed at $\xi = 0.7$ fm and σ is treated as a free parameter in the present study. The time τ_{c} that the plasma takes to cool down to $T = T_{\mathrm{c}}$ will depend on the initial conditions and on the expansion dynamics. We assume it of the order of 4 to 8 fm/c. From τ_{c} onwards, we consider longitudinal expansion if it is 4 fm/c and spherical expansion if it is between 6 to 8 fm/c.

Figure 1 shows a plot of $T_{\mathrm{m}}/T_{\mathrm{c}}$ and $T_{\mathrm{s}}/T_{\mathrm{c}}$ as a function of strength of the transition, σ/ξ, for different values of viscosity coefficient η_q at $\tau_{\mathrm{c}} = 6$ fm/c and 8 fm/c respectively in left and right panels. First consider the case of non-dissipative plasma with viscosity $\eta_q = 0$. The curves T_{m} and T_{s} show a cross-over point as the transition becomes stronger (large σ/ξ). For weak enough transition, T_{m} is well above the spinodal temperature T_{s}.

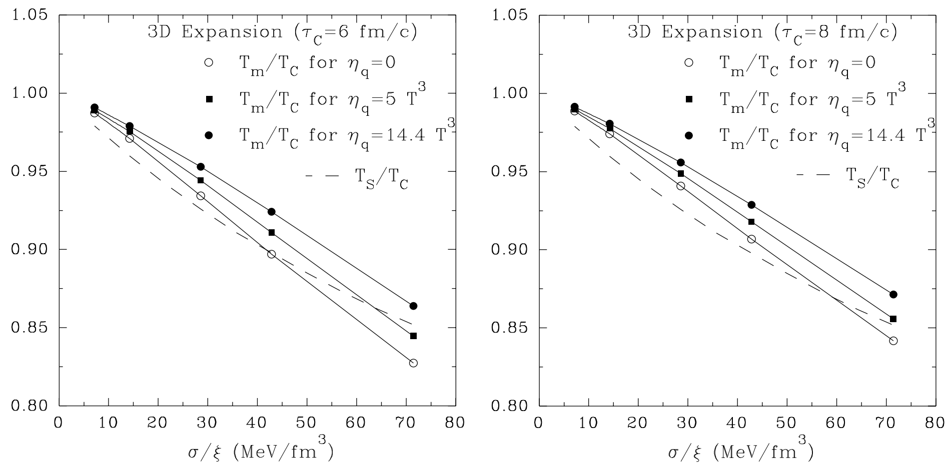

Figure 1. The spinodal temperature T_s/T_c and minimum temperature T_m/T_c reached during supercooling.

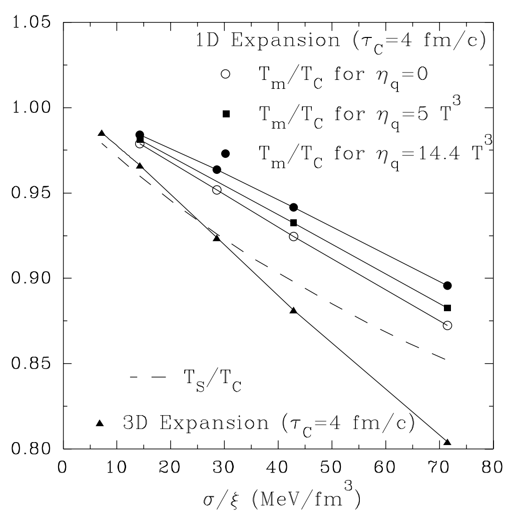

Figure 2. The spinodal temperature T_s/T_c and minimum temperature T_m/T_c reached during supercooling.

Since T_m depends on the expansion rate of the medium, the cross-over point will sensitively depend on τ_c; moving towards right for the slower expansion (higher τ_c). If the plasma is viscous (non-zero η_q), the T_m is increased further due to the slow evolution of the medium. Even the cross-over point also shifts towards right showing that the nucleation is the dominant mechanism over a wide range of σ/ξ ratios. Figure 2 shows a similar plot for a longitudinally expanding system. For comparison the calculation for spherical expansion is also shown which gives the lower limit on T_m. One can see that for longitudinal expansion, the system remains far from any spinodal instability.

Similar work has been carried out by Scavenious *et al* [5], using linear sigma model. We use bag model equation of state throughout our work and also incorporate a more relevant expansion scenario.

4. Conclusions

Although the above study depends on the choice of the parameters, a general observation is that for strong enough transition (large σ/ξ) with zero or very small amount of viscosity, $T_\mathrm{m} < T_\mathrm{s}$, the system reaches the spinodal instability before the amount of nucleated hadron bubbles become significant to begin phase conversion. The phase conversion in such a case will proceed via spinodal decomposition. If the medium is viscous or the transition is weak enough or both, the supercooling is much less and $T_\mathrm{m} > T_\mathrm{s}$. The phase conversion may still proceed through homogeneous nucleation. However, depending on the range of parameters, there could be a competition between the homogeneous nucleation and the spinodal decomposition as the nucleation and expansion time scales for relativistic heavy-ion collisions are comparable.

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