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# Quantum chromodynamics phase transition in the early Universe and quark nuggets

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**Abstract.** A first-order quark hadron phase transition in the early Universe may lead to the formation of quark nuggets. The baryon number distribution of these quark nuggets have been calculated and it has been found that there are sizeable number of quark nuggets in the stable sector. The nuggets can clump and form bigger objects in the mass range of $0.0003M_{\odot}$ to $0.12M_{\odot}$. It has been discussed that these bigger objects can be possible candidates for cold dark matter.

**Keywords.** Quark nuggets; dark matter; massive astrophysical compact halo objects.

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## 1. Introduction

The present status of the so-called standard cosmological model suggests that there is an abundance of matter which is dark. This matter is non-luminous in the sense that they do not have any other interaction except the gravitational attraction. The present consensus (for a review, see [1,2]) based on recent experimental data is that the universe is flat ($\Omega \sim 1$) and that a sizeable amount of the dark matter is 'cold', i.e., non-relativistic at the time of decoupling. This cold dark matter accounts for about 35% of the budget. Speculations as to the nature of dark matter are numerous, often bordering on the exotic, and searches for such exotic matter is a very active field of astroparticle physics. In recent years, there has been experimental evidence [3,4] for at least one form of dark matter – the massive astrophysical compact halo objects (MACHO) – detected through gravitational microlensing effects proposed by Paczynski [5] some years ago. To date, there is no clear picture as to what these objects are made of.

A lot of effort has been made to study the MACHOs after their first discovery. Based on about 13–17 Milky Way halo MACHOs detected in the direction of the large magellanic cloud (LMC) (we are not addressing the events found toward the galactic bulge), the MACHOs are expected to be in the mass range (0.15–0.95)$M_{\odot}$, with the most probable mass being in the vicinity of 0.5$M_{\odot}$ [6], substantially higher than the fusion threshold of 0.08$M_{\odot}$. The MACHO collaboration suggests that the lenses are in the galactic halo. Assuming that they are subject to the limit on the total baryon number imposed by the big bang nucleosyntheis (BBN), there have been suggestions that they could be white dwarfs [7,8]. It is difficult to reconcile this with the absence of sufficient active progenitors of appropriate masses in the galactic halo. Moreover, recent studies have shown that these objects are unlikely to be white dwarfs, even if they were as faint as blue dwarfs, since this will violate some of the very well-known results of BBN [8]. There have also been suggestions [9–11] that they could be primordial black holes (PBHs) ($\sim 1M_{\odot}$), arising from horizon scale fluctuations triggered by pre-existing density fluctuations during the cosmic quark–hadron phase transition. This suggestion requires a fine tuning of the initial density perturbation and has been criticised in the literature by other authors [12]. Alternately, Evans *et al* [13] suggested that some of the lenses are stars in the Milky Way disk which lie along the line of sight to the LMC. Gyuk and Gates [14] examined a thick disk model, which would lower the lens mass estimate. Aubourg *et al* [15] suggested that the events could arise from self-lensing of the LMC. Zaritsky and Lin [16] have argued that the lenses are probably the evidence of a tidal tail arising from the interaction of LMC and the Milky Way or even a LMC–SMC (small magellanic cloud) interaction. These explanations are primarily motivated by the difficulty of reconciling the existence of MACHOs with the known populations of low mass stars in the galactic disks.

Adopting the viewpoint that the lensing MACHOs are indeed in the Milky Way halo, we propose that they have evolved out of the quark nuggets which could have been formed in a first-order cosmic quark–hadron phase transition, at a temperature of $\sim$100 MeV during the microsecond era of the early universe.

It is believed that the universe underwent a quark hadron phase transition after a few microseconds of the big bang. In 1984, Witten [17] proposed that a first-order cosmic quark–hadron phase transition at a critical temperature $T_{\rm c} \sim$ 100–200 MeV could lead to the formation of quark nuggets (QN), made of $u$, $d$ and $s$ quarks at a density somewhat larger than normal nuclear matter density. If these primordial QNs indeed survive till the present epoch, they could be a possible candidate for the baryonic component of the dark matter.

One of the crucial questions at this juncture is the survivability of these nuggets. The first study on this issue was addressed by Alcock and Farhi [18] who argued that even the nuggets with largest possible baryon number is unstable against neutron evaporation from the surface. Madsen *et al* [19] then pointed out that, initial neutron evaporation makes the surface layer deficient in $u$ and $d$ quarks and as a result further neutron evaporation is suppressed. They found that nuggets with initial baryon number $\geq 10^{46}$ would be stable. Madsen and Olsen [20] showed that nuggets are stable against boiling. Bhattacharjee *et al* [21] and Sumiyoshi and Kajino [22] used chromoelectric flux tube model to show that, depending on the parameters, nuggets with initial baryon number $10^{39}$–$10^{42}$ would be stable against surface evaporation.

However, the issue of baryon number distribution of quark nuggets have not been addressed in detail. The baryon number distribution of quark nuggets should reveal whether there are sizeable number of nuggets in the stable sector. This information is very important in the context of the candidature of these QNs as dark matter. In this article the baryon number distribution of the QNs will be calculated in §2. Section 3 will be devoted to the discussion on the evolution of these nuggets to form dark matter and §4 will contain a brief conclusion.

## 2. The baryon number distribution of quark nuggets

The evolution of the universe during the QCD phase transition is governed by Einstein's equations

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi\rho}{3m_{\rm pl}^2}, \quad \frac{{\rm d}(\rho R^3)}{{\rm d}t} + P\frac{{\rm d}R^3}{\rm d} = 0, \tag{1}$$

where $\rho$ is the energy density, $P$ the pressure and $m_{\rm pl}$ the Planck mass. In the above equation, $R$ is the cosmological scale factor in the Robertson–Walker space-time and is defined by the relation

$$\begin{aligned} {\rm d}s^2 &= -{\rm d}t^2 + R^2{\rm d}x^2 = R^2(-{\rm d}\xi^2 + {\rm d}x^2) \\ {\rm d}x^2 &= {\rm d}X^2 + X^2(\sin^2\theta{\rm d}\phi^2 + {\rm d}\theta^2) \end{aligned} \tag{2}$$

where $X$ is the coordinate radius, i.e. the radius in the unit of the cosmological scale factor $R(t)$.

It is well-known that in a first-order phase transition, the quark and the hadron phases co-exist in a mixed phase at the critical temperature of transition. Around the critical temperature, the universe consists of leptons, photons and the massless quarks, anti-quarks and gluons, described in our case by the MIT bag model with an effective degeneracy $g_{\rm q}(\sim 51.25)$. The hadronic phase contains relativistic $\pi$-mesons, photons and leptons with a small baryon content ($\rho_{\rm B}/\rho_\gamma \sim 10^{-10}$) and is described by an equation of state corresponding to massless particles with an effective degeneracy $g_{\rm h} = 17.25$. The energy densities and the pressures of hadronic and quark matter are given by

$$\begin{aligned} \rho_{\rm h} &= \frac{\pi^2 g_{\rm h}}{30}T^4, \quad \rho_{\rm q} = \frac{\pi^2 g_{\rm q}}{30}T^4 + B \\ P_{\rm h} &= \frac{\pi^2 g_{\rm h}}{90}T^4, \quad P_{\rm q} = \frac{\pi^2 g_{\rm q}}{90}T^4 - B \end{aligned} \tag{3}$$

where $B$ is the bag constant.

The evolution of the scale factor in the mixed phase is given by (see also [23])

$$\frac{R(t)}{R(t_i)} = \frac{\left[\cos\left(\tan^{-1}\sqrt{3r} - \sqrt{\frac{3}{r-1}}(t-t_i)/t_{\rm c}\right)\right]^{2/3}}{\left[\cos\left(\tan^{-1}\sqrt{3r}\right)\right]^{2/3}}. \tag{4}$$

In the process we also get the volume fraction of the quark matter $f(t)$ in the mixed phase as

$$f(t) = \frac{1}{3(r-1)}\left[\tan\left\{\tan^{-1}\sqrt{3r} - \sqrt{\frac{3}{r-1}}\frac{t-t_i}{t_{\rm c}}\right\}\right]^2 - \frac{1}{r-1} \tag{5}$$

where $r \equiv \rho_q/\rho_h$, $t_c = \sqrt{3m_{pl}^2/8\pi B}$ is the characteristic time-scale for the QCD phase transition in the early universe and $t_i$ is the time when phase transition starts. The characteristic time-scale depends on the bag constant and hence on the critical temperature of the quark–hadron phase transition ($T_c$). In fact $t_c = 144$ $\mu$s for $T_c = 100$ MeV and $t_c = 64$ $\mu$s for $T_c = 150$ MeV.

In the coexisting phase, the temperature of the universe remains constant at $T_c$, the cooling due to expansion being compensated by the liberation of the latent heat. In the usual picture of bubble nucleation in a first-order phase transition scenario hadronic matter starts appearing as individual bubbles. With the progress of time, more and more hadronic bubbles form, coalesce and eventually percolate to form an infinite network of hadronic matter which traps the quark matter phase into finite domains. The time when the percolation takes place is usually referred to as the percolation time $t_p$, determined by a critical volume fraction $f_c$ ($f_c \equiv f(t_p)$) of the quark phase.

In an ideal first-order phase transition, the fraction of the high temperature phase decreases from the critical value $f_c$, as these domains shrink. For the QCD phase transition, however, these domains could become QNs and as such, we may assume that the lifetime of the mixed phase $t_f \sim t_p$.

As mentioned above, just after percolation one can have pockets of quark matter trapped as bubbles in the ambient hadronic matter. The probability that a spherical region of coordinate radius $X$ at time $t_p$ with nucleation rate $I(t)$ lies completely within the quark matter domain is given by [24,25]

$$P(X,t_p) = \exp\left[-\frac{4\pi}{3}\int_{t_i}^{t_p} dt I(t) R^3(t)\left(X + X(t_p,t)\right)^3\right] \tag{6}$$

where $X(t_p;t)$ is the coordinate radius of a bubble, at time $t_p$, which was nucleated at time $t$.

For convenience, let us now define a new set of variables $z = XR(t_i)/vt_c$, $x = t/t_c$ and $r(x) = R(x)/R(x_i)$ where $v$ is the radial growth velocity of the nucleating bubbles. Then

$$P(z,x_p) = \exp\left[-\frac{4\pi}{3}v^3 t_c^4 \int_{x_i}^{x_p} dx I(x)\left(zr(x) + y(x_p,x)\right)^3\right] \tag{7}$$

where

$$y(x,x') = \int_{x'}^{x} r(x')/r(x'')dx''. \tag{8}$$

So the fraction of quark matter present at time $t_p$ is

$$f_c = P(0,x_p) = \exp\left[-\frac{4\pi}{3}v^3 t_c^4 \int_{x_i}^{x_p} dx I(x) y^3(x_p,x)\right]. \tag{9}$$

Let us now look at the size distribution of the trapped quark matter domain (TQMD). In order to do so we will follow the procedure of ref. [24]. The difference of our work from that of Kodama *et al* is that we have considered exactly spherical nuggets whereas they have included a deformation factor. It should however be noted that the deformation factor, as found by Kodama *et al*, is small. Moreover, due to the presence of non-zero surface tension in the case of QCD phase transition the bubbles are likely to be spherical.

Even more importantly, we focus our attention to the percolation time $t_{\rm p}$ when the hadronic matter forms the ambient background. All these considerations allow us to consider the false vacuum domains (the quark phase) as being spherical in shape. Following ref. [24], let us assume that $F(X;t)\mathrm{d}X$ is the number of TQMDs per unit volume within the size $\{X, X+\mathrm{d}X\}$ at time $t$. Then $P(X,t)$ can be thought to be the probability that a QN of coordinate radius $X$ at a fixed position is contained in a TQMD. Now a TQMD of size $\eta$ can contain such a sphere of size $X$ only when the center of TQMD lies within the coordinate radius $\eta - X$ from the center of the sphere. If $\alpha$ is the minimum size of a TQMD, i.e., $F(X,t)$ vanishes for $X < \alpha$, then one can write

$$P(X;t) = \int_{\alpha+X}^{\infty} \frac{4\pi}{3}(\eta - X)^3 F(\eta;t)\mathrm{d}\eta. \tag{10}$$

The distribution function vanishes for $X < \alpha$. One can now solve the above equation using Laplace transformation to obtain $F(X)$ (for a detailed calculation see refs [25,26])

The result, in terms of $z$, is

$$\begin{aligned} F(z) = {} & \frac{3\theta(z-\alpha)R(t_i)^4}{4\pi\alpha^3 v^4 t_{\rm c}^4}\left[-P'(X-\alpha) - \frac{3P(X-\alpha)}{\alpha}\right. \\ & \left. + \frac{1}{\alpha^2}\int_0^{\infty}\mathrm{d}\eta P(\eta + X - \alpha)\left\{\lambda \mathrm{e}^{(-\lambda\eta/\alpha)} + \omega \mathrm{e}^{(-\omega\eta/\alpha)} + \bar{\omega}\mathrm{e}^{(-\omega\eta/\alpha)}\right\}\right] \\ = {} & \frac{R(t_i)^4}{v^4 t_{\rm c}^4} f(z). \end{aligned} \tag{11}$$

The solution of the equation $F(\alpha) = 0$ gives the minimum size of the quark nugget $\alpha$. Now, the number of nuggets per unit volume is given by

$$n_{\rm Q} = R^{-3}(t_{\rm p})\int_{\alpha}^{\infty} F(X)\mathrm{d}X = R^{-3}(t_{\rm p})\int_{\alpha}^{\infty}\frac{R^3(t_i)}{v^3 t_{\rm c}^3} f(z)\mathrm{d}z. \tag{12}$$

The volume of each quark nugget is given by $\frac{4}{3}\pi(zvt_{\rm c})^3$. Since the visible baryon constitutes only 10% of the closure density ($\Omega_{\rm B} = 0.1$ from standard big bang nucleosynthesis), a total of $10^{51}$ baryons will close the universe baryonically at $T = 100$ MeV. We emphasize at this point that these QNs would not disturb the standard primordial nucleosynthesis results to any considerable extent, as they would not participate in usual nuclear reactions. Therefore, if we assume that the total baryon content of the dark matter is carried by the quark nuggets, then

$$N_{\rm B} = 10^{51}(100/T(\mathrm{MeV}))^3 = V_{\rm H}\frac{4\pi R^3(t_i)}{3R^3(t_{\rm p})}\rho_{\rm B}\int_{\alpha}^{\infty} f(z)z^3\mathrm{d}z \tag{13}$$

where $V_{\rm H}$ is the horizon volume and $\rho_{\rm B}$ is the baryon density inside each nugget. We now solve the above equations self-consistently to obtain $\alpha$, $t_{\rm p}$ and $f_{\rm c}$. These values are then used to study the size distribution of the quark nuggets.

To calculate the distribution of QNs we need to know the rate of nucleation during the phase transition process. We use, in this article, the rate proposed by Cottingham *et al* [27] which is based on the Lee–Wick model of effective QCD. They calculated the Lee–Wick potential at finite temperature to obtain the following nucleation rate.

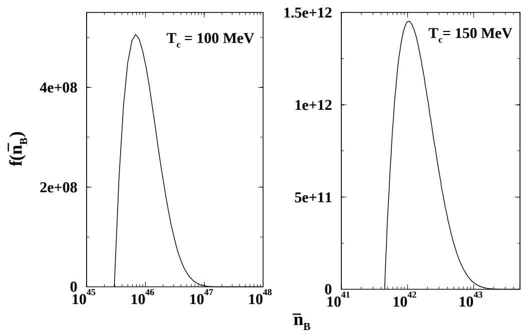


**Figure 1.** Distribution of QN, $f(\bar{n}_{\rm B})$, as a function of $\bar{n}_{\rm B}$ using nucleation rate proposed by Cottingham *et al* [27].

$$I(t) = T^4 \left(\frac{S}{2\pi T}\right)^{3/2} \exp\left(-S/T\right) \tag{14}$$

with

$$S = \frac{2\pi}{3}\left(\frac{m\sigma_0^2}{3}\right)^3 \frac{1}{P^2}$$
$$P = \frac{7\pi^2}{30}\left(T_{\rm c}^4 - T^4\right) \tag{15}$$

where $\sigma_0 = 100$ MeV and $m = 939$ MeV.

The baryon number distribution that we find is given in figure 1.

Let us now discuss the results obtained so far. In figure 1 we have plotted the distribution of QN, $f(\bar{n}_{\rm B})$, as a function of $\bar{n}_{\rm B}$ using the nucleation rate proposed by Cottingham *et al*, for different values of $T_{\rm c}$, where $\bar{n}_{\rm B}$ is the baryon number content of a single QN. We see that for $T_{\rm c} = 100$ MeV, distribution of QN peaks at baryon number $\sim 7\times 10^{45}$ and there is almost no QN with baryon number larger than $10^{47}$. For $T_{\rm c} = 150$ MeV, these values are $10^{42}$ and $10^{43}$ respectively. So one can conclude that there are large number of quark nuggets in the stable sector.

## 3. Quark nuggets as dark matter

In the previous section the baryon number distribution of the nuggets have been discussed. It has been seen that there are sizeable number of nuggets in the stable sector. These QNs behave like very unusual cold dark matter objects. Their enormous mass ($\sim 10^{44}$ GeV) and large size ($R_{\rm N} \sim 1$ m) force them to be almost static objects in the coordinate space. Even if they continue to be in kinetic equilibrium due to the radiation pressure (photons and neutrinos) acting on them, their velocity would be extremely non-relativistic. Also their mutual separation would be considerably larger than their radii; for example, at $T \sim 100$ MeV, the mutual separation between the QNs is (of size $\sim 10^{44}$ baryons) estimated to be around $\sim 300$ m. It is then obvious that the QNs cannot be treated in a hydrodynamical framework; they behave rather like discrete bodies in the background of the radiation fluid.

They thus experience the radiation pressure, quite substantial because of their large surface area as well as the gravitational potential due to the other QNs.

In such circumstances, it would be rather tempting to use the powerful tool of the Virial theorem [28,29]. Even a naive application of the Virial theorem implies that the internal energy $U$ of all the QNs contained within the event horizon ($\sim 2t$) at any instant of time is substantially less than $|\Omega|/2$, $\Omega$ being the total gravitational energy, so that these QNs would tend to undergo gravitational collapse. Such, of course, would not be the case for any other massive particles like baryons. Their kinetic energy would continue to be very large till very low temperatures. More seriously, the Virial theorem can be applied only to systems whose motion is sustained. In the presence of dissipation killing all kinetic energy, the Virial theorem becomes inapplicable. QNs have a very unusual property in that they tend to absorb baryonic matter without limit, becoming more and more bound in the process. They would therefore clump together whenever two such QNs collide, damping their motion even further.

We therefore have to estimate to what extent the radiation pressure can prevent the QNs from gravitating towards one another. It should be mentioned at this juncture that for the system of discrete QNs suspended in the radiation fluid, a detailed numerical simulation would be essential before any definite conclusion about their temporal evolution can be arrived at. This is a quite involved problem, especially since the number of QNs within the event horizon, as also their mutual separation, keeps increasing with time. In the remaining part of this section we will examine whether such an effort would indeed be justified.

Let us now consider the possibility of two nuggets coalescing together under gravity, overcoming the radiation pressure. The mean separation of these nuggets and hence their gravitational interactions are determined by the temperature of the universe. If the universe is assumed to be closed with QNs, the total baryon number contained in them within the horizon at the QCD transition temperature ($\sim$100 MeV) would be $10^{51}$. For QNs of baryon number $b_{\rm N}$ each, the number of QNs within the horizon at that time would be just $\left(10^{51}/b_{\rm N}\right)$. Then at any later time, the number of QNs within the horizon ($N_{\rm N}$) and their density ($n_{\rm N}$) as a function of temperature would be given by

$$N_{\rm N}(T) = \frac{10^{51}}{b_{\rm N}}\left(\frac{100\ \text{MeV}}{T}\right)^3 \tag{16}$$

$$n_{\rm N}(T) = \frac{N_{\rm N}}{V_{\rm H}} = \frac{3N_{\rm N}}{4\pi(2t)^3}. \tag{17}$$

In the above two equations the time $t$ and the temperature $T$ are related in the radiation dominated era by the relation

$$t = 0.3 g_*^{-1/2}\frac{m_{\rm pl}}{T^2} \tag{18}$$

with $g_*$ being $\sim$17.25 after the QCD transition.

From the above, it is obvious that the density of QNs decreases as $t^{-3/2}$ and as a result their mutual separation increases as $t^{1/2}$. Therefore, the force of their mutual gravitational pull will decrease as $t^{-1}$. On the other hand, the force due to the radiation pressure (photons and neutrinos) resisting motion under gravity would be proportional to the radiation energy density, which decreases as $T^4$ or $t^{-2}$. One can easily figure out from the above discussion that at some time the gravitational pull would win over the radiation pressure, causing the QNs to coalesce under their mutual gravitational pull.

The gravitational attraction between two nuggets is given by

$$F_{\rm grav} = \frac{GM_{\rm N}^2}{\bar{r}_{\rm NN}(T)^2} \tag{19}$$

where $M_{\rm N}$ is the mass of a nugget and $\bar{r}_{\rm NN}(T)$ is the mean separation between the two nuggets, estimated from the density of nuggets at temperature $T$.

The force due to the radiation pressure on the nuggets may be roughly estimated as follows. We consider two objects (of the size of a typical SQN) approaching each other due to gravitational interaction, overcoming the resistence due to the radiation pressure. The usual isotropic radiation pressure is $\frac{1}{3}\rho c^2$, where $\rho$ is the total energy density, including all relativistic species. The nuggets will have to overcome an additional pressure resisting their mutual motion, which is given by $\frac{1}{3}\rho c^2(\gamma - 1)$; the additional pressure arises from a compression of the radiation fluid due to the motion of the SQN. The moving SQN would become a prolate ellipsoid (with its minor axis in the direction of motion due to Lorentz contraction), whose surface area is given by $2\pi R_{\rm N}^2(1+\sin^{-1}\varepsilon/\gamma\varepsilon)$. The eccentricity $\varepsilon$ is related to the Lorentz factor $\gamma$ as $\varepsilon = \sqrt{\gamma^2 - 1}/\gamma$. For small values of $\varepsilon$ (small $\gamma$), $\sin^{-1}\varepsilon \sim \varepsilon$, so that the surface area becomes $2\pi R_{\rm N}^2 \gamma + 1/\gamma$. Thus the total radiation force resisting the motion of SQNs is

$$F_{\rm rad} = \frac{1}{3}\rho_{\rm rad} c v_{\rm fall}(\pi R_{\rm N}^2)\beta\gamma \tag{20}$$

where $\rho_{\rm rad}$ is the total energy density at temperature $T$, $v_{\rm fall}$ or $\beta_{\rm c}$ is the velocity of SQNs determined by mutual gravitational field and $\gamma$ is $1/\sqrt{1-\beta^2}$. The quantities $F_{\rm rad}$, $\beta$ and $\gamma$ all depend on the temperature of the epoch under consideration. (It is worth mentioning at this point that the $t$ dependence of $F_{\rm rad}$ is actually $t^{-5/2}$, sharper than the $t^{-2}$ estimated above, because of the $v_{\rm fall}$, which goes as $t^{-1/4}$.) The ratio of these two forces is plotted against temperature in figure 2 for two SQNs with initial baryon number $10^{42}$ each. It is obvious from the figure that ratio $F_{\rm grav}/F_{\rm rad}$ is very small initially. As a result, the nuggets will remain separated due to the radiation pressure. For temperatures lower than a critical value $T_{\rm cl}$, the gravitational force starts dominating, facilitating the coalescence of the SQNs under mutual gravity.

Let us now estimate the mass of the clumped QNs, assuming that all of them within the horizon at the critical temperature will coalesce together. This is in fact a conservative estimate, since the QNs, although starting to move towards one another at $T_{\rm cl}$, will take a finite time to actually coalesce, during which interval more QNs will arrive within the horizon.

In table 1, we show the values of $T_{\rm cl}$ for QNs of different initial baryon numbers along with the final masses of the clumped QNs under the conservative assumption mentioned above.

It is obvious that there can be no further clumping of these already clumped QNs. The density of such objects would be too small within the horizon for further clumping. Thus these objects would survive till today and perhaps manifest themselves as MACHOs. It is to be reiterated that the masses of the clumped QNs given in table 1 are the lower limits and the final masses of these MACHO candidates will be larger. (The case for $b_{\rm N} = 10^{46}$ is not of much interest, especially since such high values of $b_{\rm N}$ are unlikely for the reasonable nucleation rates [25,26]. We therefore restrict ourselves to the other cases in table 1 in

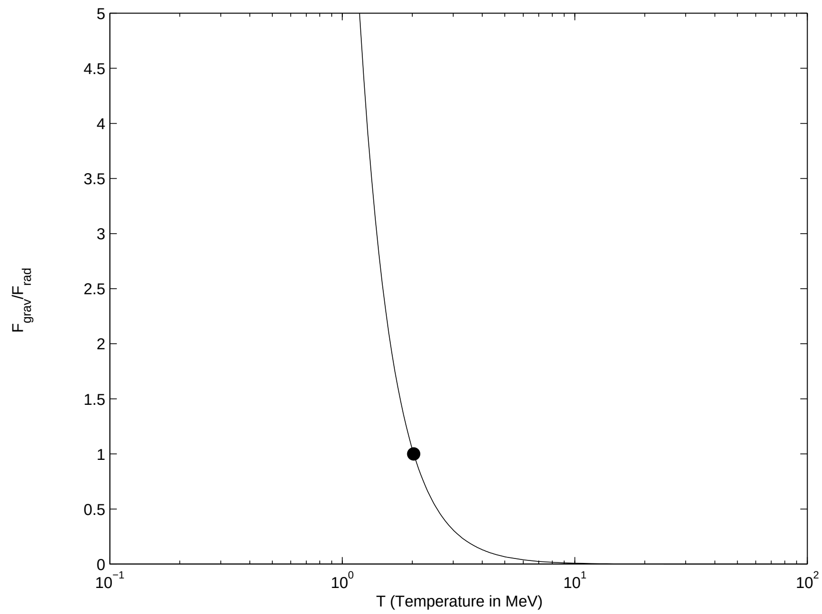


**Figure 2.** Variation of the ratio $F_{\mathrm{grav}}/F_{\mathrm{rad}}$ with temperature. The dot represents the point where the ratio assumes the value 1.

**Table 1.** Critical temperatures $(T_{\mathrm{cl}})$ of QNs of different initial sizes $b_{\mathrm{N}}$, the total number $N_{\mathrm{N}}$ of QNs that coalesce together and their total final mass in solar mass units.

| $b_{\mathrm{N}}$ | $T_{\mathrm{cl}}$ (MeV) | $N_{\mathrm{N}}$ | $M/M_{\odot}$ |
|---|---|---|---|
| $10^{42}$ | 2.03 | $1.20\times10^{14}$ | 0.12 |
| $10^{44}$ | 5.73 | $1.00\times10^{11}$ | 0.01 |
| $10^{46}$ | 15.72 | $2.57\times10^{7}$ | 0.0003 |

what follows.) A more detailed estimate of the masses will require a detailed simulation, but very preliminary estimates indicate that they could be 2–3 times bigger than the values quoted in table 1.

The total number of such clumped QNs within the horizon today would be $\sim 10^{23-24}$. We can also mention at this juncture that if the MACHOs are indeed made up of quark matter, then they cannot grow to arbitrarily large sizes. Within the (phenomenological) bag model picture [30] of QCD confinement, where a constant vacuum energy density (called the bag constant) in a cavity containing the quarks serves to keep them confined within the cavity, some of us have earlier investigated [31] the upper limit on the mass of astrophysical compact quark matter objects. It was found that for a canonical bag constant B of (145 MeV)$^4$, this limit comes out to be $1.4M_{\odot}$. The collapsed QNs are safely below this limit. (It should be remarked here that although the value of B in the original MIT bag model is taken to be $B^{1/4} = 145$ MeV from the low mass hadronic spectrum, there exist other variants of the bag model [32], where higher values of B are required. Even for $B^{1/4} = 245$ MeV, this limit comes down to $0.54 M_\odot$ [31], which would still admit such QNs.)

## 4. Conclusion

The evolution of quark nuggets (QNs) formed during the QCD phase transition in the early universe has been discussed. The baryon number distribution of QNs have been calculated, using a nucleation rate proposed by Cottingham *et al* [27], in §2. It has been seen that there are large number of quark nuggets having baryon number more than the stability limit. This shows that they can contribute to the cold dark matter budget. In §3 the clumping probability of these QNs have been calculated. The nuggets feel two forces, one is the gravitational force between the nuggets and other is the repulsion due to radiation pressure. It has been seen that the gravitational attraction starts dominating at temperatures ranging from 15.72 MeV to 2.03 MeV, depending on the initial baryon number content of the nugget. As a result the nuggets clump forming bigger objects in the mass range of $0.0003 M_\odot$ to $0.12 M_\odot$. It has been proposed that these objects could be the candidates for MACHOs observed towards LMC by two experimental groups. However, the masses of clumped objects obtained in this article are somewhat conservative as discussed in §3. To have an exact estimate of these masses one has to do a simulation. Work in that direction is in progress.

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