PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 60, No. 5
May 2003
pp. 901–907

Instability of quark matter core in a compact newborn neutron star with moderately strong magnetic field

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Abstract. It is explicitly shown that if phase transition occurs at the core of a newborn neutron star with moderately strong magnetic field strength, which populates only the electron's Landau levels, then in the β-equilibrium condition, the quark core is energetically much more unstable than the neutron matter of identical physical condition.

Keywords. Landau diamagnetism; quark matter; quark star.

PACS Nos **26.60.+c; 97.60.Jd; 76.60.Jx**

1. Introduction

One of the oldest subject – the effect of strong magnetic field on dense matter – has got a new life after the discovery of a few strongly magnetized neutron stars which are called magnetars [1]. These exotic stellar objects are also assumed to be the possible sources of soft gamma repeaters (SGR) and anomalous X-ray pulsars (AXP) [2–4]. From observations, the surface magnetic field of such objects are found to be $\sim 10^{15}$ G [5]. The field at the core region is expected to be a few orders of magnitude larger than the surface value. But there is an upper limit for magnetic field strength, beyond which the core region of the star becomes unstable [6]. This value is $\approx 10^{18}$ G. The magnetars are also thought to be the strongly magnetized young neutron stars. The studies on the effect of strong magnetic field on various physical processes, relevant for these exotic objects have been reported during the past few years. These studies are mainly related to the equation of states of dense matter [7–9], elementary processes – specially weak and electromagnetic decays and reactions [10], quark–hadron phase transition [11–13], and transport coefficients of dense matter [14]. A few years ago, we have shown explicitly that a first-order quark–hadron phase transition is absolutely forbidden in the presence of a strong magnetic field ($\geq$ a few times 10^{15} G [11,12]). However, in a recent paper, Mathews *et al* [15] have shown that such strong conclusion is not correct if one considers anomalous magnetic moment of quarks in the quark matter sector. In that case a first-order quark–hadron phase transition is possible even if the magnetic field is extremely strong. In the same publication we have also shown with certain approximation, that even if a phase transition occurs at the core region of a compact neutron star in the presence of strong magnetic fields, in the β-equilibrium condition the matter becomes energetically unstable compared to neutron matter of identical physical condition [12]. Hence we concluded that quark matter core is impossible in a strongly magnetized young neutron star. In this brief report we shall show explicitly without any approximation, that the conclusion is still valid if the magnetic field strength is moderately strong. If it is correct, then we can very strongly conclude that the quark matter is absolutely impossible at the core of a neutron star with magnetic field strength slightly greater than 4.4×10^{13} G, which is the quantum mechanical limit for electrons to populate Landau levels. We believe that such a conclusion is extremely important both from the theoretical as well as observational point of view.

In this report we have considered a young compact neutron star with moderately high magnetic field (we consider a field strength of 10^{14} G at the core region for our calculation). The density of the core region is assumed to be such that a quark–hadron phase transition (which is assumed to be first order even without the inclusion of anomalous magnetic dipole moment of quarks) can occur. Now in this case the assumed magnetic field strength at the core is about a factor of two larger than the above critical value to populate Landau levels for electrons. Further, the magnitude is not too high to affect quantum mechanically other charged components present in the system (e.g., u, d and s quarks) or populate only the zeroth Landau level for electron. We have noticed that under such a circumstance, an exact estimation of the rates of weak processes is possible. Therefore in our opinion, the uncertainty present in our previous publication is removed in the range of magnetic field strength $B_c^{(e)} < B < B_c^{(u,d)}$ [13].

2. Rate calculation

Now it is known that quark–hadron phase transition is a strong interaction phenomenon, and therefore takes place in the strong interaction time-scale. On the other hand, immediately after phase transition, the nascent quark matter is not necessarily in β-equilibrium configuration. This is achieved through weak processes in the weak interaction time-scale, which is several orders of magnitude larger than the strong interaction time-scale. The goal of the present report is to show that if quark–hadron phase transition occurs at the core of a moderately strong magnetic field, so that only electrons at the background are affected quantum mechanically, then in the β-equilibrium condition, quark matter phase becomes energetically much more unstable than the corresponding neutron matter state.

To investigate the instability of quark matter core, we solve numerically the set of kinetic equations for the nascent quark phase, which ultimately lead to chemical equilibrium configuration. We have considered the most simplified physical picture in the quark matter sector – quarks are non-interacting, at the very beginning, quark–hadron phase transition occurred from non-strange hadronic matter and neutrinos are non-degenerate – they leave the system immediately after their creation. The relevant weak process are: $d \to u + e^- + \bar{\nu}_e\ (1)$, $u + e^- \to d + \nu_e\ (2)$, $s \to u + e^- + \bar{\nu}_e\ (3)$, $u + e^- \to s + \nu_e\ (4)$, $u + d \leftrightarrow u + s\ (5)$. The approach to chemical equilibrium is governed by the following sets of kinetic equations

$$\frac{\mathrm{d}Y_u}{\mathrm{d}t} = \frac{1}{n_\mathrm{B}}[\Gamma_1 - \Gamma_2 + \Gamma_3 - \Gamma_4], \tag{1}$$

$$\frac{\mathrm{d}Y_d}{\mathrm{d}t} = \frac{1}{n_\mathrm{B}}[-\Gamma_1 + \Gamma_2 - \Gamma_5^{(\mathrm{d})} + \Gamma_5^{(\mathrm{r})}], \tag{2}$$

where $n_{\rm B}$ is the baryon number density, $Y_i = n_i/n_{\rm B}$ is the fractional abundance of the species i and Γ_js are the rates of the processes $j = 1,2,3,4,5$. The indices d and r are respectively for the direct and reverse processes for $j = 5$. The baryon number conservation and charge neutrality conditions give $Y_s = 3 - Y_u - Y_d$ and $Y_e = Y_u - 1$ respectively. To solve the kinetic equations numerically for the investigation of chemical evolution, we use these constraints as subsidiary conditions to obtain Y_e and Y_s, and further we use the numerical values for the rates Γ_1 to $\Gamma_5^{\rm (d)(r)}$ appear on the right-hand sides. Now for a neutron star of mass $\approx 1.4 M_\odot$, the baryon number density at the centre is 3–4 times normal nuclear density, temperature $\sim 10^9$ K and proton fraction is about 4%. Then the initial conditions are $Y_u(t=0) = 1.04$, $Y_d(t=0) = 1.96$. As a consequence of baryon number conservation and charge neutrality, we have $Y_s(t=0) = 0$ and $Y_e(t=0) = 0.04$.

Since the magnetic field strength is assumed to be $\approx 10^{14}$ G, the rates for the first four processes will be affected through electron spinor solution and energy eigenvalue. Further, the rates for the processes (3) and (4) can very easily be obtained from the rates of processes (1) and (2) respectively just by replacing d-quark parameters with the corresponding s-quark ones and $\cos\theta_{\rm c}$ by $\sin\theta_{\rm c}$, where $\theta_{\rm c}$ is the well-known Cabibbo angle.

Now from the definition, the transition matrix element for the weak decay processes is given by

$$T_{fi} = \frac{4iG}{\sqrt{2}} \cos\theta_{\rm c} \int {\rm d}^4x \left[\bar{\psi}_u(x)\gamma_\mu \frac{1-\gamma_5}{2} \psi_d(x) \right] \left[\psi_e(x)\gamma^\mu \frac{1-\gamma^5}{2} \psi_\nu(x) \right]. \quad (3)$$

Then the decay rate is given by ${\rm d}\Gamma = \lim_{\tau\to\infty} \mid T_{fi} \mid^2 {\rm d}\rho_{\rm f}/\tau$ where τ is the characteristic collision time and $\rho_{\rm f}$ is the final density of states, given by ${\rm d}\rho_{\rm f} = \prod_i {\rm d}^3 p_i/(2\varepsilon_i(2\pi)^3)$, where the product is over all final states i and ε_i is the single particle energy of the ith component. We have designated d, ν_e or $\bar{\nu}_e$, u and e by $i = 1,2,3$ and 4 respectively. In this moderately strong magnetic field strength, we have used conventional spinor solutions for the quarks and charge neutral neutrinos or anti-neutrinos, whereas for electron we have used [11,13,16]

$$\Psi^{(\uparrow)}(x) = \frac{1}{\sqrt{L_y L_z}} \frac{\exp(-i\varepsilon_\nu^{(i)} t + i p_y y + i p_z z)}{[2\varepsilon_\nu^{(i)}(\varepsilon_\nu^{(i)} + m_i)]^{1/2}} \begin{pmatrix} (\varepsilon_\nu^{(i)} + m_i) I_{\nu;p_y}(x) \\ 0 \\ p_z I_{\nu;p_y}(x) \\ -i(2\nu q_i B)^{1/2} I_{\nu-1;p_y}(x) \end{pmatrix} \quad (4)$$

and

$$\Psi^{(\downarrow)}(x) = \frac{1}{\sqrt{L_y L_z}} \frac{\exp(-i\varepsilon_\nu^{(i)} t + i p_y y + i p_z z)}{[2\varepsilon_\nu^{(i)}(\varepsilon_\nu^{(i)} + m_i)]^{1/2}} \begin{pmatrix} 0 \\ (\varepsilon_\nu^{(i)} + m_i) I_{\nu-1;p_y}(x) \\ i(2\nu q_i B)^{1/2} I_{\nu;p_y}(x) \\ -p_z I_{\nu-1;p_y}(x) \end{pmatrix}, \quad (5)$$

where the symbols $\uparrow$ and $\downarrow$ are used for up- and down-spin states respectively, $i = e$ and

$$I_{\nu;p_y}(x) = \left(\frac{q_i B}{\pi}\right)^{1/4} \frac{1}{\sqrt{\nu!}\, 2^{\nu/2}} \exp\left[-\frac{1}{2} q_i B \left(x - \frac{p_y}{q_i B} \right)^2 \right]$$

$$\times H_\nu\left[\sqrt{q_iB}\left(x-\frac{p_y}{q_iB}\right)\right]. \tag{6}$$

H_ν is the well-known Hermite polynomial of order ν. Then we have

$$T_{fi}=-\frac{iG\,2\pi\delta(\varepsilon_1-\varepsilon_2-\varepsilon_3-\varepsilon_4)}{\sqrt{2}V^{3/2}}\Pi', \tag{7}$$

where

$$\Pi'=[\bar{u}(p_3)\gamma_\mu(1-\gamma_5)u(p_1)][\bar{f}_e(p_4)\gamma^\mu(1-\gamma^5)v(p_2)]\cos\theta_c, \tag{8}$$

$$\bar{f}_e(p_4)=\int \mathrm{d}^3x\exp[-i(\vec{p}_1-\vec{p}_2-\vec{p}_3)\cdot\vec{r}]\bar{\psi}_e(x). \tag{9}$$

Hence we have

$$T_{fi}=-\frac{iG}{\sqrt{2}}\cos\theta_c(2\pi)^3\frac{\delta(\varepsilon_1-\varepsilon_2-\varepsilon_3-\varepsilon_4)}{V^{3/2}}$$
$$\times\delta(p_{1y}-p_{2y}-p_{3y}-p_{4y})\delta(p_{1z}-p_{2z}-p_{3z}-p_{4z})\Pi, \tag{10}$$

where

$$\Pi=[\bar{u}(p_3)\gamma_\mu(1-\gamma_5)u(p_1)][\bar{u}(p_4)\gamma^\mu(1-\gamma^5)v(p_2)] \tag{11}$$

and

$$\bar{u}(p_4)^{(\uparrow)(\downarrow)}=\int\frac{\mathrm{d}x}{\sqrt{L_yL_z}}\exp[i(p_{1x}-p_{2x}-p_{3x})\cdot x](\uparrow)(\downarrow), \tag{12}$$

where the symbols $(\uparrow)$ and $(\downarrow)$ indicate positive energy up- and down-spin states for electron. Now to obtain $\bar{u}(p_4)$ we have evaluated

$$\int_\infty^\infty \mathrm{d}x\exp(ik_xx)I_{\nu;p_y}(x). \tag{13}$$

With the substitution $X=\sqrt{q_eB}x$ and $C=p_{4y}/\sqrt{q_eB}$, the above Fourier transform reduces to

$$\int_\infty^\infty\frac{\mathrm{d}X}{\sqrt{q_eB}}\exp(ik_xx)\left(\frac{q_eB}{\pi}\right)^{1/4}\frac{1}{\sqrt{\nu!}\,2^{\nu/2}}\exp\left[-\frac{1}{2}(X-C)^2\right]H_\nu(X-C)$$
$$=\frac{1}{(q_eB)^{1/4}\sqrt{\nu!}\,2^{(\nu-1)/2}}i^\nu H_\nu(k_x)\exp\left(\frac{iCk_x}{\sqrt{q_eB}}-\frac{k_x^2}{2q_eB}\right), \tag{14}$$

where $k_x=p_{1x}-p_{2x}-p_{3x}$. Then we have after some algebraic manipulation

$$u_e^\uparrow=\sqrt{\frac{\varepsilon_4+m_e}{2\varepsilon_4}}\begin{pmatrix}C_1H_\nu(k_x)\\0\\C_3H_\nu(k_x)\\C_4H_{\nu-1}(k_x)\end{pmatrix}, \tag{15}$$

where

$$C_1 = \frac{1}{(q_eB)^{1/4}\sqrt{\nu!}\,2^{(\nu-1)/2}} i^{\nu} \exp\left(\frac{iCk_x}{\sqrt{q_eB}} - \frac{k_x^2}{2q_eB}\right), \tag{16}$$

$$C_3 = \frac{p_{4z}}{\varepsilon_4 + m_e} C_1 \tag{17}$$

and

$$C_4 = -\frac{\sqrt{q_eB}2\nu}{(\varepsilon_4 + m_e)(q_eB)^{1/4}\sqrt{\nu!}\,2^{(\nu-1)/2}} i^{\nu} \exp\left(\frac{iCk_x}{\sqrt{q_eB}} - \frac{k_x^2}{2q_eB}\right). \tag{18}$$

Similarly the down-spin state is given by

$$u_e^{\downarrow} = \sqrt{\frac{\varepsilon_4 + m_e}{2\varepsilon_4}} \begin{pmatrix} 0 \\ C_2' H_{\nu-1}(k_x) \\ C_3' H_{\nu}(k_x) \\ C_4' H_{\nu-1}(k_x) \end{pmatrix}, \tag{19}$$

where

$$C_2' = \frac{1}{(q_eB)^{1/4}\sqrt{(\nu-1)!}2^{(\nu-2)/2}} i^{\nu-1} \exp\left(\frac{iCk_x}{\sqrt{q_eB}} - \frac{k_x^2}{2q_eB}\right), \tag{20}$$

$$C_3' = -\frac{\sqrt{q_eB}}{(\varepsilon_4 + m_e)(q_eB)^{1/4}\sqrt{(\nu-1)!}2^{(\nu-2)/2}} i^{\nu-1} \exp\left(\frac{iCk_x}{\sqrt{q_eB}} - \frac{k_x^2}{2q_eB}\right) \tag{21}$$

and

$$C_4' = \frac{p_{4z}}{\varepsilon_4 + m_e} C_2'. \tag{22}$$

Now by some rearrangement, integration over p_{1x} can very easily be performed and is given by $\sqrt{\pi q_eB}$ whereas, the integrations over p_{1y}, p_{1z} and $\mathrm{d}^3 p_2$ can be evaluated trivially with the help of delta functions. Then finally, we have after substituting $(\mu_u - \varepsilon_3)/T = x_u$ and $(\mu_e - \varepsilon_4)/T = x_e$, the rate for the process (1)

$$\Gamma_1 = \frac{3G^2(q_eB)}{2\pi^6} T^4 \cos^2\theta_{\mathrm{c}} \mu_u \mu_e p_{\mathrm{Fu}} \sum_{\nu_e=0}^{[\nu_e^{\max}]} \left(\frac{1}{p_{\mathrm{Fe}}}\right) \times \int_{-\infty}^{\infty} \left(x_u + x_e - \frac{\mu_u + \mu_e - \mu_d}{T}\right)^2 f(x_u) f(x_e) \mathrm{d}x_u \mathrm{d}x_e. \tag{23}$$

Similarly, the rate for the process (2) is given by

$$\Gamma_2 = \frac{3G^2(q_eB)}{2\pi^6} T^4 \cos^2\theta_{\mathrm{c}} \mu_u \mu_e p_{\mathrm{Fu}} \sum_{\nu_e=0}^{[\nu_e^{\max}]} \left(\frac{1}{p_{\mathrm{Fe}}}\right) \times \int_{-\infty}^{\infty} \left(x_u + x_e + \frac{\mu_u + \mu_e - \mu_d}{T}\right)^2 f(x_u) f(x_e) \mathrm{d}x_u \mathrm{d}x_e. \tag{24}$$

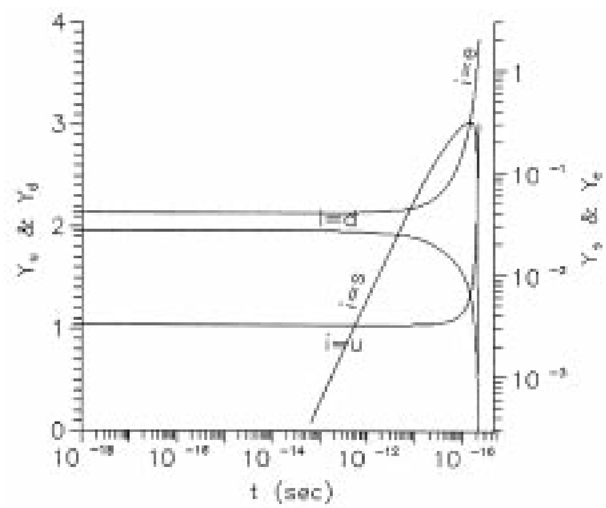

Figure 1. Fractional abundances for various species when only electrons are affected by quantizing magnetic field ($B = 10^{14}$ G).

In the above expressions, $p_{\rm Fe} = (\mu_e^2 - m_e^2 - 2\nu_e q_e B)^{1/2}$ is the electron Fermi momentum. Then as mentioned before, the rates for processes (3) and (4) are obtained from Γ_1 and Γ_2 respectively whereas, the rates for both the direct and reverse processes as shown by reaction (5) are given by the zero-field values [13].

Now knowing these rates we have solved the kinetic equations for magnetic field strength $B = 10^{14}$ G. The time evolution of the fractional abundances for various components is shown in figure 1.

This figure shows that in β-equilibrium condition there are mainly u-quarks and electrons in the quark matter system. Then from the dimensionality of the extended phase space, one can easily visualize that the system is energetically much more unstable than neutron matter of identical physical condition. We have also checked the result from explicit free energy calculation for various core densities. For a range of baryon number densities of astrophysical interest, the final quark matter system in β-equilibrium contains only u quarks and electrons, with $2n_u = 3n_e$ and $n_u = n_{\rm B}$. The result is again independent of initial proton content and also does not change significantly if there are Λ-hyperons in the hadronic system at the beginning before phase transition.

3. Conclusion

Hence we conclude that stable quark matter phase cannot exist at the core of a newborn neutron star if the magnetic field strength exceeds the critical value $\sim 4.4 \times 10^{13}$ G. In fact, we can now very strongly demand that at the core of a young neutron star even with moderately strong magnetic field, quark matter cannot exist. Hence we expect that it is also possible to extrapolate this conclusion to the quark matter system when all the charged components are affected, but the field strength is not high enough to fill only the zeroth Landau levels. Because of mathematical difficulty, of course, we are unable to show it explicitly. Finally, we do believe that if a system is energetically unstable, the nature will not allow its creation at the very beginning. Therefore, the possibility of quark–hadron phase transition at the core of a strongly magnetized young neutron star is an open question.

Acknowledgement

The author is thankful to the Department of Science and Technology, Govt. of India, for partial financial support to this work, Sanction number: SP/S2/K3/97(PRU).

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